CDM in Supersymmetric Models

Relic Density in the CMSSM
Phenomenological Constraints
  Higgs masses, $b \rightarrow s \gamma$, g-2
Relaxing the CMSSM
Prospects for Detection

with: Ellis, Falk, Ferstl, Ganis, Santoso, Srednicki
SUSY Dark Matter

MSSM and R-Parity

Stable DM candidate

1) Neutralinos
\[ \tilde{\chi}_i = \alpha_i \tilde{\tau} + \beta_i \tilde{\nu} + \gamma_i \tilde{H}_1 + \delta_i \tilde{H}_2 \]

2) Sneutrinos
Excluded (unless add L-violating terms)

3) Other:
Axinos, Gravitinos, etc
mass matrix

\[
(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)
\begin{pmatrix}
  M_1 & 0 & -\frac{g_1 v_1}{\sqrt{2}} & \frac{g_1 v_2}{\sqrt{2}} \\
  0 & M_2 & \frac{g_2 v_1}{\sqrt{2}} & -\frac{g_2 v_2}{\sqrt{2}} \\
  -\frac{g_1 v_1}{\sqrt{2}} & \frac{g_2 v_1}{\sqrt{2}} & 0 & -\mu \\
  \frac{g_1 v_2}{\sqrt{2}} & -\frac{g_2 v_2}{\sqrt{2}} & -\mu & 0
\end{pmatrix}
\begin{pmatrix}
  \tilde{B} \\
  \tilde{W}^3 \\
  \tilde{H}_1^0 \\
  \tilde{H}_2^0
\end{pmatrix}
\]

Depends on $M_{1/2}$, $\mu$, and $\tan \beta$
Assume $M_1 = M_2 = M_3 = M_{1/2}$ @ GUT Scale

Also Relic Density Depends on $m_0$, $m_A$
Parameters

Higgs mixing mass: $\mu$
Ratio of Higgs vevs: $\tan \beta$
Gaugino masses: $M_i$
Soft scalar masses: $m_o$

Bi and Trilinear Terms: $B$ and $A_i$
Phases: $\theta_\mu$, $\theta_A$
Boundary conditions

- Input parameters: $\mu$, $m_1$, $m_2$, $B$. predict $M_Z$, $\tan \beta$, $m_A$

CMSSM conditions

- Instead CMSSM:
  Input parameters: $M_Z$, $m_1$, $m_2$, $\tan \beta$  
    $(m_1 = m_2 = m_0)$
  predict $\mu$, $B$, $m_A$
CMSSM Spectra

Unification to rich spectrum + EWSB

Falk
The Relic Density

At high temperatures $T >> m_\chi$:
- $\chi$'s in equilibrium $\Gamma > H$ $n_\chi \sim n_\gamma$
- $\Gamma \sim n_\sigma v \sim T^3 \sigma v$; $HM_p \sim \sqrt{\rho} \sim T^2$
As $T < m_\chi$; annihilations drop $n_\chi$

$$n_\chi \sim e^{-m_\chi/T} n_\gamma$$

Until freeze-out, $\Gamma < H$ $n_\chi/n_\gamma \sim$ constant
How Much Dark Matter

Upper limit to $\Omega h^2$

age of the Universe

(t > 12 Gyr)

allows one to set an upper limit

$\Omega h^2 < 0.3$

Lower limit to $\Omega h^2$

Structure formation requires at least $\Omega h^2 > 0.1$ in dark matter

But this is not a strict constraint on SUSY dark matter
Bulk Region of low $m_{1/2}$, $m_0$

Upper limit to $m_{1/2}$
~ 450 GeV moved up to ~ 1400 GeV

$m_\chi \approx 0.4 \, m_{1/2}$

Ellis, Falk, Olive, Srednicki
Co-annihilation

Often, the LSP is nearly degenerate with another SUSY sparticle.

\( \chi, \chi', \chi^\pm \) nearly degenerate when \( M_2 \gg \mu \)

Enhanced annihilation  \( \rightarrow \) lower \( \Omega h^2 \)

Also, in CMSSM, \( \chi - \tilde{\tau} \) or \( \chi - \tilde{t} \)

greatly affects upper limits to LSP.

Greist + Seckel
Mizuta + Yamaguchi

EFOS
Boehm, Djouadi, Drees
Ellis, Olive, Santoso
Co-annihilation
Region of high $m_{1/2}$, low $m_0$

Ellis, Falk, Olive, Srednicki
\( \chi - \tilde{t} \) co-annihilation

Region of high \( A_0 \)

Ellis, Olive, Santoso
The CMSSM at large tan $\beta$

- Increased sensitivity to bottom quark mass - radiative corrections
- Rapid annihilation through s-channel $A$ and $H$ exchange due to:
  - $m_{A,H}$ decreases as tan $\beta$ increases
  - and $2m_\chi \sim m_{A,H}$ for a wide range in $m_{1/2}$
  - $b$ quark coupling enhanced by tan $\beta$

Drees and Nojiri
Baer et al.
Ellis, Falk, Ganis, Olive, Srednicki
Funnel region at high $\tan \beta$

\[ m_0 \text{ (GeV)} \quad m_{1/2} \text{ (GeV)} \]

$\tan \beta = 50, \mu > 0$

$m_{\chi^0} = 104 \text{ GeV}$

$m_h = 113 \text{ GeV}$
`Focus Point’ Region

When $m_0 \gg m_{1/2}$, the LSP becomes more Higgsino-like and rapid annihilation (through Z exchange) drives the density down.

Feng, Matchev, Wilczek
Constraints

- **Chargino mass limit**
  \[ M_{\chi^\pm} \geq 104 \text{ GeV} \]
  Constrains \((M_2 \text{ and } \mu)/ m_{1/2}\)

- **Higgs mass limit**
  \[ M_H \geq 114 \text{ GeV} \]
  Constrains \((m_A, M_2, A)/ m_{1/2}\)
  particularly at low \(\tan \beta\)

- **\(b\) to \(s\gamma\)**
  Constrains \((m_A)/ m_{1/2}\) at high \(\tan \beta\) and \(\mu < 0\)

- **Also sfermion mass limits from LEP and CDF**
  \[ m_f \geq 99 \text{ GeV} \text{ (roughly)} \]
  \(\chi\) is the LSP
The plane at high $\tan \beta$ and the appearance of funnels.
Another view
Boundary conditions

- Input parameters: $\mu, m_1, m_2, B$. predict $M_Z, \tan \beta, m_A$

CMSSM conditions

- Instead CMSSM:
  Input parameters: $M_Z, m_1, m_2, \tan \beta$  \((m_1 = m_2 = m_0)\)
  predict $\mu, B, m_A$

Relaxing CMSSM conditions

- Or instead NUHM:
  Input parameters: $M_Z, \mu, m_A, \tan \beta$  predict $m_1, m_2, B$
  Ellis, KO, Santoso
The $m_A - \mu$ plane

+ CMSSM value
The $M_2 - \mu$ plane

$+ \text{CMSSM value}$
The $m_0 - m_{1/2}$ plane

+ CMSSM value
Have future accelerators been saved by g-2?

- Original results from Recent BNL E821
  \[ \delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 43 \pm 16 \times 10^{-10} \]

- Strong correlations between \( a_\mu \) and \( \mu \)
  \( \mu < 0 \) excluded.

- Theoretical corrections
  \[ \delta a_\mu = 26 \pm 16 \times 10^{-10} \]
  - \( \mu > 0 \) strongly favored
  - Small \( m_0 \) \( m_{1/2} \) excluded

Chattopadhyay, Nath
Ellis, Nanopoulos, KO

Ellis, KO, Santoso
Effects of $\mu$-2

Note: $\mu < 0$ excluded

Large $m_{1/2}$, $m_0$ excluded

Labels show benchmark points

From: Battaglia et al
Effects of weaker g-2

Note: $\mu < 0$ still
Strongly constrained
Direct Detection

Elastic scattering cross sections for $\chi p$

Use only parameters which satisfy accelerator bounds and relic density

Results: low cross sections

- $< 10^{-3}$ pb spin
- $< 10^{-7}$ pb scalar

MSSM allows higher cross sections at higher $m_\chi$
\[ + \alpha_{3i} \bar{\chi} \chi \bar{q}_i q_i + \alpha_{4i} \bar{\chi} \gamma^5 \chi \bar{q}_i \gamma^5 q_i \\
+ \alpha_{5i} \bar{\chi} \chi \bar{q}_i \gamma^5 q_i + \alpha_{6i} \bar{\chi} \gamma^5 \chi \bar{q}_i q_i \]

The terms involving \(-\alpha_{1i}, -\alpha_{4i}, -\alpha_{5i}, \text{and} -\alpha_{6i}\) lead to velocity dependent elastic cross sections.

Remaining terms are:

the spin dependent coefficient \(\alpha_{2i}\)

and scalar coefficient \(\alpha_{3i}\)
Contributions to $\alpha_2$: Spin-Dependent cross-section

- Through light squark exchange
  - Dominant for binos
- Through Z exchange
  - Requires a strong Higgsino component

Contributions to $\alpha_3$: Spin-Independent cross-section

- Through light squark exchange
  - Dominant for binos
- Through Higgs exchange
  - Requires some Higgsino component
The Spin-independent Elastic cross-section

Note cancellation
When $\mu < 0$
Upper and lower limit to $\sigma$
Detectability of Benchmark points

Feng et al
CDMS: Abrams et al
astro-ph/0203500

EDELWEISS: Benoit et al
astro-ph/0206271
Summary

- **Accelerator Constraints:**
  Push towards larger values of $m_\chi$. (Trend may be halted by future g-2 results.)

  **CMSSM**

- **Spin-Dependent cross sections:**
  \[ \sigma_2 < 10^{-5} \text{ pb} \quad (>3 \times 10^{-7} \text{ pb} \text{ if g-2}) \]
  - But this is far below current sensitivites.

- **Scalar cross sections:**
  \[ \sigma_3 < 10^{-7} \text{ pb} \quad (>10^{-9} \text{ pb} \text{ if g-2}) \]
  - Perhaps within experimental reach.

  * Old g-2
MSSM

- While higher cross-sections are possible (with suitable choices of mass scales),
  (up to a few $10^{-7}$ pb for scalar cross sections
  - maybe up to $10^{-6}$ pb with nuclear uncertainties)
  much lower cross sections are also possible.

Indirect Detection

- Great for $\nu$’s, (perhaps also for exotic relics), but remains a challenge for neutralinos in the CMSSM