GRAND UNIFICATION

IN

HIGHER DIMENSIONS.

Lawrence Hall

UC Berkeley
Workshops on Grand Unification

1990

Susy Conferences

Evidence for weak scale susy

Evidence for grand unification

\[ \alpha_i \]

1. WHAT IS THE PHYSICS AT UNIFICATION?

2. HOW CAN IT BE TESTED?
(I) FEATURES OF 4D GUTs

(II) NEW VIEWPOINT FOR UNIFICATION
     hep-ph/0103125

(III) A MINIMAL MODEL
      hep-ph/0111068

(IV) EXPERIMENTAL SIGNALS
      hep-ph/0205067

with Yasunori Nomura.
THE DICHOTOMY

UNIFICATION FITS

- Light neutrinos
  -νe (Yukawa)

ABHORS UNIFICATION

- Light Higgs
  - h

Numerical Prediction

Quark-lepton masses

RECONCILIATION?
Predictions of $\alpha_s$ vs $\alpha_s (M_Z)$

$\alpha_s$, $\alpha_s^{GUT}$, $\alpha_s^{SGUT}$

Exp.

Susy

Good
But not perfect

What is this telling us?

\[ \text{GUT threshold corr.: } (S+T, \text{ unit by unit}) \]

\[ \text{Estimate of supersymmetry threshold corr.: } \]
Minimal 4D supersymmetry, SU(5): 

- No understanding of why $h_2$ light.
- Excluded by experimental limit on $Z'$

One is forced to invent mechanisms ......

... a better way forward?
A NEW PHYSICS OF UNIFICATION

OLD TOOLS

A NEW THEORY FROM $10^5 - 10^7$ GeV
Higher Dimensions

\[ x \rightarrow (x, y) \]

Low energy 3-2-1

SU(3) Bulk

3-2-1 Boundary

Suppress \( x \):

Matter Q.N.

\[ \frac{3}{2} \]

Higgs Q.N.

\[ \frac{5}{2} \]

\[ p \text{ decay} \]

\[ \alpha_s \text{ prediction} \]
II.2 ORIGIN OF 3-2-1 POINT DEF

RESTRICTED GAUGE SYMMETRY FROM BOUNDARY CONDITIONS: FIELD THEORY IN A BOX:

\[ \text{SU}(5) \rightarrow 3-2-1 \]

LOCAL EXPLICIT SU(5) BREAKING

A NEW CLASS OF PREDICTIVE GAUGE THEORIES.

- SANE UNITARITY BEHAVIOUR AS \( \text{SU}(5) \)
- NO NEED TO BREAK THE UNIFIED GAUGE SYMMETRY
The Setup

- Assume
  Fixed geometry
  Boundary conditions

- Introduce
  $\phi(y)$ & general action

"Machine"

$\phi(y)$ $\rightarrow$ \[3.2-1\] $\rightarrow$ \[M_s, M_c\]

Geometry / B.C. $\rightarrow$ KK Structure

Structure of Bulk $\rightarrow$ Experiment

- No need for mass terms or spontaneous symmetry breaking
II.

Consequences for:

- Gauge coupling unification
- Why $h_2$ light
- $p$ decay
- Quark-lepton mass ratios.

In each case the situation is quite unlike 4D:

Higher Dim GUTs \neq 4D GUTs
Consequences for Gauge Coupling Unification

Cutoff for 4+1d GUT

$\frac{1}{R}$ (Could be several)

$G$

$\frac{1}{g_{4+1d}^2} F_{4+1d}^2$

Subspaces with restricted gauge symmetry:

$\frac{1}{g_i^2} \frac{F_c^2}{g_{4+1d}^2}$

Say:

$\frac{1}{g_{i}^2} = \frac{R_{d}}{g_{4+1d}^2} + \frac{R_{d'}}{g_{i}^2}$

NO GAUGE COUP. UNIF!
**Radiative CORR.**

4D viewpoint

\[ x \]

3

2

1

usual logs.

\[ M_c \]

\[ M_s \]

power law
non-universal
sensitive to \( M_s \)

**4+1d GUT with restricted gauge symmetry**

\[ \Rightarrow \]

Explicit local breaking of G

\[ \Rightarrow \]

Unif. \( \alpha_i \)

Lost
NEW PHYSICS OF UNIFICATION

Hall, Nomura (01)

\[ \frac{1}{g^2} = \frac{R^d}{g_{4+d}^2} + \frac{R^{d'}}{g'^2} \]

1. Large vol. of bulk

\[ RM_S = \frac{M_S}{M_C} \Rightarrow 1 \]

\[ M_S \atop \text{AT} \atop \text{CLOSE TO} \atop \text{STRING COUPLING} \]

2. Local breaking to 3-2-1 at points

Relative running above \( M_c \) is log

Sensitivity to physics at \( M_S \) is \( \frac{1}{\alpha} \)
How can 2 stage uniF. be predicTive

\[
\frac{1}{\alpha_s(M_Z)} = b_s \ln \frac{M_c}{M_Z} + b_k \ln \frac{N_s}{N_c} + \frac{1}{\alpha_s}
\]

\[
\frac{M_s}{M_c} \text{ determined by strong coupling:}
\]

\[
\frac{C \frac{g^2(M_c)}{16 \pi^2} \left( \frac{M_s}{M_c} \right)^d}{ \sim 1}
\]

\[
\alpha_s = \alpha_s(d, \text{ B.C., } \Phi(y))
\]
Split Multiplets: The light $h_2$

The "machine" automatically creates split multiplets.

$$H(x,y) \rightarrow \begin{array}{c} G \\ 3-2-1 \end{array} \rightarrow M_c$$

Not New!

Candelas, Horowitz, Strominger, Witten (85)
Dixon, Harvey, Vafa, Witten (85)
Ibanez, Kim, Nilles, Quevedo (87)

Kawamura (00)

\[SU(5)\]

\[3-2-1\]
Study masses for $H_3$:

$\mathcal{L} = H_3 \gamma_2 H_3^c \rightarrow H_3 \frac{n}{R} H_3^c, \quad n \neq 0$

$N=2$ partner

No coupling by $R$ symmetry

$d=5$ Proton Decay

$d=4,5$ absent by sym.

$d=6$

$M_X = M_c = ?$
**Quark-Lepton Mass Relations**

- Depends on location:
  - Yukawas in 4d 
    \[ S^d (\bar{y} - \bar{y}_0) \] 
  - \[ \psi_1, \psi_2 H \]

- \[ m_{12} \leq \frac{1}{\sqrt{v_1}}, \frac{1}{\sqrt{v_2}} \]
  - if both touch \( \bar{y}_0 \)

- Heaviest fermions live in 4D & have unified mass rel. (if away from...)

- Lighter fermions live in bulk & don't have unified mass rel. (if touch...)

*321*
III

The Minimal Model

Conceptual framework

Calculable, predictive theories
Seek: \( \alpha_5(\text{bulk, B.C., G, H}) \to 0.117 \pm 0.002 \) (Exp. at \( M_2 \))

Recall: \( \alpha_5^{\text{SGUT}} = 0.130 \)

Find:

\[ d < 3 \]

H in bulk (not part of \( V \))

\[ S \alpha_5 = -\frac{1}{2\pi} \frac{\alpha_5^2}{7m} \ln \frac{M_5}{M_c} \]

\[ \begin{cases} T^{2/2m} \\ or \ m = 2 \end{cases} \]

Best if:

\[ d = 1 \Rightarrow \begin{cases} \frac{M_5}{M_c} = 200 \\ G = SU(5) \end{cases} \]

Simplest Model is selected over a large energy interval.

\[ \begin{array}{c}
M_5 \quad 10^{17} \text{ GeV} \\
5 \quad 3-2-1 \\
M_c \quad 5 \times 10^{14} \text{ GeV}
\end{array} \]
THE $\alpha_s$ PREDICTION

$\alpha_s(M_Z)$

- $\alpha_s^GUT$
- $\alpha_s^{SGUT}$
- $\alpha_s^{KK}$
- $\alpha_s^{\text{exp}}$

- SUSY LOG
- KK LOG

$\Box$ uncertainty from non-log corr. at high scale

$\Box$ uncertainties from susy scale

- 5D Theory with large $M_s/M_c$ is best fit by
- Other theories not excluded
The KK Modes

\[ \mathfrak{su}(5) \rightarrow \mathbb{3} - \mathbb{2} - \mathbb{1} \]

\[ V + \Sigma \]

\[ H + H^c \]

\[ m_{\text{KK}} \]

\[ \frac{3}{R} \]

\[ \frac{2}{R} \]

\[ \frac{1}{R} \]

\[ (++, +-, -, -) \]

\[ V_{321}, V_x, \Sigma_x, \Sigma_{321} \]

\[ h_2, H_3, H_3^c, h_2^c \]

cf Kawamura (2000)
**MATTER LOCATION**

**Boundary Matter**

\[ S(y) \{ TTH + TFH \} \]

**Bulk Matter**

\[ S(y) \{ TTH + TT'H + \ldots \} \]
\[ \{ TFH + T'FH + \ldots \} \]

**Unified**

\[ T, F \]

**Non-Unified**

\[ T(0, e) \]
\[ T'(q) \]
\[ F(l) \]
\[ F'(d) \]

<table>
<thead>
<tr>
<th>Boundary</th>
<th>( \frac{1}{R^{1/2}} )</th>
<th>SU(5) REL.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk</td>
<td>( \frac{1}{R^{3/2}} )</td>
<td>NO REL.</td>
</tr>
</tbody>
</table>

\( T_3 \) must be on boundary

\( F_3 \) on boundary for \( b/c \)

*Hall, Nomura ph/04.03
Hebecker, Marsh-Russell ph/01.06*
BOUNDARY INTERACTIONS

4D, N=1 allows:

FH, \bar{A}H  NO
TTH, TF\bar{H}  YES
TFF  NO
TTTF  NO

N=2 possesses continuous R sym.

eg \left[ \begin{array}{c}
H \\
H^c
\end{array} \right]_F
\begin{array}{c}
(0) \\
(2)
\end{array}

Extend to boundary interactions:

T^{(1)} F^{(1)} H^{(0)} \bar{A}^{(0)} H^c^{(2)} \bar{H}^c^{(2)}

Complete sol. to

- \Delta B, \Delta L \neq 0 \text{ at } d=4,5
- h_2 \text{ massless}
IV EXPERIMENTAL SIGNALS

- $d=6$ p decay

- Theories of quark + lepton masses.
  
  Hall, March-Russell, Okui, Smith, ph/0108161
  Hedges, March-Russell, ph/0205143

  Talk by John March-Russell

If supersymmetry breaking has high messenger scale

- Superpartner spectrum

- Lepton flavor violation
\[ d = 6 \] PROTON DECAY

1. \[ M_x = M_c \]

2. \[ \text{can give} \quad p \rightarrow e^+ \pi^0 \]

3. For large \( \frac{M_s}{M_c} \), \( T_1 \) must be in bulk

\[ \frac{T_2}{T_1(u,e)} = 321 \]

\[ p \rightarrow K^+ \pi^0 \]

via CKM mixing.

Nomura ph/0108170

Hebecker, Marsh-Russell ph/0204037
Boundary Gauge Interactions

\[ T_1(u, e) \]

\[ T_1'(q) \]

\[ \delta(y) T_1^+ T_1' \]

\[ q \rightleftharpoons e \]

Minimal model gives

\[ \tau_p \approx 10^{-34} \text{ yr} \] for \( l^+ \pi^0, l^+ K^0, \Sigma^+ \pi^0, \Sigma^+ \)

\[ e \text{ or } \mu \]
$\textbf{Susy} : M_{\text{mess}} > M_c$

Various possibilities for $T_2, F_1, F_2$

$U(3)_T \times U(3)_F \xrightarrow{\text{matter}} \xrightarrow{\text{location}} \ldots$

Expect:

- Non-universal squark/slepton masses
- FCNC from superpartner exchange
Ex: SUSY breaking B.C.

\[ \text{"SU(2)_R twist"} \]

\[
\begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix} \quad \text{on} \quad \begin{pmatrix} \frac{1}{2} \\ A \end{pmatrix}
\]

\[ \alpha \sim 10^{-13}! \]

DYNAMICS OF F

COMPONENT OF RADION

Martí Pomarol 2/1/01 06

SOFT OPS. AT \( M_c \)

All bulk superpartners:\n\[ \tilde{m} = \frac{\alpha}{R} \]

All boundary superpartners:\n\[ 0 \]

Trilinear A parameters:\n\[ 1, 2, 3 \]

(Counts \# scalars in bulk)
CONSEQUENCES

Unique matter locations:

Param.

\[ \tilde{m}, \mu, B \quad \xrightarrow{\text{ENSB}} \quad \tilde{m}, \tan \beta \]

Superpartner spectrum
**Predictions For**

**Probe matter location:**

<table>
<thead>
<tr>
<th>( \tan \beta )</th>
<th>( \bar{g} )</th>
<th>( \bar{g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>699</td>
<td>911</td>
</tr>
<tr>
<td>10</td>
<td>251</td>
<td>334</td>
</tr>
<tr>
<td>5</td>
<td>427</td>
<td>531</td>
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<tr>
<td>10</td>
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<td>175</td>
</tr>
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<td>334</td>
</tr>
<tr>
<td>10</td>
<td>417</td>
<td>518</td>
</tr>
<tr>
<td>5</td>
<td>422</td>
<td>528</td>
</tr>
<tr>
<td>10</td>
<td>701</td>
<td>915</td>
</tr>
<tr>
<td>( A )</td>
<td>675</td>
<td>880</td>
</tr>
<tr>
<td>( H^0 )</td>
<td>602</td>
<td>780</td>
</tr>
<tr>
<td>( H^\pm )</td>
<td>209</td>
<td>277</td>
</tr>
<tr>
<td>( H^\pm )</td>
<td>317</td>
<td>422</td>
</tr>
<tr>
<td>( \alpha_s(M_Z) ) { \pm 0.003 }</td>
<td>118</td>
<td>128</td>
</tr>
<tr>
<td>( m_0(M_Z) ) { \pm 0.10 }</td>
<td>552</td>
<td>690</td>
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<tr>
<td>( Br(\mu \rightarrow e\gamma) )</td>
<td>( 6 \times 10^{-12} )</td>
<td>( 8 \times 10^{-12} )</td>
</tr>
<tr>
<td>( Br(\mu \rightarrow 3\nu) )</td>
<td>( 4 \times 10^{-14} )</td>
<td>( 5 \times 10^{-14} )</td>
</tr>
<tr>
<td>( Cr(\mu \rightarrow e;^{48}Ti) )</td>
<td>( 4 \times 10^{-14} )</td>
<td>( 5 \times 10^{-14} )</td>
</tr>
<tr>
<td>( Br(\tau \rightarrow \mu\gamma) )</td>
<td>( 1 \times 10^{-8} )</td>
<td>( 1 \times 10^{-8} )</td>
</tr>
</tbody>
</table>

*Includes SUSY threshold corrections*
Complete success for moderate $\tan \beta$. 

$\frac{-28g^2 + 7y_t^2}{80\pi^2} \ln \frac{M_S}{M_c}$
LEPTON FLAVOR VIOLATION

$m^2_E = 0$, $m^2_\alpha = \begin{pmatrix} m^2_1 & m^2_2 \\ m^2_2 & m^2_3 \end{pmatrix}$, $A_E = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$

$U(3)_L \xrightarrow{\text{lepton location}} U(2)_L$

$O(1)$ tree-level effect

Much larger than $O\left(\frac{y_E^2}{16\pi^2}\right)$

4D SUSY GUT effect

Hall, Kostelecky, Roby (81)

Barbieri, Hall (94)
$\mu \rightarrow e$ example

$\frac{m_\mu \tan \beta}{W_{ee}} W_{\mu\mu}^*$

$\mu^L \rightarrow \tilde{b} \rightarrow \tilde{\tau} \rightarrow e_R$

$1.2 \times 10^{-11}$

$10^{-14}$

$4.3 \times 10^{-12}$

$10^{-16}$

$1.1 \times 10^{-6}$

$10^{-7}$

$\text{PSI} \rightarrow$

$\text{MECO} \rightarrow$

$\beta$ from factories

$\text{VERY POWERFUL PROBE}$

$\text{Br}(\mu \rightarrow e\gamma) \approx 3 \times 10^{-11} \left( \frac{200 \text{ GeV}}{m} \right)^4 \left( \frac{|W_{ee}|}{0.04} \right)^2 \left( \frac{|W_{\mu\mu}|}{0.01} \right)^2 \left( \frac{\tan \beta}{5.0} \right)^2$

$\text{Br}(\mu \rightarrow 3e) \approx 2 \times 10^{-13} \left( \frac{200 \text{ GeV}}{m} \right)^4 \left( \frac{|W_{ee}|}{0.04} \right)^2 \left( \frac{|W_{\mu\mu}|}{0.01} \right)^2 \left( \frac{\tan \beta}{5.0} \right)^2$

$\text{Cr}(\mu \rightarrow e_1^{48} Ti) \approx 5 \times 10^{-8} \left( \frac{200 \text{ GeV}}{m} \right)^4 \left( \frac{|W_{ee}|}{0.04} \right)^2 \left( \frac{|W_{\mu\mu}|}{1.0} \right)^2 \left( \frac{\tan \beta}{5.0} \right)^2$
Conclusions

- Alternative physics for unification at $10^{16}$ GeV

- Solves dichotomy of 4D susy GUTs
  - $d=5$ p decay: $u(1)_R$
  - $q-l$ mass rel. only for heavy $g$
  - $h_2 - H_3$ splitting
\[ \alpha_5(M_Z) = 0.118 \pm 0.003 \]

\[ m_b(M_Z) = 3.3 - 0.02(b_m - 10) \text{ GeV} \quad 3.0 \pm 0.2 \]

- Some of flavor from \[ \frac{M_3}{M_4} \approx 10^2 \]
- \( d = 6 \) p decay \[ p \rightarrow \ell^+ \pi^0, \ldots \]

* Predictive superpartner spectrum \( \Rightarrow \) large \( M_3 \)

* \( \mu \rightarrow e \) transitions