# ORBIFOLD GUTs (FROM THE BOTTOM-UP)

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MAINLY BASED ON

- Arthur Hebecker + JMR
  hep-ph/0106166
  hep-ph/0107039
  hep-ph/0204037
- L. Hall, T. Okui, D. Smith + JMR
  hep-ph/0108161
- Arthur Hebecker + JMR
  hep-ph/0205143
- AH, JMR, T. Yanagida
  in progress...

4d SUSY GUTs have many successes

- precision gauge coupling unif'n (if N=1 SUSY)
- explanation of quantum numbers of SM matter
- $m_b/m_\tau$ mass ratio prediction (?)

⋮

... but also have less attractive features

- Simplest 4d SUSY GUT models have trouble with dim = 5 proton decay
- analogue of $m_b/m_\tau$ prediction for lightest 2 generations badly wrong
- Higgs doublet – triplet splitting problems, at least in SU(5)
- Higgs structure in full models ugly!
- don't provide any new insight into CKM mixing and intergenerational mass ratios such as $m_c/m_t$, ...

In this talk I'll discuss some Orbifold GUT ideas that give

- new ways of thinking about origin and meaning of flavor, and mass and mixing hierarchies
- Our extra dimension is small $1/R \sim 10^{15}$ GeV so not 'large extra dimensions'

## References (briefly)

- String theory orbifolds

  Dixon, Harvey, Vafa, Witten

- Applications/constructions in string thy

  Witten

  Ibanez, Mas, Nilles, Quevedo

  Ibanez, Nilles, Quevedo

- Recent activity (basic model)

  Kawamura hep-ph/0012125

  Altarelli, Feruglio hep-ph/0102301

  Hall, Nomura hep-ph/0103125

  Hebecker, March-Russell hep-ph/0106166, hep-ph/010[illegible]

- 6d models and related SO(10)

  Asaka, Buchmuller, Covi

  Hall, Nomura, Smith

  Hall, Nomura, Smith, Okui

  Hebecker, March-Russell hep-ph/0107039

  Dermisek, Mafi

- Flavor

  Hall, Okui, March-Russell, Smith hep-ph/0108161

  Hebecker, March-Russell hep-ph/0205143

- Gauge unification, proton decay

  Hall, Nomura hep-ph/0103125

  Hebecker, March-Russell hep-ph/0106166, hep-ph/0204037

  Contino, Pilo, Rattazzi, Trincherini hep-ph/0108102

Simplest successful generalization of 4d SU(5) GUT

5d orbifold theory on

$$M_4 \times (S^1/\mathbb{Z}_2 \times \mathbb{Z}_2')$$

Kawamura
Hall, Nomura
Altarelli, Feruglio
Hebecker, JMR

with gauge group $G = SU(5)$ and $N=1$ 5d susy

In effective field theory orbifolds are defined by imposing equivalence relations in field space (restricts the configuration space – like a gauge symm does)

$\Rightarrow$ If $y \mapsto k[y]$ is geometrical action

then impose

$$\varphi_i(x,y) \sim P_{ij}(k)\,\varphi_j(x,k[y]) \qquad *$$

action on space of fields

'Orbifold' $\Longleftrightarrow$ geometrical action possesses fixed points

$k[y] = y$ for some $y_{FP}$ and some $k \neq 1$

At such a fixed point the equivalence relation $*$ restricts the allowed values of the fields

$$\varphi_i(x, y_{FP}) \sim P_{ij}(k)\,\varphi_j(x, y_{FP})$$

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What properties must the effective FT passes to be well-defined?

1) it must be non-anomalous – both in bulk and at fixed points

2) it must be unitary and local

$$\Downarrow$$

there must exist a description where no non-local restrictions are placed on fields, and therefore...

a) $\exists$ a system of well-posed boundary conditions for otherwise unrestricted fields

b) $\exists$ a self-adjoint extension of the Hamiltonian etc consistent with b.c.'s.

For a generic field $\varphi(x,y)$ on $S^1/Z_2$

if $\varphi_i(x,y) \sim -\varphi_i(x,-y) \Rightarrow \varphi_i(x,0)$ vanishes

if $\varphi_j(x,y) \sim +\varphi_j(x,-y) \Rightarrow \varphi_j(x,0) \neq 0$ but $\partial_5 \varphi_j(x,0) = 0$

Thus we can work on the interval $[0, \pi R]$

with either Neumann or Dirichlet b.c.'s on fields

Only the fields $\varphi_+$ of +ve $\mathbb{Z}_2$ parity can have zero

modes w/ vanishing 4d mass

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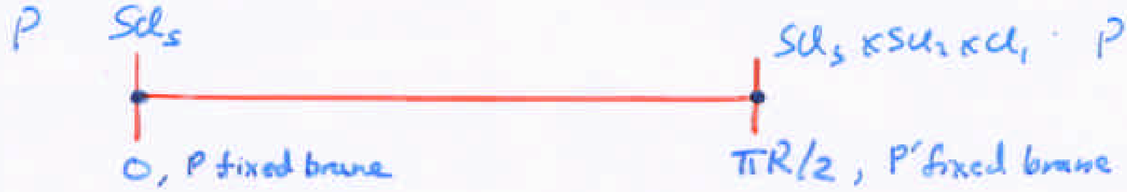
What is the symmetry structure?

The physical space is the interval $[0, \pi R/2]$

with two 'end-of-the-world' orbifold branes




For consistency w/ orbifolding

gauge transformation parameters must also have restricted form

$$\Lambda = \exp\left\{ i \left( \alpha^{a}_{++} T^{a} + \alpha^{\hat{a}}_{+-} T^{\hat{a}} + \ldots \right) \right\}$$

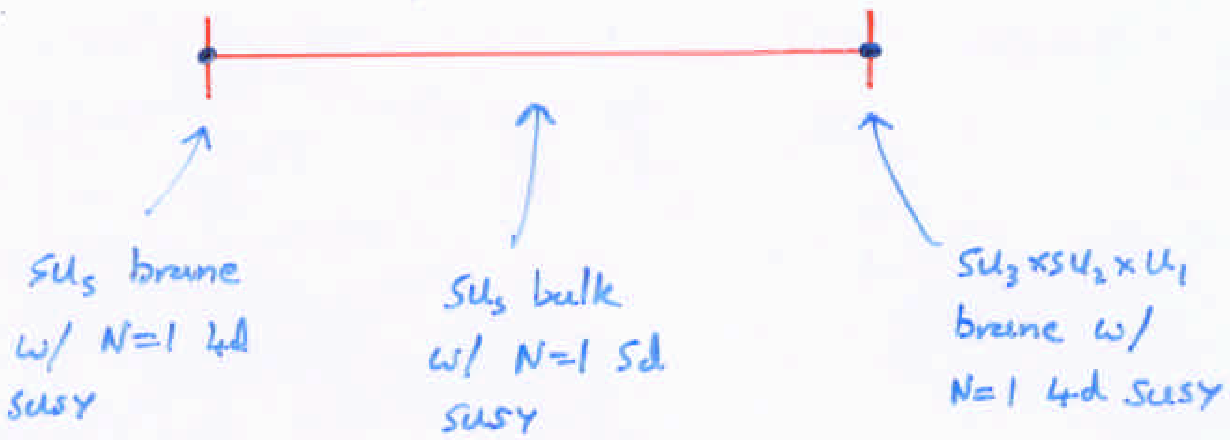
transf'n in unbroken dir'ns unrestricted $\alpha^{a}_{++}(x,y)$

broken dir'ns must vanish at $y = \pi R/2$

$\Rightarrow$ At P' fixed point $y = \pi R/2$ only $SU_3 \times SU_2 \times U_1$ gauge transformations

$\Rightarrow$ At P fixed point $y = 0$, and in bulk $y \in [0, \pi R/2)$ general $SU_5$ gauge transformations

Situation is similar for SUSY (strictly speaking needs SUGRA treatment) leading overall to




So far have just discussed gauge sector. What about SM matter and higgs?

3 possibilities a priori for location

I) On $SU_5$ brane – in which case just like normal 4d $SU_5$ case

II) In the 5d bulk – the novel possibility

III) On the SM brane at $y = \pi R/2$ – in which case just like usual 4d $SU_3 \times SU_2 \times U_1$ SM

## II. Matter fields in bulk (flavor...)

The unbroken $SU_5$ gauge symmetry of the bulk

$\Rightarrow$ bulk matter is in full $SU_5$ multiplets

Also it is $N=1$ 5d supersymmetric

minimal susy rep'n a Hypermultiplet of $N=2$ 4d from 4d perspective ...

$$\mathcal{H} \longrightarrow H + H^c$$

$N=1$ 5d

$H \downarrow$ chiral 4d

$H^c \downarrow$ antichiral 4d

eg a $\underline{5}$ of $SU_5$ $\quad \mathcal{H}_5 \to F + F^c$

$$F(x,y') \sim P' F(x,-y') \qquad P' = \mathrm{diag}(++---)$$

$$F^c(x,y') \sim -P' F^c(x,-y')$$

and similarly for $P$

This important sign is forced on us

to understand origin ....

Recall, vector s.f. in 5d decomposed as

$$V = -\theta\sigma^m\bar\theta A_m - i\theta^2\bar\theta\bar\lambda_1 + i\bar\theta^2\theta\lambda_1 + \tfrac{1}{2}D\bar\theta^2\theta^2$$

$$\Sigma'_1 = \frac{1}{\sqrt2}(\Sigma + iA_5) + \sqrt2\,\theta\lambda_2 + \theta^2 F$$

so some residual gauge transformation

Under 5d gauge transf'ms (abelian for simplicity)

$$V \to V + \Lambda + \bar\Lambda$$

$$\Sigma'_1 \to \Sigma'_1 + \sqrt2\,\partial_5\Lambda$$

and the 5d $N=1$ gauge interactions of hypermultiplets in 4d $N=1$ terms are (Arkani-Hamed, Gregoire, Wacker: Hebecker)

$$\int d^4\theta\left\{H^c e^V H^{\dagger c} + H^\dagger e^{-V} H\right\}$$

$$+ \underbrace{\int d^2\theta\, H^c\left(\partial_5 - \tfrac{1}{\sqrt2}\Sigma'_1\right)H}_{} + h.c.$$

to be invariant as $\partial_5$ and $\Sigma'_1 \to (-\partial_5, -\Sigma'_1)$ must have

$$H \sim P\,H \qquad H^c \sim -P\,H^c$$

and

$$H \sim P'H \qquad H^c \sim -P'H^c$$

Thus under the joint $Z_2 \times Z_2'$ action the spectrum of a $\underline{5}$ Hypermultiplet is

| | $(P, P')$ | mass |
|---|---|---|
| $F_{\text{doublet}}$ | $(+,+)$ | $2n/R$ |
| $F_{\text{triplet}}$ | $(+,-)$ | $(2n+1)/R$ |
| $F^c_{\text{triplet}}$ | $(-,+)$ | $(2n+1)/R$ |
| $F^c_{\text{doublet}}$ | $(-,-)$ | $(2n+2)/R$ |

$\Longrightarrow$ so a single 5d Hypermultiplet leads to a massless doublet, and NO triplet at massless level

Spectrum of MSSM higgs (with automatic doublet-triplet splitting) reproduced by

bulk hypermultiplets $F + \bar{F}$

$F + F^c$ $\qquad$ $\bar{F} + \bar{F}^c$

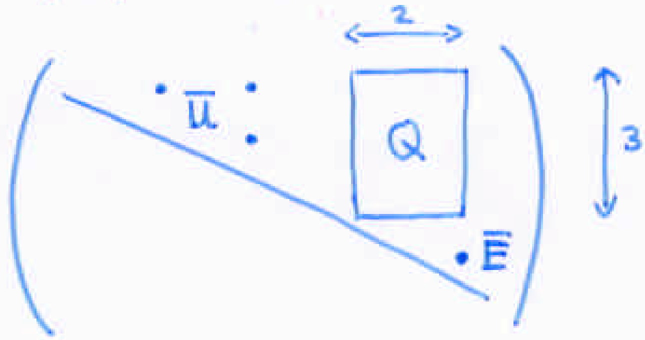
massless doublet $\qquad$ massless doublet

Split multiplets very familiar from string orbifolds
Witten, ... ; Ibanez, Quevedo; Kim, Nilles; ....

We can also put MSSM generation(s) in bulk

$$\bar{5} \sim \begin{pmatrix} L \\ \bar{D} \end{pmatrix}$$

$10 \sim$




Then under $P$ and $P'$ action a $T$ hyper ($= T + T^c$) and $\bar{F}$ hyper ($= \bar{F} + \bar{F}^c$) lead to

$$\begin{array}{lllll} T & \longrightarrow & \bar{U}_{++} & Q_{+-} & \bar{E}_{++} \\ T^c & \longrightarrow & \bar{U}^c_{--} & \bar{Q}^c_{-+} & \bar{E}^c_{--} \\ \bar{F} & \longrightarrow & \bar{D}_{+-} & L_{++} & \\ \bar{F}^c & \longrightarrow & \bar{D}^c_{-+} & L^c_{--} & \end{array}$$

massless modes

Not yet a full generation — so take another copy of $10 + \bar{5}$ hypermultiplet in bulk (w/ chiral components $T' + T'^c + F' + F'^c$) and use freedom to flip $P'$ parities on these multiplets by overall sign

$\longrightarrow$ massless modes

$$T' \longrightarrow Q_{++}$$
$$F' \longrightarrow \bar{D}_{++}$$

$\Rightarrow$ do not get usual $SU_5$ Yukawa relations for such matter

Indeed structure of Yukawa couplings ...

Because bulk is $N=2$ susy from 4d perspective only get Yukawa couplings on $y=0, \pi R/2$ branes

$$W_{\text{Yukawa}} = \int d^2\theta\, \delta(y) \Big( \sum_{IJ} h^u_{IJ}\, T'^I_Q\, T^J_{\bar{u}}\, H_u$$

generation indicies

$$+ h^d_{IJ}\, T'^I_Q\, F'^J_{\bar{D}}\, H_d$$

$$+ h^\ell_{IJ}\, T^I_E\, F^J_L\, H_d \Big)$$

since different $SU$ fields $h^d_{IJ}$, $h^\ell_{IJ}$ unrelated Yukawa couplings

On the other hand, for a generation on the $SU_5$ brane at $y=0$ do get $SU(5)$ relations ...

This suggests a very nice structure w/ further pretty features ...

Basic point: Because of $N=1$ 5d SUSY, Yukawa couplings can only arise at $y=0, \pi R/2$ branes (where $N=1$ 4d SUSY).

Then couplings involving k bulk fields are suppressed by wavefunction normalization factors

$$\left(\sqrt{\frac{2}{\pi M_* R}}\right)^k$$

$M_*$ = UV cutoff of thy (more later...)

Arkani Hamed, Dimopoulos, Dvali; J.
Dienes, Dudas, Gherghetta
Hall, Okui, Smith, JMR

Let $\left\{\begin{array}{l}\psi = \text{brane field} \\ \Phi = \text{bulk field}\end{array}\right\}$ then

$$\mathcal{L}_{Yuk} \simeq \int dy \left\{ \tilde{\lambda}_0 \delta(y) \psi^3 + \sum_{k=1}^{3} \tilde{\lambda}_k \delta(y) \psi^{3-k} \Phi^k(y) \right\}$$

canonical KE

$$\simeq \tilde{\lambda}_0 \psi^3(x) + \frac{\lambda_1}{\sqrt{M_* R}} \psi^2(x) \Phi_{(0)}(x) + \frac{\lambda_2}{(M_* R)} \psi(x) \Phi_0^2(x) + \ldots$$

from dimensions of $\tilde{\lambda}_k$ in 5d

from canonically normalizing KE term of $\Phi_{(0)}(x)$ field

Start by thinking about heaviest 2 generations

have dimensionless ratios (at $M_{GUT}$)

$$\frac{m_\mu}{m_\tau} \simeq \frac{1}{17} \qquad \frac{m_s}{m_b} \simeq \frac{1}{30}$$

$$\frac{m_c}{m_t} \simeq \frac{1}{300} \qquad V_{cb} \simeq \frac{1}{25}$$

and $\frac{m_b}{m_\tau} \simeq 1$ (maybe $= 1$)

The orbifold GUT structure allows us to give a qualitative explanation of all these from a very simple assumption

Facts we want to explain:

1. $m_b = m_\tau$ but <u>not</u> $m_c = m_\mu$

2. strong hierarchy in up-quark masses

   $m_t \ggg m_c$ w/ $m_t$ heaviest

3. lesser hierarchy in down-quark masses / leptons

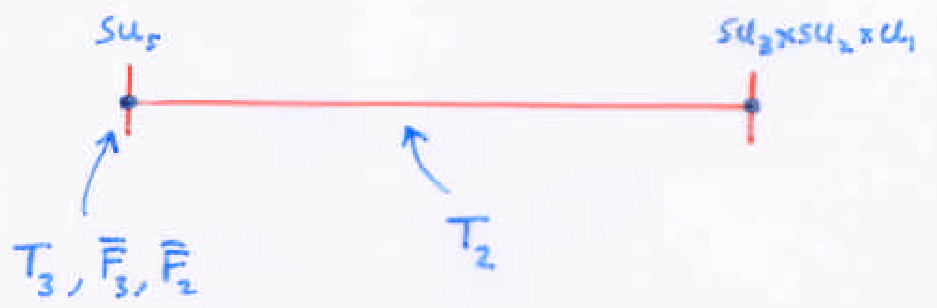
   $m_b \gg m_s$ $m_\tau \gg m_\mu$

① + ② $\Rightarrow$ $T_3$ and $\bar{F}_3$ on $SU_5$ brane

② + ③ $\Rightarrow$ $\bar{F}_2$ on $SU_5$ brane

$T_2$ ($+\,T_2'$ since now hypers) in bulk

UNIQUELY FIXED ...!




up masses

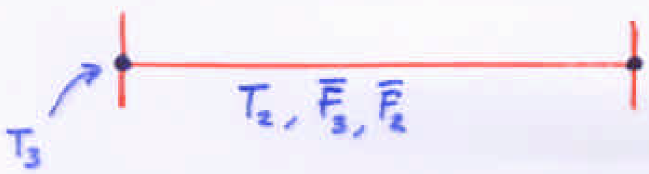
$T\,T\,H$

$T\,\bar{F}\,H$

down + charged lepton masses

'Only the <u>10</u>'s are in the bulk ....'

If <u>don't</u> require $m_b = m_\tau$ then one other possibility



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This leads to the up quark mass matrix

$$\mathcal{U} \simeq \begin{pmatrix} \delta^3 & \delta^2 \\ \delta^2 & \epsilon \end{pmatrix}$$

since Higgs in bulk to get doublet-triplet splitting

$\epsilon$ = $SU_5$ invariant coupling

$\delta$ = $SU_5$-breaking coupling (since $T$, $T'$ have indep't couplings)

$$|\epsilon| \simeq |\delta| \simeq 1/\sqrt{M_* R}$$

$$\Longrightarrow \quad m_c/m_t \simeq |\epsilon|^2$$

$V_{cb} \simeq |\epsilon|$ from diagonalization in up quark sector

(+ large $\nu_\mu - \nu_\tau$ mixing since $\bar{F}_3$, $\bar{F}_2$ treated symmetrically; rhd $N_i$ states similar...)

How about down quarks and leptons?

$$D \simeq \begin{pmatrix} \delta^2 & \delta^2 \\ 0 & \epsilon \end{pmatrix} \qquad E \simeq \begin{pmatrix} \delta^2 & 0 \\ \delta^2 & \epsilon \end{pmatrix}$$

as they arise from $\bar{F}_i T_j$ ops

$$\Longrightarrow \quad m_\mu/m_\tau \simeq m_s/m_b \simeq V_{cb} \simeq |\epsilon|$$

$$m_c/m_t \simeq |\epsilon|^2 \qquad m_b = m_\tau \qquad !$$

Gauge coupling unification and $|\epsilon|$ (roughly...)

The $SU_5$ gauge symmetry is broken explicitly on the SM brane at $y = \pi R/2$ so brane-localized gauge-kinetic terms which violate $SU_5$ are allowed

$$\int d^2\theta \int_0^{\pi R/2} dy \left\{ \frac{1}{g_5^2} W^\alpha W_\alpha + \sum_{i=1}^{3} \frac{\delta(y - \pi R/2)}{g_{4i}^2} W_i^\alpha W_{i\alpha} + h.c. + \ldots \right\}$$

What effect do they have on unification?

Let $M_*$ = scale at which theory becomes strongly coupled

Then 'naive dimensional analysis' in higher dimensions leads to

$$1/g_5^2 \sim \frac{M_*}{24\pi^3} \,, \quad 1/g_{4i}^2 \sim 1/16\pi^2$$

However the physical coupling measured at scales just below $1/R$ is that of the zero modes – its gauge coupling is given by integration $\int dy$

$$\frac{1}{g_{0i}^2} = \frac{\pi R}{2}\frac{1}{g_5^2} + \frac{1}{g_{4i}^2} \Rightarrow M_* R \lesssim \frac{12\pi}{\alpha_{GUT}}$$

$$\lesssim 10^2 \times \text{few}$$

$$\simeq \frac{M_* R}{48\pi^2} + \frac{\delta_i}{16\pi^2} \xrightarrow{} O(1)$$

so corrections small

Thus dimensionless measure of size of volume

$M_* R$ can be big, and corrections due to

brane localized ops are volume suppressed and

$\simeq$ size of usual threshold corrections

Also $\Rightarrow$ $|\epsilon| \sim 1/\sqrt{M_* R} \sim 1/\sqrt{300}$ in correct range

for flavor... $\epsilon \sim 1/17$ !

In a well-defined sense orbifold models link

smallness of $\alpha_{GUT} \simeq 1/25$

hierarchy of mass ratios, $V_{CKM}$ elements

both have common origin in size of $5^{th}$ dimension

# New features in $d=6$ proton decay

Higgsino-mediated $d=5$ p-decay is absent, but also $d=6$ X,Y gauge boson decay operators are different

Reason: Only matter on SU(5) brane (not SU(5) bulk!) has interactions with X,Y gauge bosons that lead to $\Delta B = 1$ operators at leading order

and as we've discussed flavor motivates us to put most matter in bulk or on SM brane

$a=1$ if on brane, 0 otherwise

$$O_{TF} = \frac{\pi^2}{4} \frac{g_4^2}{M_c^2} \sum_{ij} a_i b_j \, \bar{d}_{Ri} \bar{u}_{Ri} L_i Q_j$$

$M_c = 1/R < M_{GUT}$

$b=1$ if T on brane SU(5), 0 otherwise

$$O_{TT} = \frac{\pi^2}{4} \frac{g_4^2}{M_c^2} \sum_{ij} b_i b_j \, \bar{e}_{Ri} \bar{u}_{Rj} Q_i Q_j$$

Flavor models put only $T_3$ (and maybe $\bar{F}_i$'s) on SU(5) brane

$\Longrightarrow$ So physical proton decay involving light generation states only occurs via mixing!

(Nomura; Hebecker, JMR)

Dominant modes can be $p \to K^0 \mu^+$ or $K^+ \bar{\nu}_\tau$

at potentially detectable rates

## 3-generation model in 5d?

An elegant and successful model is possible if we utilize one more ingredient natural to 5d orbifold GUTs

recall that $\mathcal{L}$ for 5d hypermultiplet (in terms of 4d chiral superfields $H, H^c$) is

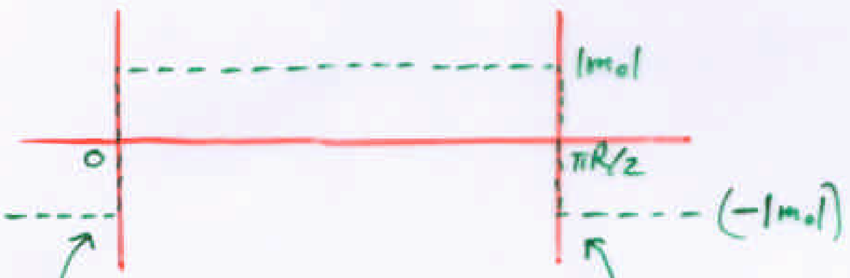
$$\mathcal{L} = \int_{\theta^2\bar\theta^2} (H^\dagger H + H^c H^{c\dagger}) + \int_{\theta^2} H^c \partial_5 H + h.c.$$

this can be supplemented by 5d Lorentz inv mass terms (if H neutral or vector-like)

$$\mathcal{L}_{mass} = m_0 \int_{\theta^2} H^c H + \frac{m_e}{2} \int_{\theta^2} (H^2 + H^{c\,2}) + h.$$

odd under $\mathbb{Z}_2$ parities

even under $\mathbb{Z}_2$ parities (forbidden for gauged multiplets)




$m_0$ jumps at orbifold fixed pts

In this case ($m_o \neq 0$, $m_e = 0$) eqn for y-dep't profile $H(y)$ is non-trivial ($m_o = m$)

$$\left(\partial_y^2 - m^2 + p_4^2 + 2m\,\delta(y) - 2m\,\delta(y-\ell)\right)H(y) =$$

effective mass of mode in 4d

from jumps of $m_o$

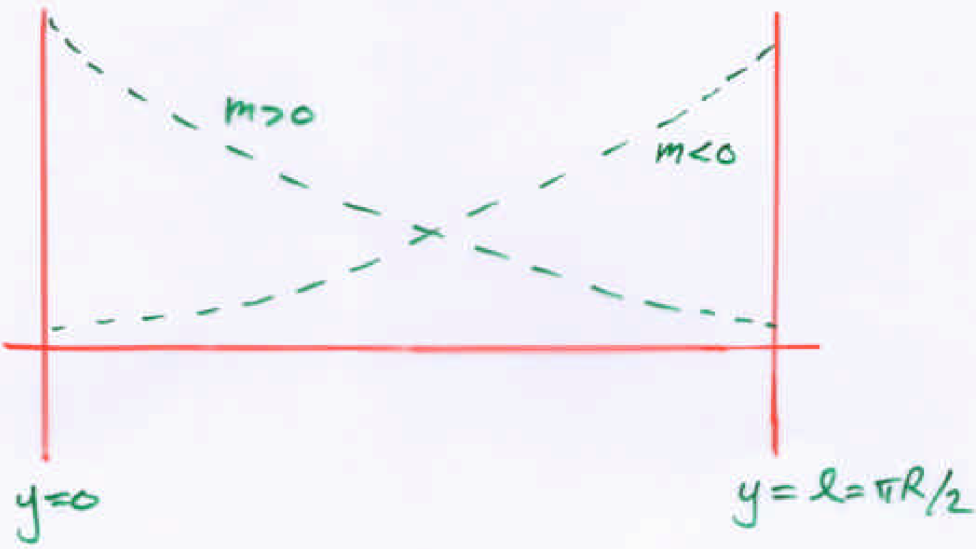
To see effect of this consider most interesting case of $(+,+)$ parity for H

$\longrightarrow$ still $\exists$ zero mode with $p_4^2 = 0$

$\longrightarrow$ $H(y) = e^{-ym}$ profile

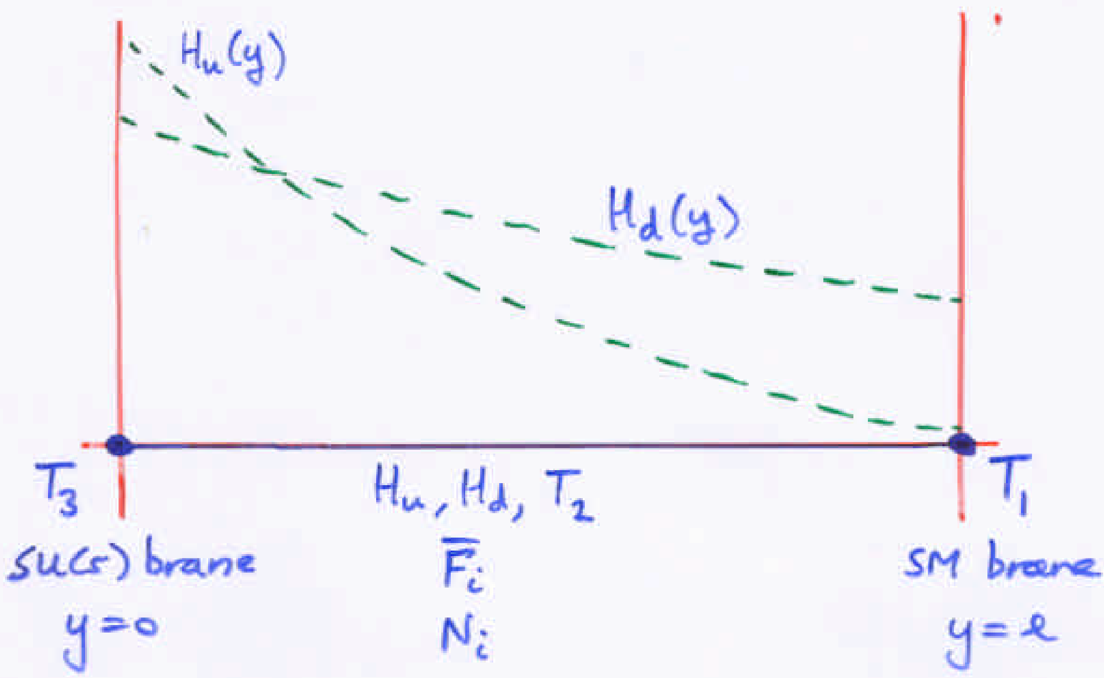
(all other KK masses $O(m) \simeq O(M_*)$ or larger)

So



A successful 3-generation model simply requires that Higgs doublets in bulk have such a mass

Pictorially




Just 3 parameters are input

$\epsilon = 1/\sqrt{M\ell}$ (but this is 'set' by $\alpha_{GUT} \sim 1/25$)

$m_u \ell$ — dimensionless measure of bulk up-Higgs m

$m_d \ell$ — " " " " down-Higgs

Because of non-trivial Higgs profiles: rescaling of 5d to 4d coupling is now

$$\lambda_{5d} \longrightarrow \lambda_{4d,eff} = \frac{\lambda_{5d}}{c(-m\ell)\sqrt{M\ell}}$$ if SU(5) brane interaction

$$\lambda_{5d} \longrightarrow \lambda_{4d,eff} = \frac{\lambda_{5d}}{c(m\ell)\sqrt{M\ell}}$$ if interaction on SM brane

with $$c(m\ell) = \sqrt{\frac{e^{2m\ell}-1}{2m\ell}}$$

$$\approx \frac{e^{m\ell}}{\sqrt{2m\ell}} \quad \text{for } m\ell > \text{few}$$

$$\approx \frac{1}{\sqrt{2m\ell}} \quad \text{for } m\ell < -\text{few}$$

* Such factors are analogue to having an O(1) anomalous dimension in a 4d QFT over a wide energy range. This correspondence is precise in AdS/CFT case.

Very non-trivial that we get these factors in weakly-coupled thy, and in computable way.

The resulting effective 4d Yukawa interactions are

$$\lambda_{TT} \simeq \lambda_t \begin{pmatrix} \delta_u & \epsilon\delta_u & 0 \\ \epsilon\delta_u & \epsilon^2 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}$$ ups

$$\lambda_{TF} \simeq \lambda_b \begin{pmatrix} \delta_d & \delta_d & \delta_d \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix}$$ downs

where

$$\lambda_t = \sqrt{\frac{2m_u}{M_*}} \qquad \lambda_b = \epsilon\sqrt{\frac{2m_d}{M_*}}$$

so most of $\lambda_t/\lambda_b$ due to $\epsilon$ again

$$\delta_u = e^{-m_u l} \qquad \delta_d = e^{-m_d l}$$

from Higgs zero mode profiles

This structure for

$$\epsilon = 1/\sqrt{M_* \ell} \simeq 1/17$$

$$m_u \ell \simeq 12$$

$$m_d \ell \simeq 7$$

is very successful for mass ratios and also gives good $V_{CKM}$

$$\begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$$

using $\delta_u \sim \delta_d^2 \sim \epsilon^4$

excellent 1-3 and 2-3 mixings, OK 1-2 (need $O(1)$ coeff to be $\sim 4$ to get $\theta_c$)

Overall very simple model!

## Neutrinos?

$\exists$ a very simple model

- introduce 3 SM-singlet bulk hypermultiplets $\mathcal{N}_i$

$$\mathcal{N}_i = N_i + N_i^c$$

take orbifold action to be

$$(+,-) \text{ for } N_i \qquad (\therefore (-,+) \text{ for } N_i^c)$$

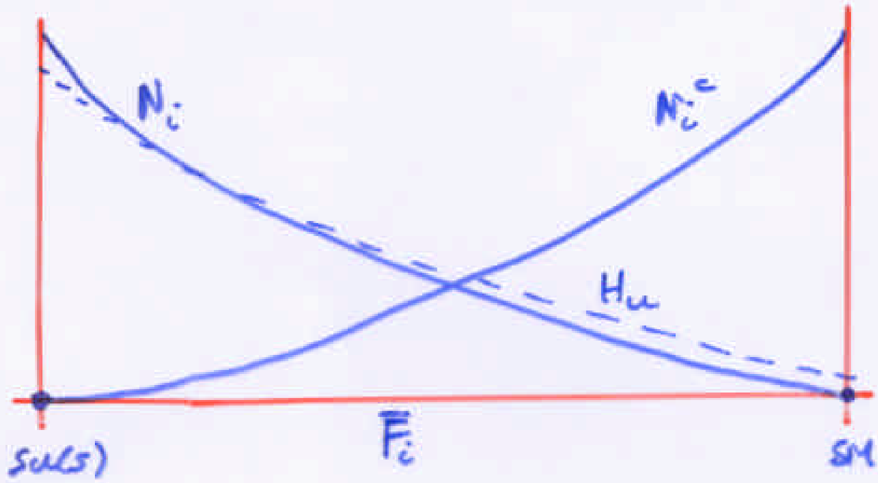
$\longrightarrow$ no zero modes

- However bulk action

$$N^{cT}(\partial_5 + m_N)\, N$$

add bulk mass

leads to exponentially localised modes




and exponential suppression of 4d mass connecting $N$ to $N^c$

$$m_4 \simeq 2 m_N e^{-m_N \ell} \qquad (\text{but still} \gg \text{TeV})$$

- Including brane Yukawa interactions

$$H_u L^T \lambda N \,\delta(y) + H_u L^T \lambda' N^c \,\delta(y-\ell)$$

and integrating out N's leads to

$$(L H_u)^2 \quad \text{neutrino mass operator}$$

- But note! $H_u$ profile is strongly peaked at $y=0$ so couplings of 4d rhd $N_i$ to $L_i$'s (from bulk $\bar{F}_i$'s) are suppressed by $\sim \delta_u \sim m_u/m_t$

$\Rightarrow$ Suitable physical $m_\nu$ scale

but with 'light' rhd N with small coupling $\longrightarrow$ interesting for leptogenesis!
(see T. Yanagida's talk...)

- What about mixing angles?

We already treated $\bar{F}_i$'s in bulk symmetrically wrt $i$

Natural to assume same for $N_i$'s in bulk ($m_N$)

but break this intergenerational symm of bulk

by only brane interactions $\lambda, \lambda'$

large mixing angles!

$$\rightarrow \quad m_\nu \sim \frac{v^2 \delta_u e^{2m_N \ell}}{M_*} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \rightarrow O(1)$$

# CONCLUSIONS

The 5d SU(5) orbifold GUT model does very well ...

i) It breaks $SU_5 \longrightarrow SU_3 \times SU_2 \times U_1$ in an elegant and unusual fashion

ii) Corrections to gauge coupling unification are $\simeq$ same size as usual SUSY GUT thresholds

iii) automatic doublet – triplet splitting

iv) much reduced (or depending on flavor locations absent) proton decay

v) simple assumption of $T_2$ in bulk gives large number of flavor relations which work well

vi) reason why $3^{rd}$ generation $m_b = m_\tau$ OK, but not $m_c = m_\mu$ ....

vii) simple 3-generation models in 5d that account for all flavor hierarchies

... an interesting theory ...