Nucleon Decay

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SUSY '02

21 June, 2002
DESY, Hamburg, Germany

Outline

- Experimental Bounds & Dimensional Analysis
- 4 Dim. SUSY GUTs Dead or Alive ??
- 5 Dim. SUSY GUTs Miracles from the Bulk ??
- Conclusions

The Ohio State University

Super-K Bounds
(Kearns, Snowmass 2001)

mode	exposure (kt · yr)	τ/B limit (10^{32} yrs)
$p \to \pi^0 + e^+$	79	50
$p \to \pi^0 + \mu^+$	79	37
$p \to K^+ + \bar{\nu}$	79	16
$p \to K^0 + e^+$	70	5.4
$p \to K^0 + \mu^+$	70	10
$n \to K^0 + \bar{\nu}$	79	3.0

Generic Nucleon Decay Operator

- 4 Fermion Operator $\sim \frac{1}{\Lambda^2}\ q\ q\ q\ l$

$$\Longrightarrow \quad \Gamma_p \sim 10^{-3}\ m_p^5/\Lambda^4$$

- $\tau_p > 5 \times 10^{33}$yrs

$$\Longrightarrow \quad \Lambda > 4 \times 10^{15}\text{GeV}$$

SUSY Desert ??

4 Dimensional SUSY GUTs

- Charge Quantization*
- Gauge Coupling Unification*
- Yukawa Coupling Unification
- + Family Symmetry $\Longrightarrow$ Hierarchy of Fermion Masses
- Neutrino Masses via See - Saw scale $\sim 10^{-3}\ M_G$
- LSP – Dark Matter Candidate
- Baryogenesis

Charge Quantization *

— Georgi & Glashow; Pati & Salam; Georgi; Fritzsch & Minkowski

$$\mathbf{SU_5} \qquad \{Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad e^c \quad u^c\} \subset \mathbf{10}$$

$$\{d^c \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix}\} \subset \bar{\mathbf{5}}$$

$$\begin{pmatrix} H_u \\ T \end{pmatrix}, \begin{pmatrix} H_d \\ \bar{T} \end{pmatrix} \subset \mathbf{5_H}, \; \bar{\mathbf{5}}_\mathbf{H}$$

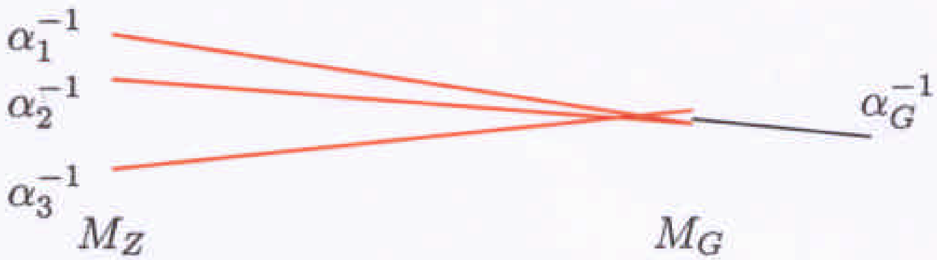
$$\mathbf{SO_{10}} \qquad \mathbf{10} + \bar{\mathbf{5}} + \bar{\nu}_{sterile} \subset \mathbf{16}$$

$$\mathbf{5_H}, \; \bar{\mathbf{5}}_\mathbf{H} \subset \mathbf{10_H}$$

Gauge coupling unification *

— Dimopoulos, S.R. & Wilczek; Dimopoulos & Georgi ; Ibañez + Ross

* Only evidence for SUSY



• Significant GUT threshold corrections from Higgs and GUT breaking sectors

Def: $M_G \iff \alpha_1(M_G) = \alpha_2(M_G) \equiv \tilde{\alpha}_G$

Good fit requires: $\epsilon_3 \equiv \frac{(\alpha_3(M_G) - \tilde{\alpha}_G)}{\tilde{\alpha}_G} \sim -4\%$

Yukawa unification

$SU_5 \qquad \lambda_b = \lambda_\tau$

Small $\tan\beta \sim 4$ or large $\tan\beta \sim 50$

$SO_{10} \qquad \lambda_b = \lambda_t = \lambda_\tau = \lambda_{\nu_\tau} = \lambda$

Minimal model gives $\tan\beta \sim 50$

Nucleon Decay

- Dimension 6 operators

$$\Lambda \approx M_G$$

$M_G \sim 3 \times 10^{16}$ GeV $\Longrightarrow \tau_p \sim 5 \times 10^{37}$ yrs.

$p \to \pi^0 + e^+$ – dominant decay mode

- Dimension 5 operators

Weinberg; Sakai and Yanagida

$$\frac{c^2}{M_T^{eff}} \left((Q\,Q\,Q\,L) + (U^c\,U^c\,D^c\,E^c) \right)$$

L L L L + R R R R operators

- Dimension 4 operators Dangerous

Weinberg; Sakai and Yanagida

$$(U^c\,D^c\,D^c) + (Q\,L\,D^c) + (E^c\,L\,L)$$

R_p

R parity forbids all dimension 3 and 4 (and even one dimension 5) baryon and lepton number violating operators. It is a necessary ingredient of any "natural" SUSY GUT.

Nucleon Decay – Dimension 5 Operators

Sakai and Yanagida; Dimopoulos, S.R. and Wilczek; Ellis, Nanopoulos and Rudaz

$$T(p \to K^+ + \bar{\nu}) \propto$$

$$\frac{c^2}{M_T^{eff}}(\text{Loop Factor})\ (RG)\ \langle K^+ \bar{\nu} | qqql | p \rangle$$

$$\sim \frac{c^2}{M_T^{eff}}(\text{Loop Factor})\ (RG)\ \frac{\beta_{lattice}}{f_\pi}\ m_p$$

Chiral Lagrangian analysis

- $\beta_{lattice}$

  3 Quark matrix element — QCD

- c^2

  Model dependence

- Loop Factor

  SUSY Breaking

- M_T^{eff}

  Effective color triplet Higgs mass/GUT breaking

$\beta_{lattice}$ - Strong Interaction Matrix Element

S. Aoki et al. (2000)

$$\beta_{lattice} = \langle 0|qqq|N\rangle = 0.015(1)\ \text{GeV}^3$$

Also $\alpha_{lattice} \approx -\beta_{lattice}$

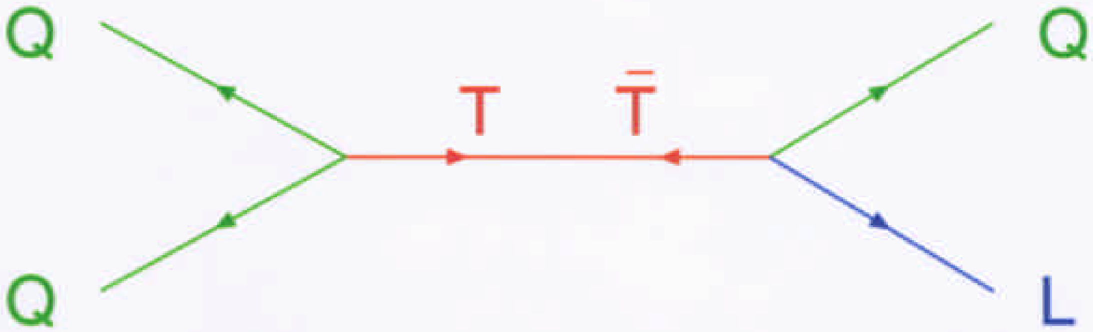
NOTE: Previously $0.003\text{GeV}^3 < \beta_{lattice} < 0.03\text{GeV}^3$

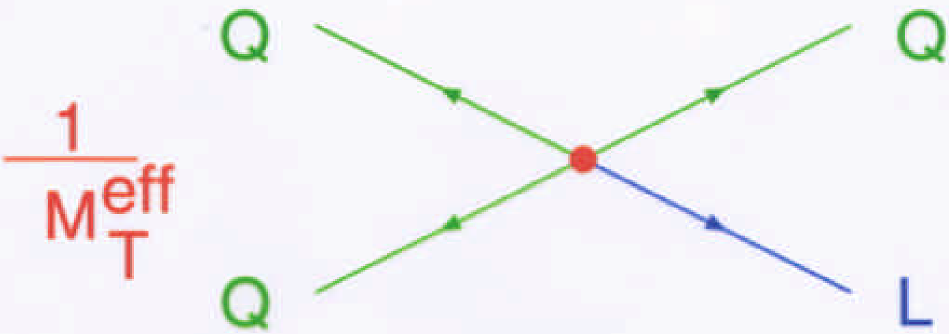
- New lattice result = $5\times$ "conservative lower bound"
- Systematic uncertainties (quenched + chiral Lagrangian)

± 50 % (my estimate)

Dimension 5 Operators

$$W \supset H_u\, Q Y_u \overline{U} + H_d (Q Y_d \overline{D} + L Y_e \overline{E}) +$$
$$T (Q \tfrac{1}{2} c_{qq} Q + \overline{U} c_{ue} \overline{E}) + \bar{T} (Q c_{ql} L + \overline{U} c_{ud} \overline{D})$$





$$\frac{1}{M_T^{eff}} \left[\, Q \tfrac{1}{2} c_{qq} Q \;\, Q c_{ql} L + \overline{U} c_{ud} \overline{D} \;\, \overline{U} c_{ue} \overline{E} \,\right]$$

c^2 - Model Dependence

SU_5 $\quad \lambda(\langle\Phi\rangle)\, 10\, 10\, 5_H + \lambda'(\langle\Phi\rangle)\, 10\, \bar{5}\, \bar{5}_H$

or

SO_{10} $\quad \lambda(\langle\Phi\rangle)\, 16\, 16\, 10_H$

gives

$$H_u\, Q\, Y_u\, \overline{U} + H_d\, (\, Q\, Y_d\, \overline{D} + L\, Y_e\, \overline{E}) +$$

$$\frac{1}{M_T^{eff}}\, [\, Q\, \tfrac{1}{2} c_{qq}\, Q\, Q\, c_{ql}\, L + \overline{U}\, c_{ud}\, \overline{D}\, \overline{U}\, c_{ue}\, \overline{E}\,]$$

- NOTE: $Y_u \neq c_{qq} \neq c_{ue}$ and $Y_d \neq Y_e \neq c_{ud} \neq c_{ql}$

Example: $\lambda_b = \lambda_\tau$ OK

BUT $\lambda_s = \lambda_\mu$ and $\lambda_d = \lambda_e \Longrightarrow 20 \sim \frac{m_s}{m_d} = \frac{m_\mu}{m_e} \sim 200$

- $c_{qq}\, c_{ql},\ c_{ud}\, c_{ue} \propto m_u\, m_d\ \tan\beta$
- If $16_H,\ 10_H$ mix and have $\frac{1}{M}(16\ 16\ 16_H\ 16_H)$ for neutrino masses $\Longrightarrow$ new terms Babu, Pati and Wilczek. However this is not required for neutrino masses Blažek, Tobe and S.R.
- Family symmetries affect texture of $c_{qq},\ c_{ql},\ c_{ud},\ c_{ue}$

Nucleon Decay in Realistic SUSY GUTs

Babu, Pati and Wilczek; Dermíšek, Mafi and S.R.; Albright and Barr; Altarelli, Feruglio and Masina

- Vary parameters at GUT scale $\tilde{\alpha}_G$, M_G, Y_u, Y_d, Y_e and SOFT SUSY breaking parameters until FIT precision electroweak data, including fermion masses and mixing angles.

- Now c_{qq}, c_{ql}, c_{ud}, c_{ue} at M_G are FIXED.

- Renormalize Dimension 5 operators from $M_G \to M_Z$ in MSSM; evaluate Loop Factor at M_Z and renormalize Dimension 6 operator from $M_Z \to 1$ GeV. The latter gives renormalization constant $A_3 \sim 1.3$ (NOT $A_L \sim .22$). Finally calculate decay amplitudes using chiral Lagrangian approach or direct lattice gauge calculation.

- The Loop Factor depends on squark, slepton and gaugino spectrum.

$SO_{10} \times SU_2 \times U_1^n$
Features of the Model

Barbieri, Hall, S.R. and Romanino; Blažek, S.R. and Tobe

1. Family Hierarchy

$$SU_2 \times U_1 \quad \underset{\epsilon}{\longrightarrow} \quad U_1 \quad \underset{\epsilon'}{\longrightarrow} \quad \text{nothing}$$

   3rd family >> 2nd family >> 1st family

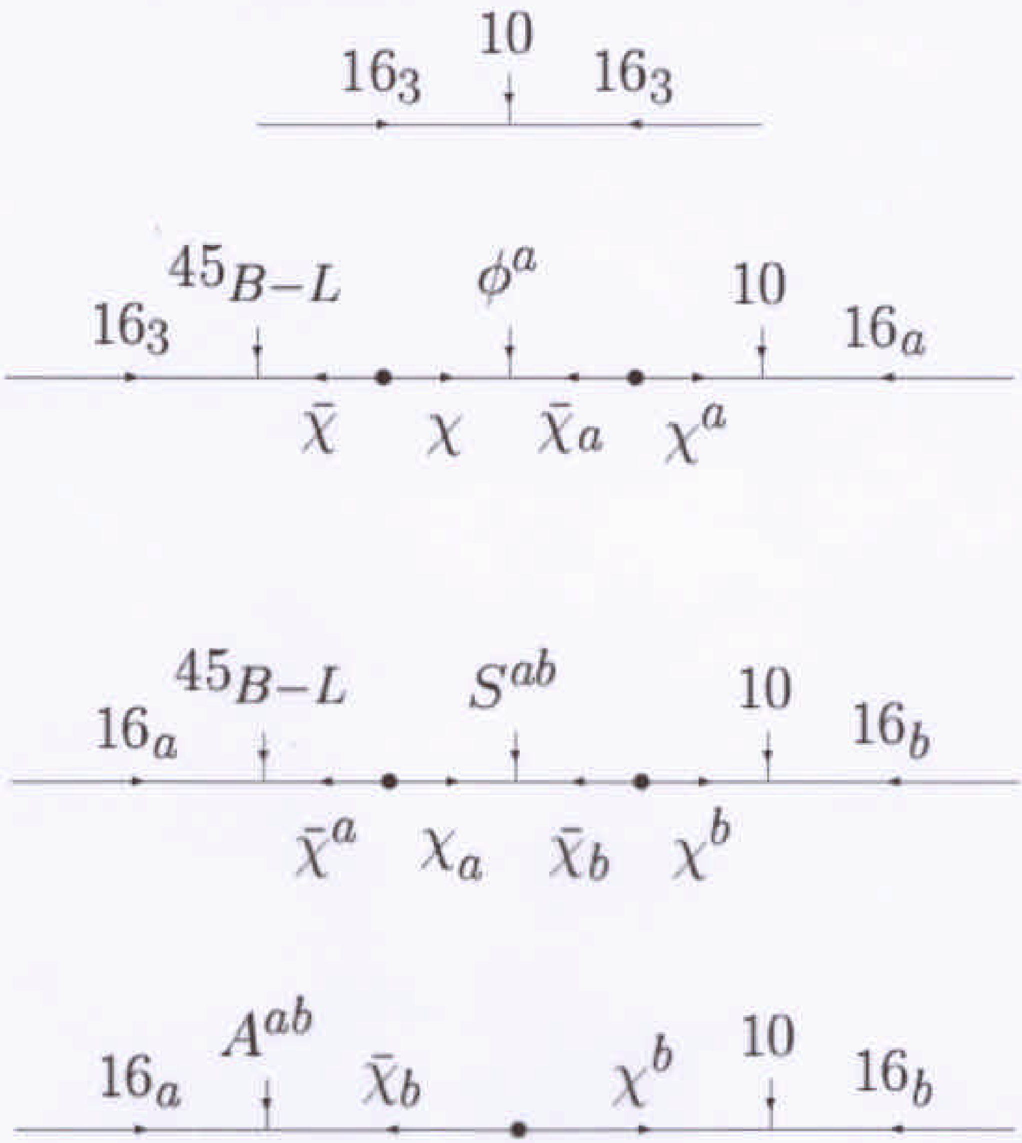
2. Patterns - approximate Georgi - Jarlskog "natural"

$$\langle 45 \rangle = (B - L) M_G$$

$$m_s \sim \tfrac{1}{3} m_\mu$$
$$m_d \sim 3 m_e$$

3. $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} = \lambda \ @M_G$
4. $m_u < m_d$ even though $m_t >> m_b$
5. Gauge Coupling Unification
6. SU_2 suppresses flavor violation such as $\mu \to e\gamma$
7. 9 Yukawa parameters fits 13 fermion masses and mixing angles

Effective Fermion Mass Operators



Effective Yukawa Couplings

$$Y_u = \begin{pmatrix} 0 & \epsilon' \rho & 0 \\ -\epsilon' \rho & -\frac{1}{3}\,\epsilon\,\rho & r\,\epsilon \\ 0 & \frac{1}{3}\,r\,\epsilon & 1 \end{pmatrix} \lambda$$

$$Y_d = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & -\frac{1}{3}\,r\,\sigma\,\epsilon \\ 0 & \frac{1}{3}\,r\,\epsilon & 1 \end{pmatrix} \lambda$$

$$Y_e = \begin{pmatrix} 0 & -\epsilon' & 0 \\ \epsilon' & 3\,\epsilon & r\,\epsilon \\ 0 & -r\,\sigma\,\epsilon & 1 \end{pmatrix} \lambda$$

$$Y_\nu = \begin{pmatrix} 0 & -\omega\,\epsilon' & 0 \\ \omega\,\epsilon' & 3\,\omega\,\epsilon & \frac{1}{2}\,\omega\,r\,\epsilon \\ 0 & -r\,\sigma\,\epsilon & 1 \end{pmatrix} \lambda$$

Observable	Data(σ) (masses	*Theory* in GeV)
M_Z	91.188 (0.091)	91.20
M_W	80.419 (0.080)	80.41
$G_\mu \cdot 10^5$	1.1664 (0.0012)	1.166
α_{EM}^{-1}	137.04 (0.14)	137.00
$\alpha_s(M_Z)$	0.1181 (0.002)	0.1171
$\rho_{new} \cdot 10^3$	−0.20 (1.1)	−0.0085
M_t	174.3 (5.1)	173.3
$m_b(M_b)$	4.20 (0.20)	4.46
$M_b - M_c$	3.400 (0.200)	3.378
$m_s(2\text{GeV})$	0.109 (0.031)	0.1195
m_d/m_s	0.050 (0.010)	0.0591
Q^{-2}	0.00194 (0.00015)	0.00193
M_τ	1.777 (0.0018)	1.777
M_μ	0.10566 (0.00011)	.1057
$M_e \cdot 10^3$	0.5110 (0.00051)	0.5110
V_{us}	0.2196 (0.0023)	0.2197
V_{cb}	0.0402 (0.0019)	0.04037
V_{ub}/V_{cb}	0.0900 (0.025)	0.0685
$\hat{B}_K$	0.860 (0.080)	0.865
$B(b \to s\gamma) \cdot 10^4$	3.150 (0.54)	3.193
TOTAL χ^2	3.75	

$$T(p \to K^+ + \bar{\nu}) \propto$$

$$\sim \frac{c^2}{M_T^{eff}} (\text{Loop Factor})\ (RG)\ \frac{\beta_{lattice}}{f_\pi}\ m_p$$

- $\beta_{lattice}$

  3 Quark matrix element — QCD

- c^2

  Model dependence

- Loop Factor

  SUSY Breaking

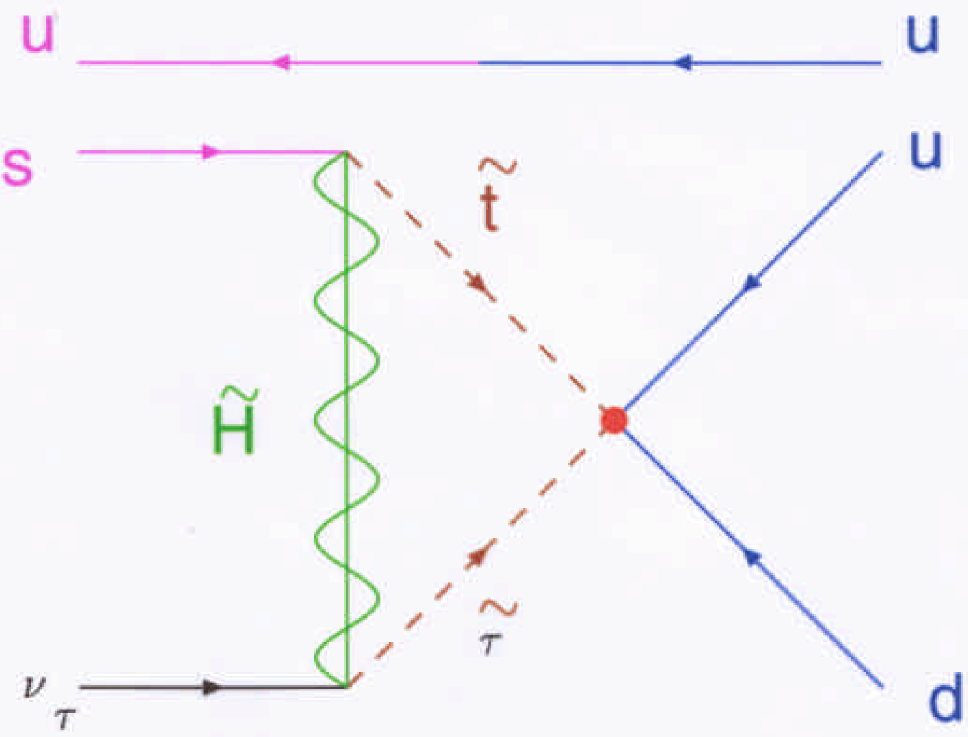
- M_T^{eff}

  Effective color triplet Higgs mass/GUT breaking

Loop Factor and Sparticle Masses

$$p \to K^+ + \bar{\nu}_\tau$$

Lucas and S.R.; Goto and Nihei; Babu and Strassler; Murayama and Pierce



$$\text{Loop Factor} = \frac{\lambda_t \lambda_\tau}{16\pi^2} \frac{\sqrt{\mu^2 + M_{1/2}^2}}{m_{16}^2}$$

$\Longrightarrow \mu, M_{1/2}$ SMALL; m_{16} Large

Is this reasonable ??
"Naturalness" ??

- Inverted Mass Hierarchy

Bagger, Feng, Polonsky, and Zhang, *Phys. Lett.* **B473**, 264 (2000)

Demand: Heavy 1st & 2nd generation squarks and sleptons >> TeV ; Light 3rd generation scalars $\leq$ TeV

Why ?? Suppresses flavor & CP violation

Find from RG running conditions

$$A_0^2 = 2m_{10}^2 = 4m_{16}^2, \quad m_{16} >> 1 \text{ TeV}$$

- Soft SUSY breaking parameters at M_G

m_{16} : Universal squark and slepton mass

A_0 : Universal trilinear scalar coupling

$M_{1/2}$: Universal gaugino mass

m_{10} : Universal Higgs mass

Constraints from SO_{10}
Yukawa unification
$\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} = \lambda$

Blažek, Dermíšek and S.R. — *Phys. Rev. Lett.* **88**, 111804 (2002) and hep-ph/0201081

- Good fits require

$\delta m_b/m_b = \Delta m_b^{\tilde{g}} + \Delta m_b^{\tilde{\chi}} + \Delta m_b^{\log} + \cdots < -2\%$

$\Delta m_b^{\tilde{g}} \approx \frac{2\alpha_3}{3\pi} \frac{\mu m_{\tilde{g}}}{m_{\tilde{b}}^2} \, tan\beta \qquad \Delta m_b^{\tilde{\chi}^+} \approx \frac{\lambda_t^2}{16\pi^2} \frac{\mu A_t}{m_{\tilde{t}}^2} \, tan\beta$

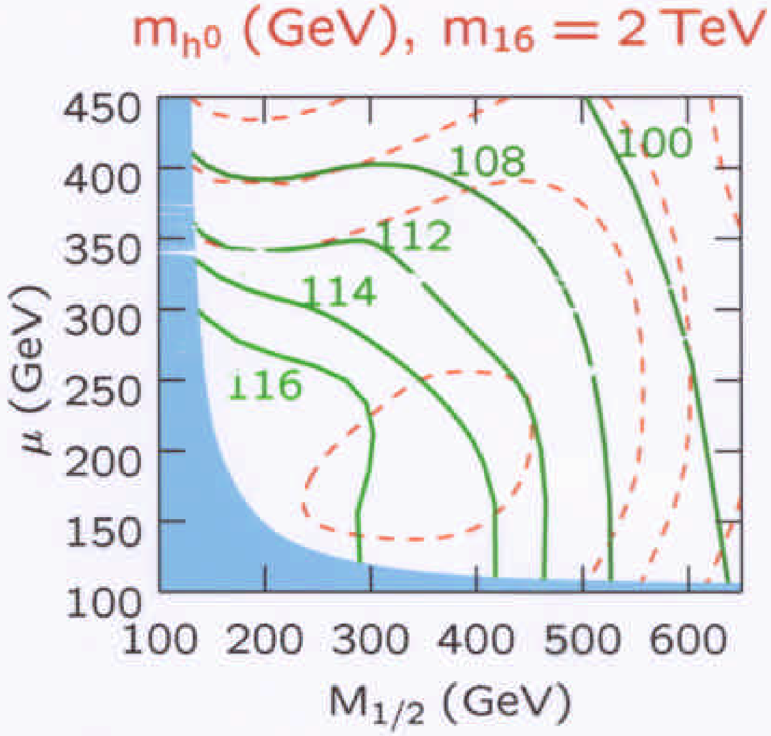
- Require $\mu > 0$: preferred by $b \to s\gamma$ and a_μ^{NEW}
- Yukawa unification possible only in a narrow region of SUSY parameter space

$$A_0 \sim -1.9 \, m_{16}$$
$$m_{10} \sim 1.35 \, m_{16}$$
$$m_{16} > 1200 \text{ GeV}$$
$$\mu, M_{1/2} \sim 100 - 500 \text{ GeV}$$

- Fits improve as m_{16} increases.

Find $\tan\beta \sim 50; \quad m_{\tilde{t}} << m_{\tilde{b}}$

- $m_h \sim 114 \pm 5 \pm 3$ GeV
- $a_\mu^{SUSY} < 16 \times 10^{-10}$



Contours of constant h^0 mass (solid) with χ^2 contours (dashed).

M_T^{eff} and Gauge Coupling Unification

Lucas and S.R.; Goto and Nihei; Babu, Pati and Wilczek; Dermíšek, Mafi and S.R.; Altarelli, Feruglio and Masina; Murayama and Pierce

Recall : $\epsilon_3 \equiv \frac{(\alpha_3(M_G)-\tilde{\alpha}_G)}{\tilde{\alpha}_G} \sim -4\%$

$$\epsilon_3 = \epsilon_3^{\text{Higgs}} + \epsilon_3^{\text{GUT breaking}} + \cdots$$

- $\epsilon_3^{\text{Higgs}} = \frac{3\alpha_G}{5\pi} \ln(\frac{M_T^{eff}}{M_G})$

Model	Minimal SU_5	SU_5 "Natural" D/T	Minimal SO_{10}
$\epsilon_3^{\text{GUTbreaking}}$	≈ 0	-7.7%	-10%
$\epsilon_3^{\text{Higgs}}$	-4%	$+3.7\%$	$+6\%$
M_T^{eff} [GeV]	2×10^{14}	3×10^{18}	6×10^{19}

- $\frac{1}{M_T^{eff}} = (M_T^{-1})_{11}$

M_T : Higgs color triplet mass matrix

Example: $M_T = \begin{pmatrix} 0 & M_G \\ M_G & X \end{pmatrix} \Longrightarrow \frac{1}{M_T^{eff}} \equiv \frac{X}{M_G^2}$

- $X << M_G \Longrightarrow M_T^{eff} >> M_G$
- NO particle with mass greater than M_G

Minimal SU_5 Excluded

Goto and Nihei;
Murayama and Pierce

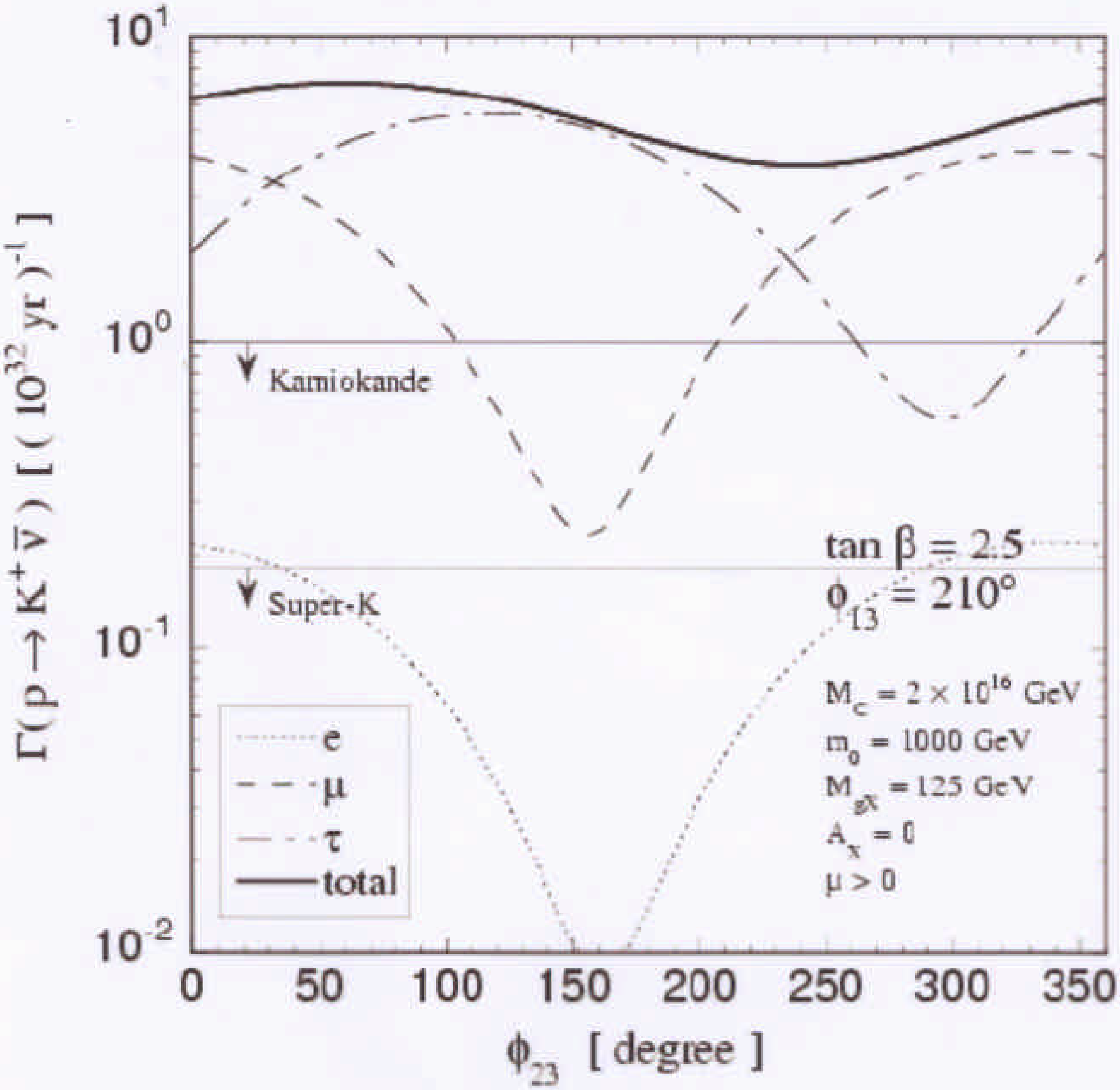


Figure 2: Decay rates $\Gamma(p \to K^+ \overline{\nu}_i)$ ($i = e, \mu$ and τ) as functions of the phase ϕ_{23} for $\tan\beta = 2.5$. The other phase ϕ_{13} is fixed at 210°. The CKM phase is taken as $\delta_{13} = 90^\circ$. We fix the soft SUSY breaking parameters as $m_0 = 1\,\mathrm{TeV}$, $M_{gX} = 125\,\mathrm{GeV}$ and $A_X = 0$. The sign of the supersymmetric Higgsino mass μ is taken to be positive. The colored Higgs mass M_C and the heavy gauge boson mass M_V are assumed as $M_C = M_V = 2 \times 10^{16}\,\mathrm{GeV}$. The horizontal lower line corresponds to the Super-Kamiokande limit $\tau(p \to K^+ \overline{\nu}) > 5.5 \times 10^{32}$ years, and the horizontal upper line corresponds to the Kamiokande limit $\tau(p \to K^+ \overline{\nu}) > 1.0 \times 10^{32}$ years.

Summary

$$T(p \to K^+ + \bar{\nu}) \sim \frac{c^2}{M_T^{eff}} (\text{Loop Factor}) \frac{\beta_{lattice}}{f_\pi} m_p$$

- c^2: model dependent but constrained by fermion masses and mixing angles
- $\beta_{lattice}$: JLQCD central value 5 times larger than previous "conservative lower bound". Still need to reduce the systematic uncertainties of quenching and chiral Lagrangian analyses
- Loop Factor $\propto \frac{\alpha}{4\pi} \frac{\sqrt{\mu^2 + M_{1/2}^2}}{m_{16}^2}$

  $\Longrightarrow$ Gauginos light; 1st & 2nd generation squarks and sleptons > TeV.

  "Naturalness" $\Longrightarrow$ Stops, sbottoms and stau mass < 1 TeV.
- M_T^{eff}: constrained by gauge coupling unification and GUT breaking sectors

Bottom Line

- "Upper bound"

$\tau(p \to K^+ + \bar{\nu}) < (\frac{1}{3} - 3) \times 10^{34}$ yrs

Blažek, Dermíšek and S.R.; Pati; Altarelli, Feruglio and Masina

- $\tau(n \to K^0 + \bar{\nu}) < \tau(p \to K^+ + \bar{\nu})$

- Other decay modes may be significant, eg.

$$p \to \quad K^0 + \mu^+, \quad \pi^0 + e^+$$

Can we eliminate Dimension 5 operators by symmetries ?

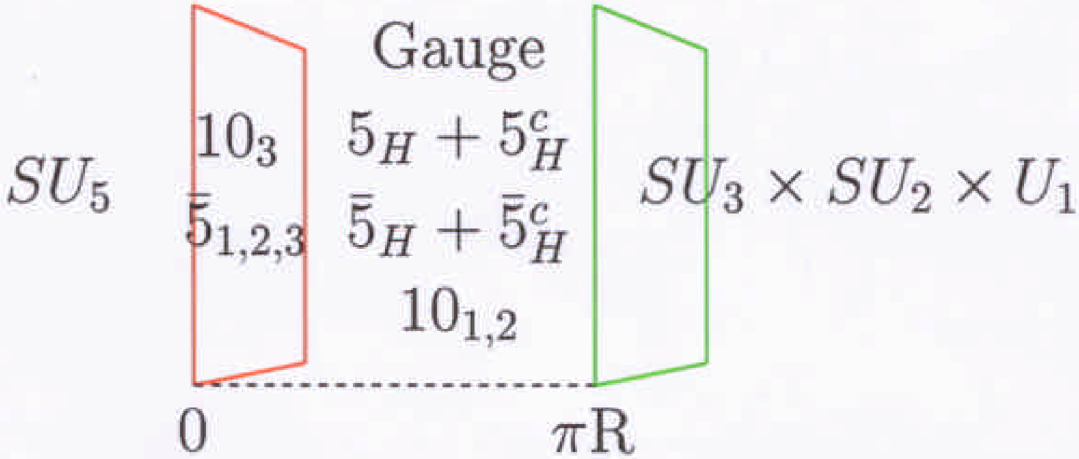
- This is non-trivial in 4 Dim., but possible Babu and Barr, hep-ph/0201130 Shadmi, parallel session

- "Natural" in Extra Dim. with GUT symmetry breaking by orbifold boundary conditions. Kawamura; Hall and Nomura; Hebecker and March-Russell; Dermíšek and Mafi, Altarelli and Feruglio, ...

+ String literature

5 Dim. SUSY GUTs

Hall and Nomura, hep-ph/0205067



- NO Unification on πR Brane

$\Longrightarrow$ Gauge coupling unification requires

$$M_C = \frac{1}{R} \sim 10^{15} \text{ GeV} << M^* = M_G$$

- NO explanation of charge quantization. Bulk states are derived from many different 10s.

On a Positive Note

- $\lambda_b = \lambda_\tau$
- "Natural" D/T splitting with $U_1(R)$ symmetry
- R_p subgroup of $U_1(R)$
- NO Dimension 5 baryon # violating operators
- Dim. 6 operators enhanced. $\frac{\bar{g}}{M_C^2}$ $\boxed{\bar{g} \sim 1}$

$$\Longrightarrow \tau(p \to \pi^0 + e^+) \sim 10^{34}\text{yrs.}$$

Proton stability in 6 Dim.

Appelquist et. al hep-ph/0107056

- Suppress decay with symmetry

$$SU_3 \otimes SU_2 \otimes U_1 \otimes \mathbb{Z}_8$$

↑ Lorentz

- Massless modes

	Q	u^c	d^c	L	e^c	ν^c
Q_{45}	-1	$+1$	$+1$	∓ 1	± 1	± 1

$$\Rightarrow \mathbb{Z}_8 : \quad \omega = \exp i\frac{\pi}{8} Q_{45}$$

↑ rot'n in 4–5 plane

- $\frac{C_{17}}{M_*^{11}} (d^c L)^3 H_u$

Eff. Dim 17 operator

$$(M_* R) \sim 5 \qquad 1/R \simeq .5\,\text{TeV}$$

$$\Rightarrow \tau(p \to e^- \pi^+ \pi^+ \nu\nu) \simeq 10^{35}\,\text{yr.}$$

for $Q^L_{45} = -1$

- Suppressed by 5 body phasespace + Large volume factor. ~ 5

$$\tau(p \to e^+ e^+ (\mu^+) \pi^- \bar{\nu})$$

$$\sim 10^{26} \text{ yrs.}$$

$$Q^L_{45} = +1$$

Eff. Dim 15 operator

$$C_{15} \lesssim 10^{-2}$$

Conclusions

- 4D SUSY GUTs still very much alive.
- Prefers light gauginos; Heavy 1st & 2nd Generation squarks and sleptons
- Consistent with SO_{10} Yukawa unification. + SUSY Flavor + CP problems
- Favors $m_h \sim 114 \pm 5 \pm 3$ GeV
- $a_\mu^{SUSY} < 16 \times 10^{-10}$
- "Upper bound"

$\tau(p \to K^+ + \bar{\nu}) < (\frac{1}{3} - 3) \times 10^{34}$ yrs

SUSY GUTs in Extra Dimensions

- NO miracles in Extra dimensions.

- Dimension 5 operators can easily be eliminated in extra dimensions at the expense of NOT understanding charge quantization.

- In 5D GUT, have "Natural" D/T splitting and conserved R_p; adhoc GUT breaking sector and dimension 5 baryon number violating operators eliminated.

- Dimension 6 operators are enhanced.

- $\tau(p \to \pi^0 + e^+) \sim 10^{34}$ yrs.

- "Natural" suppression of Nucleon decay in 6 D with

$$SU_3 \otimes SU_2 \otimes U_1 \otimes \mathbb{Z}_8$$

↑ subgroup of rotations in 2 extra dim.

- Novel decay modes

$$p \to e^- \pi^+ \pi^+ \nu \nu$$

or

$$e^+ e^+ \pi^- \bar{\nu}$$

$$e^+ \mu^+ \pi^- \bar{\nu}$$

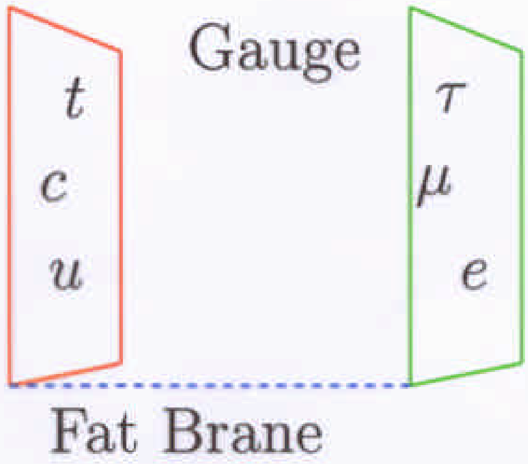
Nucleon Decay in Large Extra Dimensions

- Assume the fundamental scale

$$M^* \sim 10 - 100 \text{ TeV}$$

$\Longrightarrow \Lambda \sim M^*$

$\Longrightarrow$ Rapid Proton Decay ???



Dali'lean