Proton Decay

Stuart Raby

Department of Physics, The Ohio State University
174 W. 18th Ave., Columbus, Ohio 43210

Abstract

We discuss the status of supersymmetric grand unified theories [SUSY GUTs] with regards to the observation of proton decay. In this talk we focus on SUSY GUTs in 4 dimensions. We outline the major theoretical uncertainties present in the calculation of the proton lifetime and then present our best estimate of an absolute upper bound on the predicted proton lifetime. Towards the end, we consider some new results in higher dimensional GUTs and the ramifications for proton decay.

1 Introduction

Preliminary Super-K bounds [1] on the proton and neutron lifetimes provide stringent constraints on grand unified theories [2, 3].

mode	exposure (kt $\cdot$ yr)	τ/B limit (10^{32} yrs)
$p \to \pi^0 + e^+$	79	50
$p \to \pi^0 + \mu^+$	79	37
$p \to K^+ + \bar{\nu}$	79	16
$p \to K^0 + e^+$	70	5.4
$p \to K^0 + \mu^+$	70	10
$n \to K^0 + \bar{\nu}$	79	3.0

These constraints place bounds on the GUT scale. For example, a generic 4 Fermion baryon and lepton number violating operator of the form $\frac{1}{\Lambda^2}\ q\ q\ q\ l$ results in a proton decay rate, typically of order $\Gamma_p \sim 10^{-3}\ m_p^5/\Lambda^4$. Thus a bound on the proton lifetime $\tau_p > 5 \times 10^{33}$yrs roughly constrains the scale $\Lambda > 4 \times 10^{15}$ GeV.

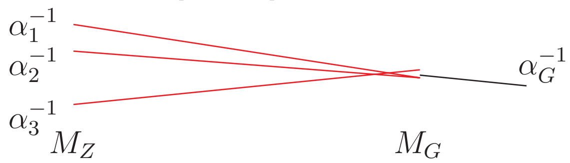
4D SUSY GUTs have many notable virtues.

- GUTs [2, 3] explain the standard model charge assignments for the observed families.
- They predict the unification of the three gauge couplings at the GUT scale [4] and the prediction of SUSY GUTs [5, 6, 7] agrees quite well with the low energy data.
- Bottom-Tau or Top-Bottom-Tau Yukawa coupling unification is predicted in simple SU_5 or SO_{10}, resp.

- Including additional family symmetries relating different generations leads to simple models of fermion masses and mixing angles.
- Neutrino oscillations governed by a see-saw scale of order $10^{-3}\ M_G$ are easily included.
- The lightest SUSY particle [LSP] is a natural dark matter candidate, and
- SUSY GUTs provide a natural framework for understanding baryogenesis and/or leptogenesis.

In order to set the notation recall, in $SU(5)$, quarks, leptons and Higgs fields are contained in the following GUT representations[2]: $\{Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad e^c \quad u^c\} \subset \mathbf{10}$, $\{d^c \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix}\} \subset \bar{\mathbf{5}}$ and $(H_u,\ T)$, $\left(H_d,\ \bar{T}\right) \subset \mathbf{5_H},\ \bar{\mathbf{5}}_\mathbf{H}$. While for SO_{10}[3] we have all quarks and leptons of one family (including one additional state, a "sterile" neutrino) $\mathbf{10} + \bar{\mathbf{5}} + \bar{\nu}_{sterile} \subset \mathbf{16}$ and the Higgs in one irreducible representation $\mathbf{5_H},\ \bar{\mathbf{5}}_\mathbf{H} \subset \mathbf{10_H}$.

At the moment, the only experimental evidence we have for low energy SUSY comes from gauge coupling unification [5, 6, 7].



The current status of this analysis uses two loop renormalization group running from M_G to M_Z with one loop threshold corrections at the weak scale. It is important to note that there are significant one loop, GUT scale, threshold corrections from the Higgs and GUT breaking sectors. Hence at two loops the three gauge couplings do not meet at M_G. Nevertheless, the GUT scale can be defined as the point where two couplings meet; $\alpha_1(M_G) = \alpha_2(M_G) \equiv \tilde{\alpha}_G$. Then we define $\epsilon_3 \equiv \frac{(\alpha_3(M_G) - \tilde{\alpha}_G)}{\tilde{\alpha}_G}$. A negative 4% correction at the GUT scale is sufficient to precisely fit the low energy data.

2 Nucleon Decay

In SUSY GUTs, nucleon decay is affected by dimension 4, 5 and 6 operators.

• For dimension 6 operators $\Lambda \approx M_G$ and for $M_G \sim 3 \times 10^{16}$ GeV we find $\tau_p \sim 10^{35\pm1}$ yrs. with the dominant decay mode $p \to \pi^0 + e^+$.

• Dimension 5 operators [8] are of the form $\frac{c^2}{M_T^{eff}}\left((Q\ Q\ Q\ L) + (U^c\ U^c\ D^c\ E^c)\right)$ or commonly described as L L L L + R R R R operators.

• Dimension 4 operators given by $(U^c\ D^c\ D^c) + (Q\ L\ D^c) + (E^c\ L\ L)$ are very dangerous [8]. Fortunately the symmetry R_p or R parity forbids all dimension 3 and 4 (and even one dimension 5) baryon and lepton number violating operators. It is thus a necessary ingredient of any "natural" SUSY GUT.

In summary, dimension 4, 5 and 6 operators may contribute to nucleon decay. The proton lifetime as a result of dimension 6 operators is very long due to the large value of

M_G [5]. Dimension 4 operators are necessarily forbidden by incorporating R parity. We are thus lead to consider dimension 5 operators which are the dominant contribution to nucleon decay in SUSY GUTs [9].

The proton decay amplitude depends on four main theoretical factors. We have

$$\begin{aligned} T(p \to K^+ + \bar{\nu}) \propto & \ \tfrac{c^2}{M_T^{eff}} (\text{Loop Factor}) \ (\text{RG}) \ \langle K^+ \bar{\nu} | qqql | p \rangle \\ & \sim \tfrac{c^2}{M_T^{eff}} (\text{Loop Factor}) \ (\text{RG}) \ \tfrac{\beta_{lattice}}{f_K} \ m_p \end{aligned} \tag{1}$$

where the latter equation results from a chiral Lagrangian analysis. Let's consider each factor in detail[10].

2.1 $\beta_{lattice}$

$\beta_{lattice}$ is a 3 quark, strong interaction matrix element between the vacuum and a nucleon. A recent lattice calculation gives[11]

$$\beta_{lattice} = \langle 0 | qqq | N \rangle = 0.015(1) \ \text{GeV}^3. \tag{2}$$

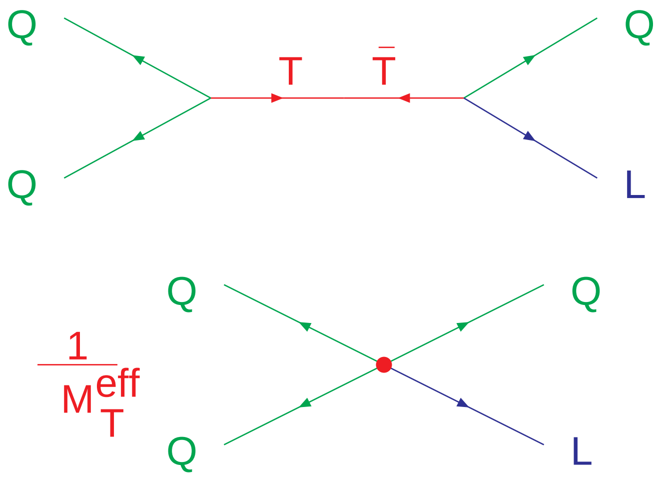
Without going into detail there is also an $\alpha_{lattice}$ satisfying $\alpha_{lattice} \approx -\beta_{lattice}$[11]. NOTE, in previous theoretical analyses, a conservative lower bound $\beta_{lattice} > 0.003\text{GeV}^3$ has been used to obtain an upper bound on the proton lifetime. The new lattice result is 5× larger than this "conservative lower bound." This has the effect of decreasing the upper bound on the proton lifetime by a factor of 25. The error on the lattice result represents statistical errors only. Systematic uncertainties due to quenching and using a chiral Lagrangian analysis may be as large as ±50 % (my estimate).

2.2 c^2

c^2 is a model dependent factor; calculable within a theory of fermion masses and mixing angles. Dimension 5 baryon and lepton number violating operators due to color triplet Higgsino exchange are derived from the superpotential

$$\begin{aligned} W \supset \quad & H_u \ Q Y_u \overline{U} + \quad H_d (Q Y_d \overline{D} + L Y_e \overline{E}) \\ & + T (Q \tfrac{1}{2} c_{qq} Q + \overline{U} c_{ue} \overline{E}) + \ \bar{T} (Q c_{ql} L + \overline{U} c_{ud} \overline{D}) \end{aligned} \tag{3}$$

obtained by integrating out the Higgs color triplets (T, $\bar{T}$) as in the figures below.



We then obtain

$$W \supset H_u\, Q\, Y_u\, \overline{U} + H_d\, (\, Q\, Y_d\, \overline{D} + L\, Y_e\, \overline{E}) \\ + \frac{1}{M_T^{eff}} \left[Q\, \frac{1}{2} c_{qq}\, Q\, Q\, c_{ql}\, L + \overline{U}\, c_{ud}\, \overline{D}\, \overline{U}\, c_{ue}\, \overline{E} \right]. \tag{4}$$

The matrix structure of the factor c^2 depends on a theory of charged fermion masses[12, 13, 14, 15, 16]. For example, in any realistic GUT model which fits charged fermion masses we have either, for SU_5, an effective superpotential term $W \supset \lambda(\langle\Phi\rangle)\ 10\ 10\ 5_H + \lambda'(\langle\Phi\rangle)\ 10\ \bar{5}\ \bar{5}_H$ or, for SO_{10}, $W \supset \lambda(\langle\Phi\rangle)\ 16\ 16\ 10_H$, where $\langle\Phi\rangle$ represents the vacuum expectation value [vev] of scalars in non-trivial GUT representations. These vevs are absolutely necessary in order to fix bad GUT mass relations as discussed below. As a consequence, the 3×3 Yukawa matrices $\{Y_u,\ Y_d,\ Y_e\}$ and the c matrices $\{c_{qq},\ c_{ql},\ c_{ud},\ c_{ue}\}$ are related by GUT symmetry relations. But, in general, $Y_u \neq c_{qq} \neq c_{ue}$ and $Y_d \neq Y_e \neq c_{ud} \neq c_{ql}$.

As noted above, the effective superpotential with scalar vevs $\langle\Phi\rangle$ are needed to correct bad GUT Yukawa relations. For example, consider the good GUT relation $\lambda_b = \lambda_\tau$. Assuming it also works for the first two families gives $\lambda_s = \lambda_\mu$ and $\lambda_d = \lambda_e$. Combining the two we find $20 \sim \frac{m_s}{m_d} = \frac{m_\mu}{m_e} \sim 200$ which is a bad mass relation.

In general we also have

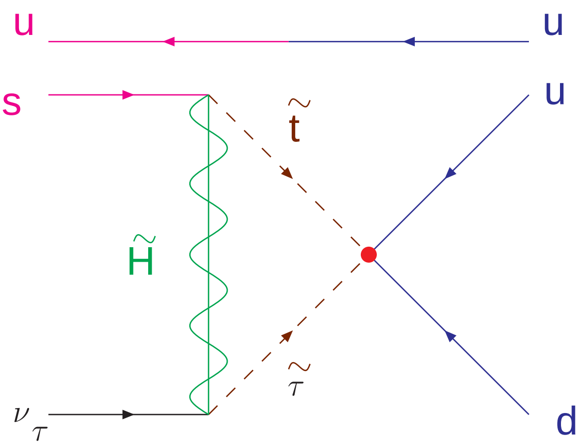
- $\{c_{qq}\ c_{ql},\ c_{ud}\ c_{ue}\} \propto m_u\ m_d\ \tan\beta$.

- If, for example in SO_{10}, we allow for Higgs in 16_H and 10_H which mix and in addition we have higher dimension operators such as $\frac{1}{M}(16\ 16\ 16_H\ 16_H)$ for neutrino masses then this can lead to new proton decay operators[14]. However this is not required for neutrino masses (see [13]).

- Finally, family symmetries affect the texture of $c_{qq},\ c_{ql},\ c_{ud},\ c_{ue}$.

2.3 Loop Factor

The Loop Factor depends on the squark, slepton and gaugino spectrum. For large $\tan\beta$, it has been shown that proton decay is significantly constrained by RRRR operators[12, 17, 18]. The dominant decay mode for the proton, $p \rightarrow K^+ + \bar{\nu}_\tau$, is given by the following graph.



This leads to a Loop Factor given approximately by $\frac{\lambda_t\,\lambda_\tau}{16\pi^2}\frac{\sqrt{\mu^2+M_{1/2}^2}}{m_{16}^2}$. Minimizing the Loop Factor requires taking the limit $\mu, M_{1/2}$ SMALL; m_{16} Large.

Is this limit reasonable and what about "Naturalness" constraints?? In partial answer to this question let's consider two additional motivations for being in this particular region of SUSY breaking parameter space.

An inverted scalar mass hierarchy [with heavy 1st & 2nd generation squarks and sleptons (>> TeV) and light 3rd generation scalars ($\leq$ TeV)] is useful for ameliorating the SUSY CP and flavor problems. Such a hierarchy can be obtained "naturally" via renormalization group running from M_G to M_Z if one assumes the following boundary conditions at the GUT scale[19]. One needs a universal A parameter A_0, gaugino mass $M_{1/2}$, Higgs mass m_{10} and squark and slepton masses m_{16}, consistent with SO_{10} boundary conditions. In addition they must satisfy the constraint: $A_0^2 = 2m_{10}^2 = 4m_{16}^2, \quad m_{16} >> 1$ TeV.

On the other hand we can assume SO_{10} Yukawa unification with $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} = \lambda$ at M_G and see if consistency with the low energy data constrains the SUSY breaking parameter space[20]. Good fits require the weak scale threshold correction to the bottom quark mass satisfy $\delta m_b/m_b = \Delta m_b^{\tilde{g}} + \Delta m_b^{\tilde{\chi}} + \Delta m_b^{\log} + \cdots < -2\%$ with $\Delta m_b^{\tilde{g}} \approx \frac{2\alpha_3}{3\pi}\frac{\mu m_{\tilde{g}}}{m_{\tilde{b}}^2}\, tan\beta$ and $\Delta m_b^{\tilde{\chi}^+} \approx \frac{\lambda_t^2}{16\pi^2}\frac{\mu A_t}{m_{\tilde{t}}^2}\, tan\beta$. Requiring $\mu > 0$, which is preferred by $b \rightarrow s\gamma$ and a_μ^{NEW}, we find that Yukawa unification is possible only in a narrow region of SUSY parameter space given by $A_0 \sim -1.9\, m_{16}$, $m_{10} \sim 1.35\, m_{16}$, $m_{16} > 1200$ GeV and $\mu, M_{1/2} \sim 100 - 500$ GeV. Moreover the fits improve as m_{16} increases. So from a completely independent perspective we find the same region of SUSY breaking parameter space. Perhaps there is something to this?

In this region of parameter space we find the predictions[20]

- $\tan\beta \sim 50$;
- $m_{\tilde{t}} << m_{\tilde{b}}$;
- $m_h \sim 114 \pm 5 \pm 3$ GeV, and
- $a_\mu^{SUSY} < 16 \times 10^{-10}$

2.4 M_T^{eff}

M_T^{eff} is an effective color triplet Higgs mass which is intimately connected to doublet-triplet splitting and GUT symmetry breaking. It may be constrained by requiring perturbative corrections to the prediction for gauge coupling unification[21, 17, 10, 14, 22, 15].

Recall the GUT threshold correction $\epsilon_3 \equiv \frac{(\alpha_3(M_G)-\tilde{\alpha}_G)}{\tilde{\alpha}_G} \sim -4\%$ needed to fit the low energy data. This correction has two main contributions given by $\epsilon_3 = \epsilon_3^{\text{Higgs}} + \epsilon_3^{\text{GUT breaking}} + \cdots$. The Higgs contribution, in minimal models, is of the form $\epsilon_3^{\text{Higgs}} = \frac{3\alpha_G}{5\pi}\ln\left(\frac{M_T^{eff}}{M_G}\right)$. In the following table, we list the values obtained from the GUT symmetry breaking sectors of the theory. Note, minimal SU_5 has a negligible contribution from the GUT breaking sector. Hence in order to fit the low energy data, a very low value of M_T^{eff} is needed. As a result, minimal SU_5 is excluded by proton decay[17, 22]. In the following figure, taken from the paper of Goto and Nihei[17], it is clear that minimal SU_5 is excluded by Super-Kamiokande bounds on the proton lifetime. Murayama and Pierce[22] have shown that this result cannot be saved by an inverted scalar mass hierarchy.
In the case of SU_5 with "natural" doublet-triplet splitting[15] and minimal SO_{10}[21], on the other hand, it is possible to get significant corrections from the GUT breaking sector. For SO_{10} we have taken a 10 % correction as an (albeit ad hoc) upper limit consistent with perturbativity[10].

Model	Minimal SU_5	SU_5 "Natural" D/T	Minimal SO_{10}
$\epsilon_3^{\text{GUTbreaking}}$	≈ 0	-7.7%	-10%
$\epsilon_3^{\text{Higgs}}$	-4%	$+3.7\%$	$+6\%$
M_T^{eff}[GeV]	2×10^{14}	3×10^{18}	6×10^{19}

Note, that in some cases the value of M_T^{eff} is very large. However in these cases, there is no particle with mass greater than M_G. By definition we have $\frac{1}{M_T^{eff}} = (M_T^{-1})_{11}$ where M_T is the Higgs color triplet mass matrix. In the cases of natural doublet-triplet splitting in SU_5 and minimal SO_{10} we have M_T schematically given by $M_T = \begin{pmatrix} 0 & M_G \\ M_G & X \end{pmatrix}$ with $\frac{1}{M_T^{eff}} \equiv \frac{X}{M_G^2}$. Hence for $X << M_G$ we obtain $M_T^{eff} >> M_G$ and the heaviest Higgs has mass of order M_G.

To summarize, given a realistic GUT with a small set of effective parameters at the GUT scale, we use the following procedure to evaluate the proton lifetime[10]. We first vary the parameters at the GUT scale – $\tilde{\alpha}_G$, M_G, the 3×3 Yukawa matrices Y_u, Y_d, Y_e

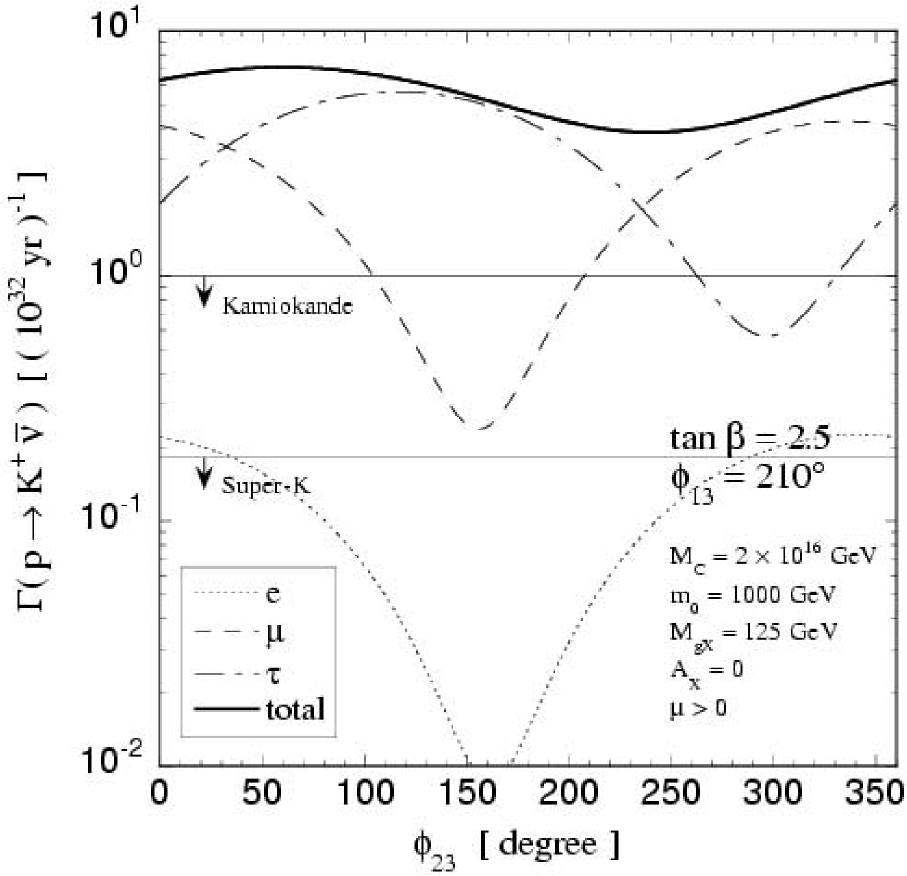


Figure 2: Decay rates $\Gamma(p \to K^+\overline{\nu}_i)$ ($i = e$, μ and τ) as functions of the phase ϕ_{23} for $\tan\beta = 2.5$. The other phase ϕ_{13} is fixed at 210°. The CKM phase is taken as $\delta_{13} = 90°$. We fix the soft SUSY breaking parameters as $m_0 = 1$ TeV, $M_{gX} = 125$GeV and $A_X = 0$. The sign of the supersymmetric Higgsino mass μ is taken to be positive. The colored Higgs mass M_C and the heavy gauge boson mass M_V are assumed as $M_C = M_V = 2 \times 10^{16}$ GeV. The horizontal lower line corresponds to the Super-Kamiokande limit $\tau(p \to K^+\overline{\nu}) > 5.5 \times 10^{32}$ years, and the horizontal upper line corresponds to the Kamiokande limit $\tau(p \to K^+\overline{\nu}) > 1.0 \times 10^{32}$ years.

28

and the soft SUSY breaking parameters until we FIT the precision electroweak data, *including fermion masses and mixing angles.*

As a result of this analysis, c_{qq}, c_{ql}, c_{ud}, c_{ue} at M_G are FIXED. We then renormalize the dimension 5 operators from $M_G \to M_Z$ within the MSSM; evaluate the Loop Factor at M_Z and renormalize the effective dimension 6 operator from $M_Z \to 1$ GeV. The latter gives the renormalization constant $A_3 \sim 1.3$ (NOT $A_L \sim .22$)[10]. Finally we calculate the decay amplitudes using a chiral Lagrangian approach or a direct lattice gauge calculation of the appropriate matrix elements. As discussed above:

- c^2 is model dependent but constrained by fermion masses and mixing angles.
- $\beta_{lattice}$ given by the recent JLQCD central value is 5 times larger than previous "conservative lower bound". The systematic uncertainties of quenching and chiral Lagrangian analyses need to be evaluated.
- Loop Factor $\propto \frac{\alpha}{4\pi} \frac{\sqrt{\mu^2 + M_{1/2}^2}}{m_{16}^2} \Longrightarrow$ Gauginos light; 1st & 2nd generation squarks and sleptons $>$ TeV and "naturalness" requires the stops, sbottoms and stau mass $<$ 1 TeV.
- Finally, M_T^{eff} is constrained by gauge coupling unification and the GUT breaking sectors.

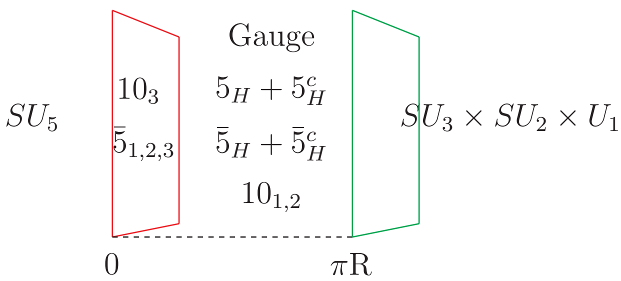
2.5 The Bottom Line

- The proton lifetime due to dimension 6 operators is in the range $\tau(p \to e^+ + \pi^0) \sim 10^{35 \pm 1}$ yrs.
- The "upper bound" from dimension 5 operators is roughly given by $\tau(p \to K^+ + \bar{\nu}) < (\frac{1}{3} - 3) \times 10^{34}$ yrs[10, 14, 15].
- In general, $\tau(n \to K^0 + \bar{\nu}) < \tau(p \to K^+ + \bar{\nu})$, and
- Other decay modes may be significant, eg. $p \to K^0 + \mu^+$, $\pi^0 + e^+$, but this is very model dependent.

Can we eliminate dimension 5 operators by symmetries? This is non-trivial in 4 dimensions, but possible[23]. It is however "natural" in extra dimensions with GUT symmetry breaking by orbifold boundary conditions[24].

3 5 Dim. SUSY GUTs

Let's consider one recent construction of a complete SU_5 SUSY GUT in five dimensions[25]. The picture below represents flat 3 + 1 dimensional end of the world branes separated by a fifth dimensional line segment which runs from 0 to πR. The fifth dimension is an orbifold which has an SU_5 gauge symmetry on the 0 Brane and in the bulk. However the symmetry on the πR Brane is only the standard model $SU_3 \times SU_2 \times U_1$. Quarks and leptons in the third family sit on the 0 Brane as indicated in the figure. On the other hand, the quarks and leptons in the first two families are partially on the 0 Brane and partially in the bulk. Finally the two Higgs doublets sit in the bulk.



In order to preserve the predictions of gauge coupling unification whereby bulk symmetry relations dominate over the broken symmetry on the πR Brane one must have a "large" extra dimension with a compactification scale M_C satisfying $M_C = \frac{1}{R} \sim 10^{15}$ GeV $<< M^*$ with $M^* \sim 10^{17}$ GeV, the cut-off scale. The three gauge couplings unify at M^*, but the baryon and lepton number violating gauge bosons obtain mass at M_C.

The resulting 5d theory has the following virtues:

- Threshold corrections to gauge coupling unification, coming from Kaluza-Klein modes with mass between M_C and M^*, are just right to give a good fit to the low energy data.
- The theory preserves the good Yukawa relation: $\lambda_b = \lambda_\tau$.
- The orbifold symmetry breaking results in "natural" doublet-triplet Higgs splitting with a conserved $U_1(R)$ symmetry.
- R parity (R_p) is a subgroup of $U_1(R)$ and is thus conserved.
- Dimension 5 baryon number violating operators are eliminated.

On the other hand,

- there is NO explanation of charge quantization, since weak hypercharge is not quantized on the πR Brane.
- Since the physical quarks and leptons, coming from bulk states, are derived from many different 10s the GUT explanation for families of quarks and leptons is lost.
- Proton decay due to dimension 6 operators may be enhanced. The decay amplitude depends on new effective operators with an unknown dimensionful coupling given by $\frac{\bar{g}}{M_C^2}$. Assuming $\bar{g} \sim 1$, we obtain a proton lifetime $\tau(p \to \pi^0 + e^+) \sim 10^{34}$ yrs.

4 Conclusions

4D SUSY GUTs are still very much alive.

- The proton lifetime due to dimension 6 operators is in the range $\tau(p \to e^+ + \pi^0) \sim 10^{35\pm1}$ yrs.
- The "upper bound" due to dimension 5 operators is given by $\tau(p \to K^+ + \bar{\nu}) < (\frac{1}{3} - 3) \times 10^{34}$ yrs.
- Suppressing proton decay due to dimension 5 operators requires light gauginos and heavy 1st & 2nd generation squarks and sleptons.

  This range of soft SUSY breaking parameters is consistent with SO_{10} Yukawa unification which also favors $m_h \sim 114 \pm 5 \pm 3$ GeV.

  It is also consistent with an inverted scalar mass hierarchy which ameliorates the SUSY CP and flavor problem.

  Finally for $m_{16} > 1200$ GeV, we find $a_\mu^{SUSY} < 16 \times 10^{-10}$.

SUSY GUTs in Extra Dimensions

- There are NO miracles in extra dimensions.
- Dimension 5 operators can easily be eliminated in extra dimensions *at the expense of NOT understanding charge quantization.*
- In 5D GUTs, we can have "natural" doublet-triplet Higgs splitting and a conserved R_p.
- Finally, proton decay due to dimension 6 operators may be enhanced giving $\tau(p \to \pi^0 + e^+)$ roughly of order 10^{34} yrs.

In conclusion, SUSY GUTs in four or even higher dimensions lead to proton decay rates which may easily be observable in a future proton decay experiment. Moreover an observation of proton and/or neutron decay would provide a tantalizing window to new physics "way" beyond the standard model.

References

[1] Kearns, Snowmass 2001, `http://hep.bu.edu/`

[2] J.C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974); H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[3] H. Georgi, Particles and Fields, Proceedings of the APS Div. of Particles and Fields, ed C. Carlson, p. 575 (1975); H. Fritzsch and P. Minkowski, Ann. Phys. **93**, 193 (1975).

[4] H. Georgi, H.R. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974); S. Weinberg, Phys. Lett. **B91**, 51 (1980).

[5] S. Dimopoulos, S. Raby, and F. Wilczek, *Phys. Rev.* **D24**, 1681 (1981).

[6] S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193**, 150 (1981); L. Ibanez and G.G. Ross, *Phys. Lett.* **105B**, 439 (1981); N. Sakai, *Z. Phys.* **C11**, 153 (1981); M. B. Einhorn, and D. R. T. Jones, *Nucl. Phys.* **B196**, 475 (1982); W. J. Marciano and G. Senjanovic, *Phys. Rev.* **D 25**, 3092 (1982).

[7] U. Amaldi, W. de Boer, and H. Fürstenau, *Phys. Lett.* **B260**, 447 (1991); J. Ellis, S. Kelly, and D.V. Nanopoulos, *Phys. Lett.* **B260**, 131 (1991); P. Langacker and M. Luo, *Phys. Rev.* **D44**, 817 (1991); P. Langacker and N. Polonsky, *Phys. Rev.* **D47**, 4028 (1993); M. Carena, S. Pokorski, and C.E.M. Wagner, *Nucl. Phys.* **B406**, 59 (1993); see also the review by S. Dimopoulos, S. Raby and F. Wilczek, Physics Today, 25–33, October (1991).

[8] S. Weinberg, *Phys. Rev.* **D26**, 287 (1982); N. Sakai and T Yanagida, *Nucl. Phys.* **B197**, 533 (1982).

[9] N. Sakai and T Yanagida, *Nucl. Phys.* **B197**, 533 (1982); S. Dimopoulos, S. Raby and F. Wilczek, *Phys. Lett.* **112B** 133 (1982); J. Ellis, D.V. Nanopoulos, and S. Rudaz, *Nucl. Phys.* **B202** 43 (1982).

[10] R. Dermíšek, A. Mafi, and S. Raby, Phys. Rev. **D63**, 035001, (2001).

[11] S. Aoki et al., Phys. Rev. **D62**, 014506 (2000).

[12] V. Lucas and S. Raby, Phys. Rev. **D55**, 6986 (1997).

[13] T. Blažek, S. Raby, and K. Tobe, Phys. Rev. **D60**, 113001 (1999); ibid., Phys. Rev. **D62**, 055001 (2000); R. Dermíšek and S. Raby, Phys. Rev. **D62**, 015007 (2000).

[14] K.S. Babu, J.C. Pati, and F. Wilczek, Nucl. Phys. **B566**, 33 (2000); J.C. Pati, hep-ph/0204240.

[15] G. Altarelli, F. Feruglio, and I. Masina, *JHEP* **0011**, 040 (2000).

[16] C.H. Albright and S.M. Barr, *Phys. Rev. Lett.* **85**, 244 (2000).

[17] T. Goto and T. Nihei, Phys. Rev. **D59**, 115009 (1999).

[18] K.S. Babu and M.J. Strassler, hep-ph/9808447.

[19] J. Bagger, J. Feng, N. Polonsky, and R. Zhang, Phys. Lett. **B473**, 264 (2000).

[20] T. Blažek, R. Dermíšek, and S. Raby, Phys. Rev. Lett. **88**, 111804 (2002), Phys. Rev. **D65**, 115004 (2002).

[21] V. Lucas and S. Raby, Phys. Rev. **D54**, 2261 (1996).

[22] H. Murayama and A. Pierce, Phys. Rev. **D65**, 055009 (2002).

[23] K.S. Babu and S.M. Barr, Phys. Rev. **D65**, 095009 (2002).

[24] Y. Kawamura,*Prog. Theor. Phys.* **105** 999 (2001); L.J. Hall and Y. Nomura, Phys. Rev. **D64**, 055003 (2001), Phys. Rev. **D65**, 125012 (2002); A. Hebecker and J. March-Russell, Nucl. Phys. **B613**, 3 (2001), Phys. Lett. **B539**, 119 (2002); R. Dermíšek and A. Mafi, Phys. Rev. **D65**, 055002 (2002); G. Altarelli and F. Feruglio, Phys. Lett. **B511**, 257 (2001).

[25] L.J. Hall and Y. Nomura, Phys. Rev. **D66**, 075004 (2002).