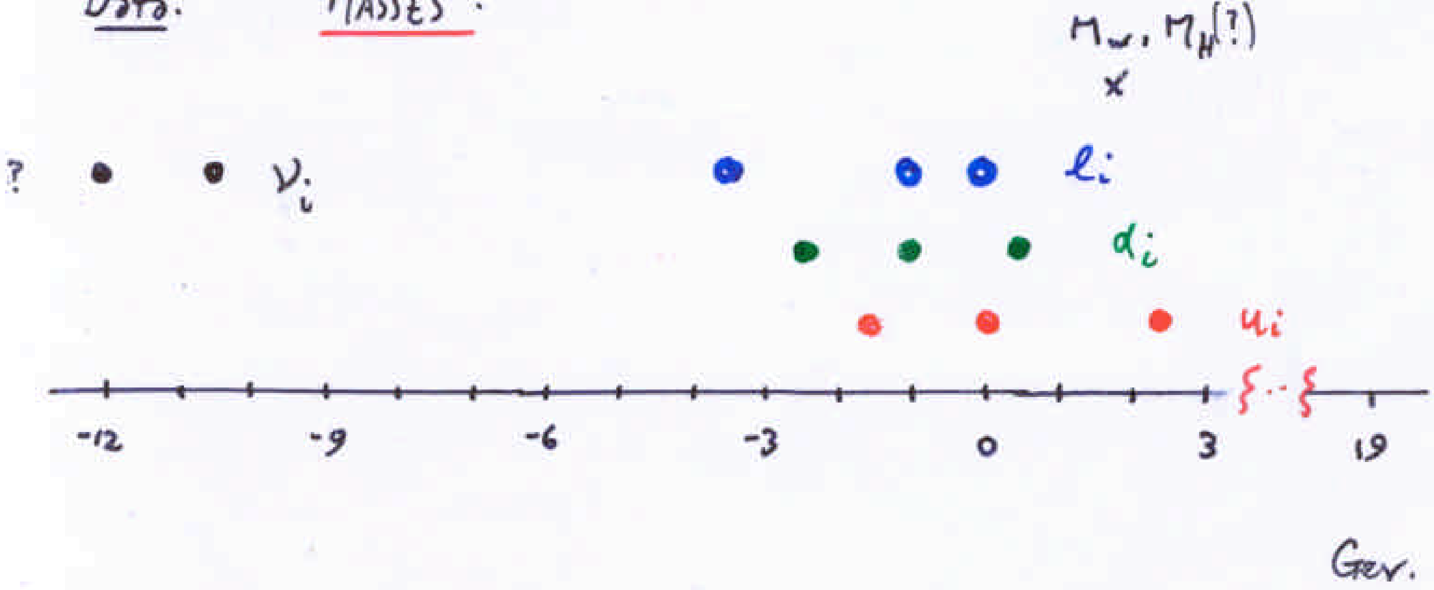
Quark and lepton mass textures

G.Ross, DESY, June 02

Data: MASSES :



MIXING :

$$V_{CKM} \approx \begin{pmatrix} 1 & 0.218-0.224 & 0.002-0.005 \\ 0.218-0.224 & 1 & 0.032-0.048 \\ 0.004-0.015 & 0.03-0.048 & 1 \end{pmatrix}$$

MASS MATRICES

$$\mathcal{L}^{mass} = h^u_{ij}\, \bar{\psi}_{iL}\, u_{jR} \langle H^0 \rangle + h^d_{ij}\, \bar{\psi}_{iL}\, d_{jR} \langle H^d \rangle$$

$$= \bar{\psi}_L\, m^u\, u_R + \bar{\psi}_L\, m^d\, d_R$$

$$\boxed{\begin{aligned} m^u &= V_L^\dagger\, \underline{m}^u_{Diag}\, V_R \\ m^d &= U_L^\dagger\, \underline{m}^d_{Diag}\, U_R \end{aligned} \qquad \underline{V_{CKM}} \equiv V_L^\dagger U_L .}$$

Mass matrix determination

Not democratic : but see Fritzsch, Taixeira

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$$

$$V_{23} \simeq \frac{m_{23}}{m_{33}} + \frac{m_{32}}{m_{33}} \quad \frac{m_s}{m_b}$$

$$V_{12} \simeq \frac{m_{12}}{\tilde{m}_{22}} + \frac{m_{21}}{\tilde{m}_{22}} \quad \frac{m_d}{m_s}$$

$$V_{13} \simeq \frac{m_{13}}{m_{33}} + \frac{m_{31}}{m_{33}} \quad \frac{m_d}{m_b}$$

Symmetric fit

Roberts, Romanino, GGR, Velasco-Sevil

$$\frac{m^d}{m_b} = \begin{pmatrix} 0 & 1.5\,\varepsilon^3 & 0.4 e^{i20^\circ} \varepsilon^3 \\ 1.5\,\varepsilon^3 & \varepsilon^2 & 1.3\,\varepsilon^2 \\ 0.4 e^{i20^\circ} \varepsilon^3 & 1.3\,\varepsilon^2 & 1 \end{pmatrix} \qquad \varepsilon = 0.15$$

$$\frac{m^u}{m_t} = \begin{pmatrix} 0 & \varepsilon'^3 & ?\,\varepsilon'^3 \\ \varepsilon'^3 & \varepsilon'^2 & ?\,\varepsilon'^2 \\ ?\,\varepsilon'^3 & ?\,\varepsilon'^2 & 1 \end{pmatrix} \qquad \varepsilon' = 0.05$$

Asymmetric fit

$$\frac{m^d}{m_b} = \begin{pmatrix} 0 & 1.7\,\varepsilon^3 & 0 \\ 1.7\,\varepsilon^3 & 0 & 5\,\varepsilon^2 \\ 0 & 0.3 & 1 \end{pmatrix} \qquad \frac{m_u}{m_t} = \begin{pmatrix} 0 & 2\,\varepsilon'^3 & 0 \\ 2\,\varepsilon'^3 & 0 & 2\,\varepsilon' \\ 0 & 0.6\,\varepsilon' & 1 \end{pmatrix}$$

SUSY Textures

V_{ub}

m_d V_{us}

V_{cb}

$$\frac{M^d}{m_b} = \begin{pmatrix} \leq \varepsilon^4 & \varepsilon^3 & \leq \varepsilon^3 \\ ? & \varepsilon^2 & \varepsilon^2 \\ ? & ? & 1 \end{pmatrix}$$

m_s

SUSY Textures : FCNC, $\cancel{CP}$ constraints

$(m_d, \Delta S \neq 0 \; \Delta Q = 0)$

$$\frac{M^d}{m_b} = \begin{pmatrix} \leq \varepsilon^4 & \varepsilon^3 & \leq \varepsilon^3 \\ \leq \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \leq \varepsilon^2 & \leq \varepsilon^2 & 1 \end{pmatrix}$$

$\Delta S \neq 0 \; \Delta Q = 0$

$(A_{\alpha\beta\gamma})$

$\cancel{CP}$

$(A_{\alpha\beta\gamma})$

Abel, Kahil, Lebedev
Servant

SUGRA ×
Gauge mediation ?
but Froggatt Nielsen × (?!)

VIVES, GGR ; Masiero, Murayama
Gabbiani, Gabrielli, Masiero, Silvestrini
Hisano, Moroi, Tobe, Yamaguchi, Nomura.
Feng, Nir, Shadmi
Lavignac, Masina, Savoy
Davidson, Ibarra
Babu, Dutta, Mohapatra
...

$\mu \to e\gamma$
$\tau \to \mu$

SUSY Textures

GATTO, SARTORI, TONIN

$$\frac{M^d}{m_b} = \begin{pmatrix} 0 & \varepsilon^3 & \leq \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \leq \varepsilon^2 & \leq \varepsilon^2 & 1 \end{pmatrix}$$

$\leq \varepsilon^5$

CP SM phase

Fritzsch, Weinberg

$$\boxed{\sin\theta_C = \sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}}}$$

$$0.217 - 0.222 \quad c.f. \; |(0.216 - 0.214) - (0.07 - 0.076)e^{i\delta}|$$

$$= 0.213 - 0.223, \; \delta \approx 90^0$$

Extension to charged leptons

$$Det(M^l) = Det(M^d)|_{M_X}$$

$$\frac{M^l}{m_\tau} = \begin{pmatrix} 0 & \varepsilon^3 & ?\varepsilon^3 \\ \varepsilon^3 & 3\varepsilon^2 & ?\varepsilon^2 \\ ?\varepsilon^2 & ?\varepsilon^2 & 1 \end{pmatrix}$$

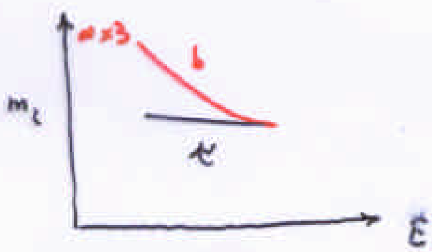
$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

$$\frac{m_d}{m_e}(M_X) = 3$$

Georgi Jarlskog

$$\frac{m_b}{m_\tau}(M_X) = 1$$

Buras, Ellis, Gaillard, Nanopoulos



Symmetries and textures

Hierarchical structure strongly suggests a broken symmetry

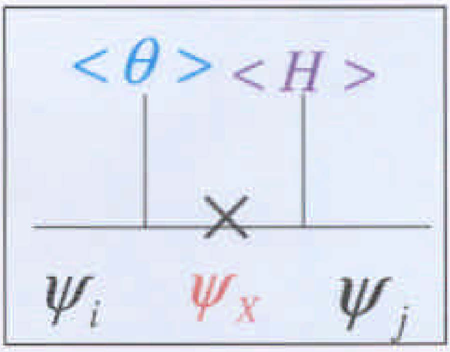
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{<\theta>\neq 0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & a\varepsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$a = O(1), \quad \varepsilon = \frac{<\theta>}{M}$$

SYMMETRY?

Abelian, Non-Abelian

MESSENGER SECTOR?



$$m_{ij} = \frac{<\theta><H>}{M_X}$$

Froggatt, Nielsen

Abelian Family Symmetry $\tilde{U}(1)$

$$\begin{array}{c} \\ \dfrac{\bar{\psi}_R M^d \psi_L}{m_b} = \end{array} \begin{array}{c} 4 \quad\; 1 \quad -5 \\ (\bar{d}_L \;\; \bar{s}_L \;\; \bar{b}_L) \end{array} \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon & 1 & 1 \end{pmatrix} \begin{array}{cc} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} & \begin{array}{c} 2 \\ -1 \\ -1 \end{array} \end{array}$$

? ?

Elwood, Irges, Ramond

$$\varepsilon = \frac{\overset{3}{\langle\theta\rangle}\;\overset{6}{\langle H\rangle}}{M_X}$$

- Anomalous $\tilde{U}(1)$ $\quad D: \; (\xi - g\theta^\dagger\theta + \dots)^2$

+ Green-Schwarz anomaly cancellation — Ibanez.

$(g_i \rightleftharpoons Q_i)$

$$U_{MNS} = \begin{pmatrix} 1 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & 1 & 1 \\ \varepsilon^3 & 1 & 1 \end{pmatrix}$$

} Large mixing due to equal family charges (S…

$\tilde{U}(1)$ horizontal symmetries

Froggatt, Nielsen
Hall, Randall,
Dine, Leigh, Kagan
Nir, Seiberg,
Pouliot, Seiberg
Leurer, Nir, Seiberg
Kaplan, Schmaltz,
Pomarol, Tommasini
Hall, Murayama,
Frampton, Kong,
Dudas, Pokorski, Savoy
Carone, Hall, Murayama
Barbieri, Dvali, Hall,
Kawamura, Murayama, Yamaguchi
Arkani-Hamed, Cheng, Hall
Barbieri, Hall,
Blazek, Dermisek, Raby
Anderson, Raby, Dimopoulos, Hall, Starkman
Binetruy, Lavignac, Ramond
Berezhiani, Tavartkiladze
Jezabek, Sumino
Grossman, Nir, Shadmi
Buchmuller, Yanagida
Froggatt, Gibson, Nielsen
Choi, Hwang, Chun
Lola, Ross
Nir, Shadmi,
Altarelli, Feruglio, Masina
Jack, Jones, Wild
King, Oliveira
Tanimoto ; Steel
Bando, Maekawa
Nielsen, Takanishi
Kane, Everett, King . . . . .

Abelian Family symmetry $\tilde{U}(1)$

$$\frac{\bar{\psi}_R M^d \psi_L}{m_b} = \begin{array}{ccc} -3 & 2 & 1 \\ (\bar{d}_R & \bar{s}_R & \bar{b}_R) \end{array} \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^4 \\ \varepsilon^3 & \varepsilon^2 & \frac{1}{2}\varepsilon \\ \varepsilon^4 & \frac{1}{2}\varepsilon & 1 \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \begin{array}{c} -3 \\ 2 \\ 1 \end{array}$$

$$O(\varepsilon^8)$$

$$\Uparrow$$

$$<\theta> = <\bar{\theta}>$$

$$\varepsilon = \frac{<\overset{1}{\theta}><\overset{-1}{H}>}{M_X} = \frac{<\overset{-1}{\bar{\theta}}><\overset{-1}{H}>}{M_X}$$

- Anomaly cancellation ⇒ $\sin^2\theta_w = \frac{3}{8}$

Ibanez GGR

- Origin of charges ... strings ?

Cvetic, Everett, Langacker
Faraggi ...

Non-Abelian family symmetry

- Maximal $U(3)^5$
- GUTs, eg $SO(10)$, $\longrightarrow U(3)$.
- F.C.N.C. $m^2_{\tilde{u}_L}(\tilde{u}^2_L + \tilde{c}^2_L + \tilde{t}^2_L) + m^2_{\tilde{u}_R}(\tilde{u}^2_R + \tilde{c}^2_R + \tilde{t}^2_R)$

$$+ m^2_{\tilde{d}_L}(\tilde{d}^2_L + \tilde{s}^2_L + \tilde{b}^2_L) + m^2_{\tilde{d}_R}(\tilde{d}^2_R + \tilde{s}^2_R + \tilde{b}^2_R) \checkmark$$

(but beware D-terms)

⇨ Expect $U(3)$ to be strongly broken to $U(2)$ ...

still good for $\Delta S \neq 0, \Delta Q = 0$ processes

⇨ Full $U(3)$ Structure may be of interest

(~ near bi-maximal neutrino mixing)

eg $\begin{pmatrix} u \\ d \end{pmatrix}_{L,i}$ $u^c_{L,i}$ $d^c_{L,i}$ $\begin{pmatrix} \nu \\ e \end{pmatrix}_{L,i}$, $e^c_{L,i}$, $\nu^c_{L,i}$ $\quad$ $SU(3)$: 3

$$\boxed{P = d^c_{L_i} \underbrace{\phi^i_3}_{\begin{pmatrix}0\\0\\1\end{pmatrix}} \phi^i_3 \psi_{L,i} + d^c_i \underbrace{\phi^i_{23}}_{\begin{pmatrix}0\\1\\1\end{pmatrix}\epsilon} \phi^i_{23} \psi_{i,i} + \epsilon^{ijk} d^c_{L_i} \psi_{L_j} \phi^j_{23} (\phi^k_{23} \ldots}$$

$+ d^c_L \ldots$

King - GGR

$v(2)$: Barbieri, Geminelli, Romin.
Barbieri, Hall, Romanino
Barbieri, Hall, Raby, Romanino
Blazek, Raby, Tobe.
Berezhiani, Rossi
Aranda, Carone, Lebed.
Ramond.
...

$v(3)$: Chkareuli
Berezhiani, Chkareuli
Berezhiani, Rossi
Anselm, Berezhiani
Soldate, Rao, Hill
Koide, Oneda
Kitano, Mimura
...

Mass matrices.

$$\frac{m^d}{m_b} = \begin{pmatrix} 0 & -\varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}$$

$$m^u \longleftrightarrow m^d \quad ; \quad \varepsilon \longleftrightarrow \varepsilon'$$

Vacuum alignment

$$\langle \phi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ a_3 \end{pmatrix} \qquad \langle \bar{\phi}_3 \rangle = (0, 0, a_3)$$

$$\langle \bar{\phi}_2 \rangle = (0, a_2, 0)^\dagger$$

$$P \supset Y(\phi_{23}\bar{\phi}_2 \, \phi_{23}\bar{\phi}_3 - \mu^4) + Z(\phi_{23}\bar{\phi}_{23}) + X \phi_3 \phi_2^{+\dagger}$$

$$V = \sum_i \left| \frac{\partial P}{\partial \phi_i} \right|^2 + m_{23}^2 (\phi_{23,1}^2 + \phi_{23,2}^2 + \phi_{23,3}^2)$$

$$\langle \phi_3 \rangle = \begin{pmatrix} 0 \\ b \\ b \end{pmatrix} \qquad \bar{\phi}_3 = (0, b, -b)$$

Textures and symmetry

Liliana Velasco-Sevilla,

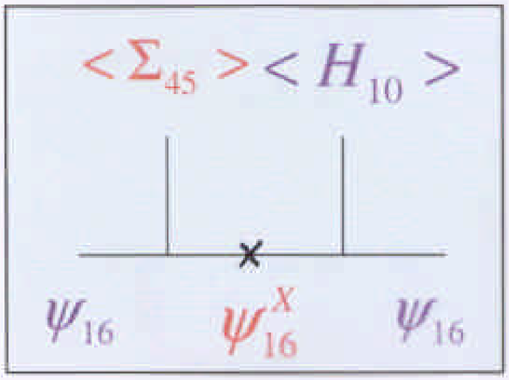
$SU(3)_f \otimes GUT \otimes SU(2)_R$ ⇨ M^d, M^u, M^e, M^ν

$$\frac{M^d}{m_b} = \begin{pmatrix} \varepsilon^8 & \varepsilon^3(z+(x+y)\varepsilon) & \varepsilon^3(z+(x-y)\varepsilon \\ -\varepsilon^3(z+y\varepsilon) & \varepsilon^2(w+u\varepsilon) & \varepsilon^2(w-u\varepsilon) \\ -\varepsilon^3(z\pm y\varepsilon) & \varepsilon^2(w-u\varepsilon) & 1 \end{pmatrix}$$

$GUT \quad q \leftrightarrow l \quad :$

$$\frac{M^l}{m_\tau} = \begin{pmatrix} \varepsilon^8 & \varepsilon^3(z+(x+y)\varepsilon) & \varepsilon^3(z+(x-y)\varepsilon \\ -\varepsilon^3(z+y\varepsilon) & a\varepsilon^2(w+u\varepsilon) & a\varepsilon^2(w-u\varepsilon) \\ -\varepsilon^3(z+y\varepsilon) & a\varepsilon^2(w-u\varepsilon) & 1 \end{pmatrix}$$

$$SU(2)_R \quad : \quad \underbrace{M^d \leftrightarrow M^u \quad M^l}_{\neq} \leftrightarrow M^\nu \quad \varepsilon \leftrightarrow \varepsilon'$$



$$<\Sigma_{45}> \propto (B-L) + k\,T_3^R$$

Barbieri, Hall, Raby, Romanino

➡ $k=0 \quad a_l = a_\nu = -3$

➡ $k=2 \quad a_l = 3 \quad a_\nu = 0$

Near bi-maximal mixing

$\varepsilon' = 0.05$

$$\frac{M_D^\nu}{m_3} = \begin{pmatrix} \varepsilon'^8 & \varepsilon'^3(z+(x+y)\varepsilon') & \varepsilon'^3(z+(x-y)\varepsilon') \\ -\varepsilon'^3(z+y\varepsilon') & a_\nu \varepsilon'^2(w+u\varepsilon') & a_\nu \varepsilon'^2(w-u\varepsilon') \\ -\varepsilon'^3(z \cancel{+} - y\varepsilon') & a_\nu \varepsilon'^2(w-u\varepsilon') & 1 \end{pmatrix}$$

Majorana Mass $\quad \varepsilon_M << \varepsilon'$

$$M_M \approx \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} \qquad m_1 << m_2 << m_3$$

"See Saw" $\quad M_\nu = \; M_D^{\nu\,T} M_M^{-1} M_D^\nu$

Gell-Mann
Ramond
Slansky
Yanagida.

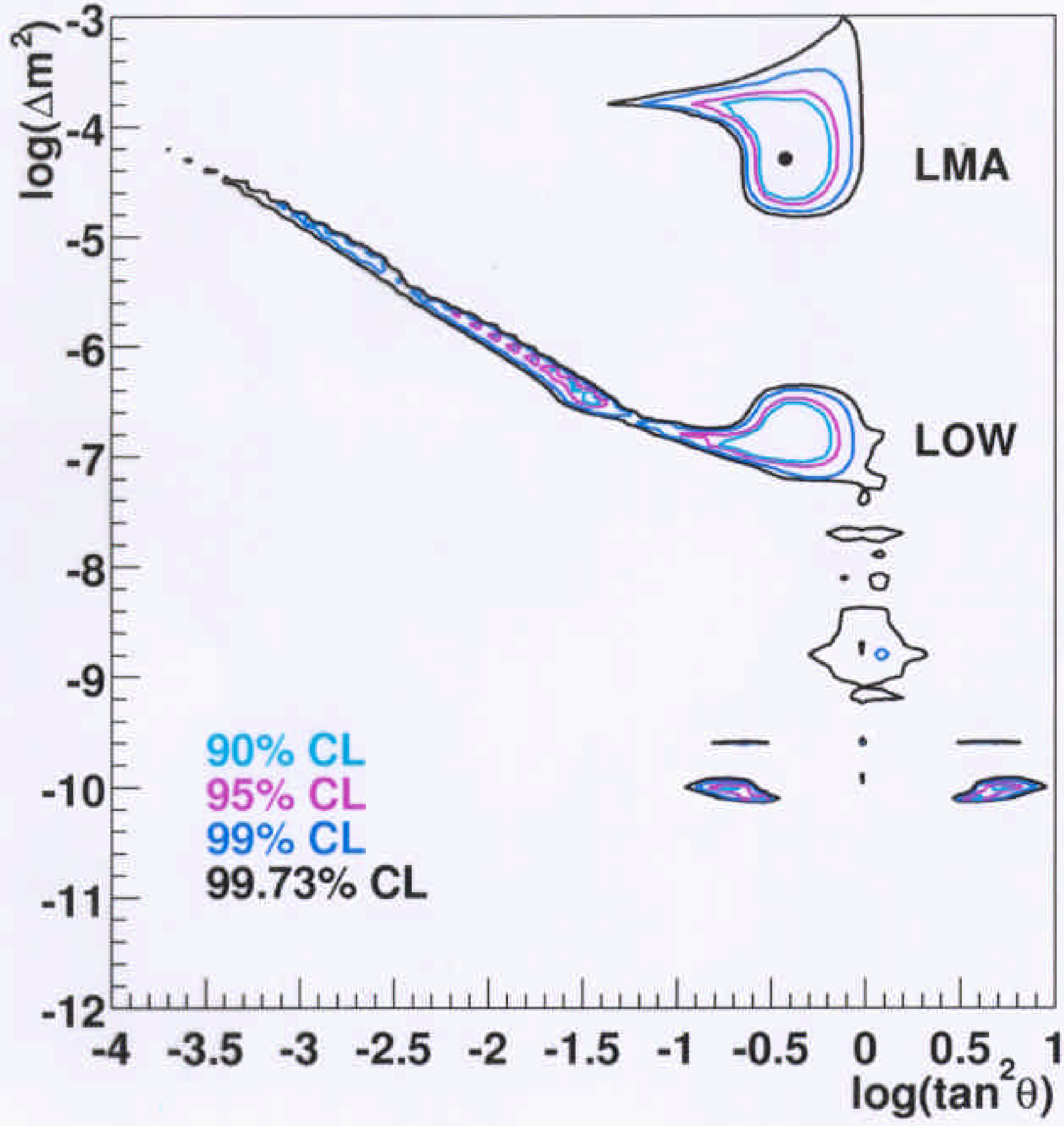
$$\nu_a \propto \quad z(\nu_\mu + \nu_\tau) + y\varepsilon'(\nu_\mu - \nu_\tau)$$

$\frac{1}{m_1}$ domin

$$\nu_b \propto \quad z\nu_e + r(\nu_\mu - \nu_\tau - y\varepsilon'(\nu_\mu + \nu_\tau)/z)$$

$\frac{1}{m_2}$ next

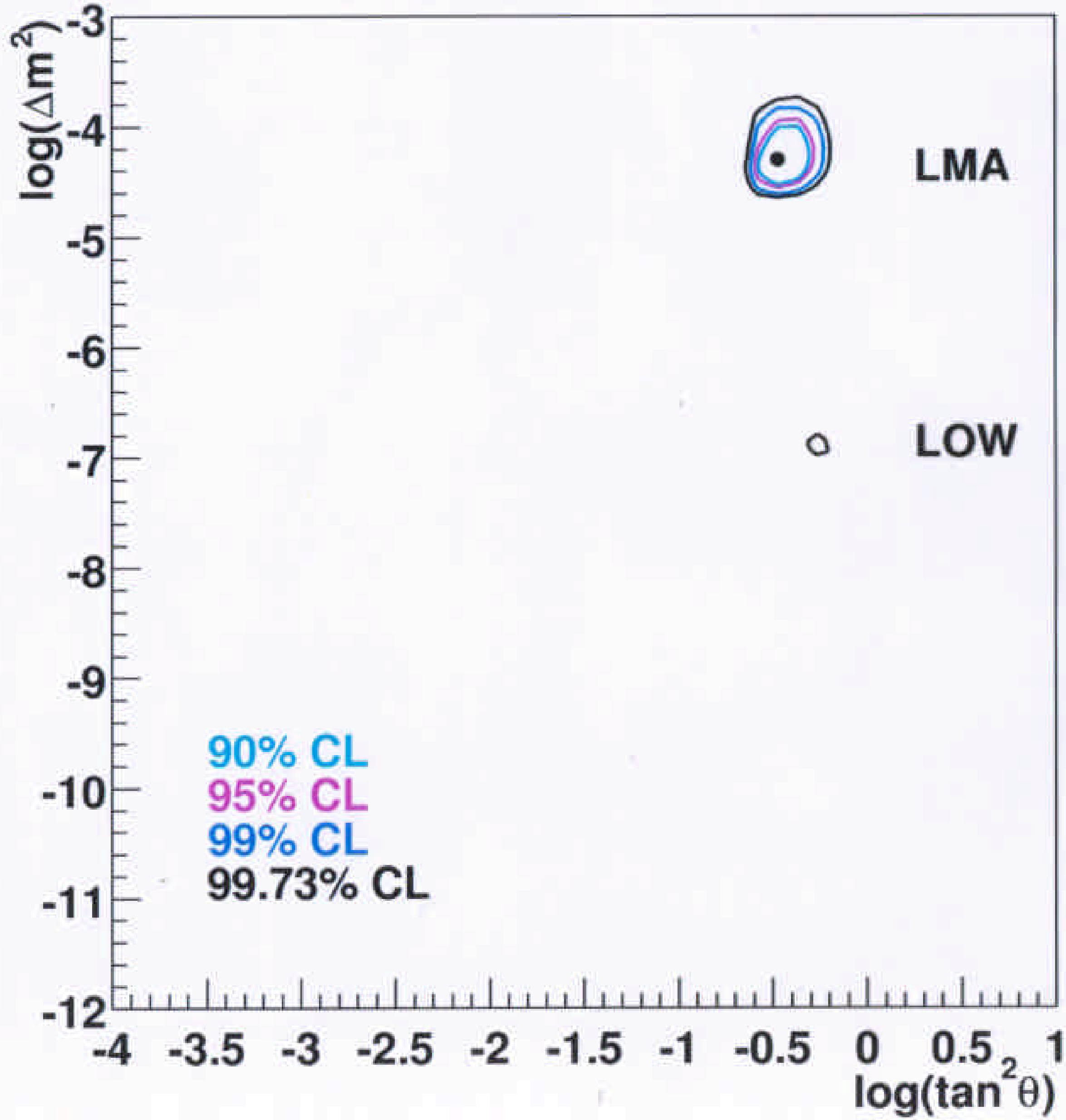
$$z, \quad r = \sqrt{2}(u - a_\nu wy/z) \quad = O(1)$$



log(Δm²)
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
log(tan²θ)
LMA
LOW
90% CL
95% CL
99% CL
99.73% CL

LATEST FROM SNO

(via S. Biller & N. Jelley)



k : 0

LOW solution

z	0.69 ± 0.046
x	$-(1.98 \pm 0.07)$
y	0.73 ± 0.07
ϕ_x	0.07 ± 0.01
ϕ_y	0.19 ± 0.01
ω	0.56 ± 0.05
u	$-(0.55 \pm 0.12)$
$\bar{\epsilon}$	0.21 ± 0.01
$\epsilon(M_X)$	0.067 ± 0.012
$\epsilon(M_Z)$	0.096 ± 0.017

Table 4:

	LMA	LOW	VAC
$\tan^2\theta_{12}$	0.16	0.71 †	2.37
$\left\lvert\frac{M_2}{M_3}\right\rvert^2 <$	$(0.8 \pm 0.5) \times 10^{-3}$	$(13 \pm 9) \times 10^{-5}$	$(6 \pm 4)10^{-5}$
$\frac{\lvert\Delta m^2_{12}\rvert}{\lvert\Delta m^2_{23}\rvert}_{\mathrm{exp}}$	$(1.6, 71) \times 10^{-3}$	$(0.82, 1.8) \times 10^{-5}$	$(0.57, 4.01) \times 10^{-7}$

Table 5:

$$t_{23}^2 = 1.84 \pm 0.1.$$

$$\begin{aligned} \lvert (U_{\mathrm{MNS}})_{13} \rvert &= \lvert -(s^l_{13} c^{L\prime}_{23} e^{i\phi_{sl}} - s^{\nu}_{13} c^{l}_{13} e^{i(\gamma^{L\prime}_{12} - \gamma^{\nu\prime}_{12})}) + s^{l}_{12} s^{L\prime}_{23} e^{-i(\gamma^{\nu\prime}_{13} - \gamma^{\nu\prime}_{12})} \rvert \\ &= 7.10^{-2} \end{aligned}$$

† cf $(4-10).10^{-1}$

LMA solution

$k=2.$

Parameters
$z = 0.78 \pm 0.14$
$x = -1.86 \pm 0.51$
$\phi_x = 0.14 \pm 0.04$
$y = 1.37 \pm 0.21$
$\phi_y = 0.19 \pm 0.05$
$w = 0.77 \pm 0.06$
$u = -0.87 \pm 0.15$
$\bar{\epsilon} = 0.18 \pm 0.02$
$t_{23}^2 = 0.67 \pm 0.05$ ✓
$\frac{b'}{b} = 1.1 \pm 0.1$ … H^0 ✓
$\frac{M_2^2}{M_3^2} = 3.1 \frac{m_1^2}{m_2^2}$ ✓

$t_{12}^2 = \underline{4.2 \times 10^{-1}}$ + fit.

LMA, solar information					
Δm_{12}^2 (eV)^2		t_{12}^2		g.o.f.	Ref.
BFP	$at 3\sigma$ %CL	BFP	$at 3\sigma$%CL		
5.5×10^{-5}	$(0.23, 3.7) \times 10^{-4}$	4.2×10^{-1} +	$(2.4, 8.9) \times 10^{-1}$	49%	[?]
5.6×10^{-5}	$(0.22, 2.2) \times 10^{-4}$	0.39×10^{-1}	$(2.0, 6.4) \times 10^{-1}$		[?]
6.2×10^{-5}	$(0.23, 3.4) \times 10^{-5}$	0.4×10^{-1}	$(2.3, 7.9) \times 10^{-1}$	84%	[?]
	$at 1\sigma \% CL$		$at 1\sigma \% CL$		
4.5×10^{-5}	$(3.1, 7.2) \times 10^{-1}$	0.4×10^{-1}	$(3.2, 4.8) \times 10^{-1}$		[?]

$(U_{MNS})_{13} = 7 . 10^{-2}$

SUMMARY

The new data for both the quark and lepton sectors is testing our ideas for the origin of the fermion masses and mixings and raises interesting questions :

- Is there a family symmetry generating the hierarchy?

  U(1)? SU(3)? ..

- Is there a connection between quarks and leptons?

  SU(5), SO(10), strings?

- Is there an extension of the flavour symmetry?

  $SU(2)_R$?

  $(+GUTs \;:\; \theta^q \Rightarrow \theta^l, \quad M^d \Rightarrow M^\nu)$

- FCNC, CP

  Processes related to family structure

  Observable signals close