Quark and lepton mass textures

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Abstract

We review ideas for explaining the pattern of fermion masses and mixing angles based on a spontaneously broken family symmetry.

1 The form of the quark mass matrices.

The fundamental parameters of the Standard Model or its supersymmetric extension that are responsible for fermion masses and mixings are the Yukawa couplings or,equivalently, the fermion mass matrix. Unfortunately we do not have sufficient experimental information to construct the full mass matrices because we only measure the mass eigenvalues and the CKM matrix. The latter is the combination $V^{u\dagger}V^d$ of the unitary matrices which express the left-handed current quark eigenstates in terms of the mass eigenstates. We do not know V^u or V^dseparately and we have no information about the right-handed matrices needed to diagonalise the mass matrix. As a result our knowledge of the full mass matrices is limited. A fit to all the data, including the new data coming from the $b-$factories, assuming that small CKM mixing angles implies small mixing angles in both the up and the down mass matrices, gives [1]

$$\frac{M^u}{m_t} = \begin{pmatrix} 0 & b'\epsilon^3 & c'\epsilon^3 \\ ? & \epsilon^2 & a'\epsilon^2 \\ ? & ? & 1 \end{pmatrix} \tag{1}$$

and

$$\frac{M^d}{m_b} = \begin{pmatrix} \leq \bar{\epsilon}^4 & b\bar{\epsilon}^3 & c\bar{\epsilon}^3 \\ ? & \bar{\epsilon}^2 & a\bar{\epsilon}^2 \\ ? & ? & 1 \end{pmatrix} \tag{2}$$

The parameters below the diagonal are only weakly constrained because, as discussed above, measurement of the quark masses and the CKM matrix does not provide enough information to determine the full quark mass matrices. The parameters of the up quark mass matrix are given by $\epsilon = 0.05$, $b' \simeq 1$ while a' and c' are very weakly constrained. The parameters of the down quark mass matrix are much better determined with

$$\bar{\epsilon} = 0.15 \pm 0.01 \quad |b| = 1.5 \pm 0.1 \quad a = 1.31 \pm 0.14 \tag{3}$$

$$|c| = 0.4 \pm 0.05 \quad \psi = -24^0 \pm 3^0 \quad \text{or} \tag{4}$$

$$|c| = 0.9 \pm 0.05 \quad \psi = 60^0 \pm 5^0 \tag{5}$$

where $c = |c|\,e^{i\psi}$. The $(2,2)$ matrix element is mainly determined by m_s, while the $(1,2)$ and $(1,3)$ matrix elements are mainly determined by V_{us} and V_{ub} respectively. The fact that the $(2,2)$ and $(2,3)$ matrix elements are very similar in magnitude is required by the smallness of V_{cb}. Finally the bound on $(1,1)$ element comes from m_d. The recent data now requires that the $(1,2)$ and $(1,3)$ are also quite similar [1].

The uncertainty in the matrix elements below the diagonal gives rise to a wide variety of models capable of explaining the fermion masses [2]. One particularly popular choice is to make them large giving rise to large right handed down quark mixing angles. The gauge group $SU(5)$ relates these angles to the mixing of the left-handed neutrinos, offering the possibility of explaining the large angles needed for neutrino oscillation in a manner consistent with the structure of the quark mass matrices. Clearly it is of interest to try to determine the elements below the diagonal in order to test these ideas.

In the context of supersymmetry, these elements also determine the coupling of the squarks and large elements will give rise to large flavour changing effects via virtual squark exchange. In the case of supersymmetry breaking driven by supergravity these effect can be large. In particular, for the case that fermion mass structure is due to a family symmetry spontaneously broken by the vacuum expectation value of a familon field one can place strong bounds on the down quark mass matrix below the diagonal. The reason is that the familon necessarily acquires an $F-term$ proportional to the gravitino mass and this in turn generates soft trilinear couplings which are not diagonalised with the fermion mass matrix. The resulting mass matrix has the form [3]

$$\frac{M^d}{m_b} = \begin{pmatrix} \leq \bar{\epsilon}^4 & b\bar{\epsilon}^3 & c\bar{\epsilon}^3 \\ \leq \bar{\epsilon}^3 & \bar{\epsilon}^2 & a\bar{\epsilon}^2 \\ \leq \bar{\epsilon}^2 & \leq \bar{\epsilon}^2 & 1 \end{pmatrix}$$

where the bound on the $(2,1)$ element comes from m_d and also from the bounds on $\Delta S \neq 0$, $\Delta Q = 0$ processes. The bound on the $(3,1)$ element comes from the bounds on $\Delta S \neq 0$, $\Delta Q = 0$ processes while the bound on the $(3,2)$ element comes from the bound on the electric dipole moments. Clearly these bounds do not allow large right-handed mixing and point rather at a near symmetric form for the mass matrix[2].

What other evidence is there for a symmetric form for the mass matrices? Perhaps the most convincing is the success of the Gatto, Sartori, Tonin (GST) relation [4] which predicts the Cabibbo angle (the $(1,2)$ mixing) in terms of quark masses

$$\sin\theta_c = \sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}} \tag{6}$$

[1] If one allows for an asymmetric form of the quark mass matrices with large entries below the diagonal it is still possible to have a $(1,3)$ zero [1].

[2] One should note that the bounds do not apply in gauge mediated schemes of supersymmetry breaking where the gravitino mass is significantly reduced.

where δ is the CP violating phase of the Standard model. This predicts a value 0.218 ± 0.005 to be compared to the experimental value 0.220 ± 0.003. This relation follows if the $(1,1)$ element is too small to affect the quark masses in leading order (a "texture zero") and the $(1,2)$ block is symmetric :

$$\frac{M^d}{m_b} = \begin{pmatrix} \leq \bar{\epsilon}^5 & b\bar{\epsilon}^3 & c\bar{\epsilon}^3 \\ b\bar{\epsilon}^3 & \bar{\epsilon}^2 & a\bar{\epsilon}^2 \\ \leq \bar{\epsilon}^2 & \leq \bar{\epsilon}^2 & 1 \end{pmatrix}. \tag{7}$$

We have few enough clues as to the physics beyond the Standard model that I am loath to treat the success of this relation as an accident and prefer to require that a theory of fermion masses should reproduce it. In parenthesis I note that models based on a spontaneously broken family symmetry require familons with both signs of family charge to achieve this relation. The popular class of models [2] based on charges of a single sign to give holomorphic zeros replace the equality of eq.(6) by an order of magnitude equality.

Of course we would like to describe charged lepton masses too. Surprisingly a very similar form describes the lepton mass eigenvalues very accurately :

$$\frac{M^l}{m_\tau} = \begin{pmatrix} \leq \bar{\epsilon}^5 & b\bar{\epsilon}^3 & ?\bar{\epsilon}^3 \\ b\bar{\epsilon}^3 & 3\bar{\epsilon}^2 & ?\bar{\epsilon}^2 \\ ?\bar{\epsilon}^2 & ?\bar{\epsilon}^2 & 1 \end{pmatrix} \tag{8}$$

In this the equality of the expansion parameter and the coefficients in the $(1,1)$, $(1,2)$ and $(2,1)$ elements give the relations $m_b/m_\tau(M_X) = 1/3$, $Det(M^l) = Det(M^d)|_{M_X}$ which are in excellent agreement with experiment after including radiative corrections in running from a high (unification) scale to the low energy quark mass scale. This strongly suggests an underlying quark-lepton unification. Of course experiment tell us one does not have exact equality between down quark and lepton masses and to allow for this in eq.(8) the $(2,2)$ element has been chosen be 3 time larger than the equivalent entry in the down quark sector. Such a factor readily arises as a Clebsch–Gordon coefficient in a Grand Unified theory [5]. With it one has the phenomenologically successful relations $m_s/m_\mu(M_X) = 1/3$, $m_d/m_e(M_X) = 3$.

2 Symmetries and Textures

How can one obtain mass matrices of the form discussed here? In my opinion the hierarchical structure for the fermion mass matrices strongly suggests it originates from a spontaneously broken family symmetry. In this approach, when the family symmetry is exact, only the third generation will be massive corresponding to only the (3,3) entry of the mass matrix being non-zero. When the symmetry is spontaneously broken, the zero elements are filled in at a level determined by the symmetry. Suppose a field θ which transforms non-trivially under the family symmetry acquires a vacuum expectation value, thus spontaneously breaking the family symmetry. The zero elements in the mass matrix will now become non-zero at some order in $< \theta >$. If only the $(2,3)$ and $(3,2)$ elements are allowed by the symmetry at order θ/M, where M is a mass scale to be determined, then a second fermion mass will be generated at $O((\theta/M)^2)$. In this way one may build up an hierarchy of masses.

$$\mathcal{M} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & <\theta>/M \\ 0 & <\theta>/M & 1 \end{pmatrix} \tag{9}$$

2.1 Symmetry breaking in the mass matrix

An important question is how do these elements at $O(\theta/M)$ arise? A wide variety of models have been constructed in which the mechanism for communicating the breaking to the mass matrix, and generating the hierarchy takes on different forms. It can be due to a heavy messenger sector, to radiative corrections or even to propagation in additional space dimensions [2]. Here I will just discuss the first case in which symmetry breaking is communicated via an extension of the "see-saw" mechanism mixing light to heavy "messenger" states - in this context it is known as the Froggatt–Nielsen mechanism [6]. To illustrate the mechanism, suppose there is a vector-like pair of quark messenger states X and $\overline{X}$ with mass M and carrying the same Standard Model quantum numbers as the c_R quark, but transforming differently under the family symmetry, so that the Yukawa coupling $h\overline{c_L}XH$ is allowed. Here H is the Standard Model Higgs responsible for giving up quarks a mass. When H acquires a vacuum expectation value (vev), there will be mixing between $\overline{c_L}$ and $\overline{X}$. If in addition there is a gauge singlet field θ transforming non-trivially under the family symmetry so that the coupling $h'\overline{X}c_R\theta$ is allowed, then the mixing with heavy states will generate the mass matrix.

$$\begin{pmatrix} \overline{c_L} & \overline{X} \end{pmatrix} \begin{pmatrix} 0 & h<H> \\ h'<\theta> & M \end{pmatrix} \begin{pmatrix} c_R \\ X \end{pmatrix}$$

Diagonalising this gives a see-saw mass formula

$$m_c \simeq \frac{hh'<H><\theta>}{M} \tag{10}$$

This mass arises through mixing of the light with heavy quarks.

A similar mechanism can generate the mass through mixing of the light Higgs with heavy Higgs states. Suppose H_X, $\overline{H}_X$ are Higgs doublets with mass M. If H_X has family quantum numbers allowing the coupling $H\overline{H}_X\theta$, there will be mixing between H and H_X. If the family symmetry also allows the coupling $\overline{c_L}c_RH_X$, the light-heavy Higgs mixing induces a mass for the charm quark of the form given in eq.(10).

3 Identification of the Family symmetry

The nature of the spontaneously broken family symmetry is hard to identify because the available data on quark masses and mixings is insufficient uniquely to pin it down. The kinetic terms and gauge interactions of the Standard Model has a very large family symmetry group, namely $U(3)^5$, where the $U(3)$ factors act on the left- and right- handed multiplets of quarks and leptons respectively. The group is extended to $U(3)^6$ if three right-handed neutrinos are added. Any family group should be contained in $U(3)^6$ but this leaves very many possibilities. In what follows I shall illustrate the possibilities by discussing two characteristic possibilities. In the first I consider the simplest case of an Abelian family symmetry. Such symmetries abound in compactified string theories so are quite natural extensions of the Standard Model. The second is a non-Abelian family symmetry, a subgroup of the $U(3)^6$.

3.1 An Abelian family symmetry for quark masses

How difficult is it to find a family symmetry capable of generating an acceptable fermion mass matrix? The surprising answer is "Not at all difficult" as I will illustrate by a very simple example utilising an Abelian family symmetry group [7]. We assign the same family charges $(-3, 2, 1)$ to (d_L, s_L, b_L) and to (d_L^c, s_L^c, b_L^c). This guarantees the symmetric form of the mass matrices. We assume the symmetry is spontaneous via Standard Model singlet fields, θ, $\bar{\theta}$, with $U(1)$ charges -1, +1 respectively, which acquire vacuum expectation values (vevs), $<\theta>=<\bar{\theta}>$, along a "D-flat" direction The resultant form of the down quark mass matrix is

$$\frac{M^d}{m_b} = \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^3 & \bar{\epsilon}^4 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon}^4 & \bar{\epsilon} & 1 \end{pmatrix}$$

where $\bar{\epsilon} = (<\theta>/M)$. This form is in good agreement with eq.(7) apart from the fact that the $(2, 3)$ element is of $O(\bar{\epsilon})$ rather than the $O(\bar{\epsilon}^2)$ required by the value of V_{cb}. To correct this requires a choice of the coefficients which are not determined by the Abelian symmetry. For example good agreement is obtained if the $(2, 3)$ and $(3, 2)$ elements are $1/2$ while the other elements are near unity. Note that the texture zero in the $(1, 1)$ element is required by the Abelian symmetry and the magnitude predicted for the $(1, 3)$ element is in good agreement with the recent measurements requiring a non-zero entry for a symmetric mass matrix. Since the up quarks have the same family charge as the down quarks, their mass matrix has a similar form to that of the down quarks. If $SU(2)_R$ is also an exact symmetry of the theory even the expansion parameters and operator coefficients are equal and the mass matrices are identical. This is clearly not acceptable and, if there is an underlying $SU(2)_R$ symmetry, it must be spontaneously broken so that the equality of the mass matrices will be lost through soft symmetry breaking terms. These enter through the expansion parameter ϵ which is determined by θ/M where θ is the field spontaneously breaking the symmetry and M is the messenger mass of the state responsible for communicating the symmetry breaking and generating the higher dimension operators. Due to $SU(2)_R$ breaking the messenger mass may be different for the up and down quark sectors and hence the expansion parameters may differ. It is also possible that the family symmetry breaking field, θ, is not a singlet under $SU(2)_R$ and its vev breaks $SU(2)_R$, again leading to a different expansion parameter for the up and the down sectors and giving a form in agreement with eq.(1). A similar form applies to the charged leptons with the same expansion parameter as for the down quarks and one can readily obtain a form in agreement with eq.(8).

3.2 Non-Abelian family symmetry

While a very simple Abelian symmetry provides a remarkable consistent description of fermion masses it does fall short of a complete theory, mainly because the relative coefficients of the various matrix elements are not determined. To do this requires a non-Abelian family symmetry. As discussed above any family group should be contained in $U(3)^6$ but this leaves very many possibilities for non-Abelian family symmetries. If there is a GUT symmetry the possible family group will be smaller; for example if the GUT is $SO(10)$ the maximal group is $U(3)$.

There is a strong motivation for a non-Abelian family symmetry in supersymmetric theories [8] because it can explain why the squarks and sleptons are nearly degenerate as is required to suppress flavour changing neutral currents. For the case of a $SU(3)$ family symmetry commuting with $SO(10)$ the 16 dimensional representation of $SO(10)$ transforms as a triplet under $SU(3)$ giving the three generations of quarks and leptons. As a result, after $SO(10)$ breaking but while the family symmetry is unbroken, the soft squark masses are degenerate as are the soft sleptons. The degeneracy is lifted once the family symmetry is broken, but this breaking is through higher dimension operators and may be small. The structure of fermion masses is particularly sensitive to this breaking, as they vanish in the symmetry limit. It is through the vacuum structure that the fermion hierarchy is established. Here I discuss a specific model which illustrates the interesting phenomenological possibilities in such a scheme [9, 10].

3.2.1 Vacuum alignment

The family group has a breaking pattern given by

$$SU(3) \quad \underset{<\theta_1>}{\overset{V_1}{\longrightarrow}} \quad SU(2) \quad \underset{<\theta_2>}{\overset{V_2}{\longrightarrow}} \quad Nothing$$

where θ_1 and θ_2 are scalar fields transforming as $\overline{3}$ under the family symmetry. Vacuum expectation values (vevs) for θ_1 and θ_2 may be readily driven by negative soft masses squared or by other terms in the potential[3]. Clearly one may always perform an SU(3) rotation so that

$$<\theta_1^T>= V_1 \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

At the second stage of breaking one may always use the residual $SU(2)$ symmetry to rotate the vev of the second field to have the form

$$<\theta_2^T>= V_2 \begin{pmatrix} 0 & \cos\varphi & \sin\varphi \end{pmatrix}$$

It turns out [9] that it is straightforward to align the breaking such that $\varphi = 45^0$. As we shall see this has the effect of making the $(1,2)$ and $(1,3)$ and the $(2,2)$ and $(2,3)$ mass matrix elements equal respectively in leading order explaining the structure of eq.(7). It also can lead quite naturally to large neutrino mixing angles.

[3] I assume that the vevs develop along D-flat directions $<\theta_i>=<\overline{\theta}^i>$.

In leading order, with an additional Abelian family symmetry restricting the allowed terms, the superpotential has the form

$$W_1 = Q_i\theta_1^i d_i^c \theta_1^i \frac{H_1}{M_1^2} + Q_i\theta_2^i d_i^c \theta_2^i \frac{H_1}{M_1^2}$$

The resulting mass matrix has the form

$$M_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \overline{\epsilon}^2 & \overline{\epsilon}^2 \\ 0 & \overline{\epsilon}^2 & 1+\overline{\epsilon}^2 \end{pmatrix} \frac{V_1^2}{M_1^2} < H_1 >, \quad \overline{\epsilon}^2 = \frac{V_2^2}{V_1^2}$$

where I have taken $\cos\varphi \approx \sin\phi$. Similarly one obtains the same form for the up quark mass matrix but with expansion parameter $\epsilon^2 = \frac{V_2^3}{V_1^2 M_2}$ where one must allow for a different messenger mass M_2 in the up quark sector. Finally the remaining elements of the mass matrix are generated by the next terms of higher order allowed in the superpotential[4]

$$W_2 = (\epsilon^{ijk} Q_i \overline{\theta}_{1j}\overline{\theta}_{2k}\ d_i^c \theta_2^i + Q_i\theta_2^i\ \epsilon^{i'j'k'} d_{i'}^c \overline{\theta}_{1j'}\overline{\theta}_{2k'})\frac{S}{M_1}\frac{H_1}{M_1^3}$$

where $\overline{\theta}_{1,2}$ are family triplet chiral superfields whose scalar components are aligned with those of $\theta_{1,2}$ fields to maintain $D-flatness$. Including these terms gives

$$M_d = \begin{pmatrix} 0 & \overline{\epsilon}^2\alpha & -\overline{\epsilon}^2\alpha \\ \overline{\epsilon}^2\alpha & \overline{\epsilon}^2 & \overline{\epsilon}^2 \\ -\overline{\epsilon}^2\alpha & \overline{\epsilon}^2 & 1+\overline{\epsilon}^2 \end{pmatrix} \frac{V_1^2}{M_1^2} < H_1 >$$

This illustrates the main structure one can obtain from a non-Abelian symmetry.

4 Non-Abelian symmetry and neutrino mixing

One may readily extend the non Abelian symmetry to obtain a description of lepton masses and mixing. Because the expansion parameter $\overline{\epsilon}$ is not very small, one must include higher dimension operators before attempting a detailed comparison with data. Including these effects one obtains a mass matrix of the form [10]

$$M/M_{3,3} = \begin{pmatrix} \varepsilon^8 & \varepsilon^3(z+(x+y)\varepsilon) & \varepsilon^3(z+(x-y)\varepsilon) \\ -\varepsilon^3(z+(x+y)\varepsilon) & \varepsilon^2(aw+u\varepsilon) & \varepsilon^2(aw-u\varepsilon) \\ -\varepsilon^3(z+(x-y)\varepsilon) & \varepsilon^2(aw-u\varepsilon) & 1 \end{pmatrix} \quad (11)$$

Here z, w and u are real coefficients and x and y complex coefficients of order 1. Given that the symmetry properties of the up and the down quarks , the charged leptons and the neutrinos are the same the form of the mass matrices will also be the same. For the

[4]It is necessary to forbid the lower dimension terms $\epsilon^{ijk} Q_i \overline{\theta}_{1,2\ j}\ d_k^c \frac{H_1}{M_1}$ and this requires at least an additional discrete Z_N symmetry under which only the θ, $\overline{\theta}$ fields transform. If $\overline{\theta}_{1,2}$ transforms nontrivially while $\overline{\theta}_1\overline{\theta}_2\ \theta_2$ is invariant the unwanted term is not present. One may readily check that all other unwanted terms can similarly be eliminated.

case of the down quarks and leptons the expansion parameter is $\varepsilon = \bar{\epsilon}$ and for neutrinos and up quarks $\varepsilon = \epsilon$ and the higher order terms are necessary to fit the data for M_d of eq(2). The coefficients w, u, z, x, y of $O(1)$ are not determined by the symmetry alone but can be determined by a fit to the down quark mass matrix. The factor a is the result of the Clebsch–Gordon factor of an underlying GUT. A simple example explaining the origin of this factor is provided in the context of an $SO(10)$ theory. Suppose there is a field Σ transforming as a 45 responsible for $SO(10)$ breaking and communicating this breaking to the mass matrix through its coupling to quarks and leptons. Writing the vacuum expectation value

$$< \Sigma >= B - L + \kappa T_{R,3}, \tag{12}$$

the contribution of these graphs to the down quarks and leptons is respectively $(1/3-\kappa/2)$ and $(-1-\kappa/2)$ when coupling to the right-handed states and 1/3 and -1 when coupling to the left-handed states. We consider the case that the first class has the lighter messengers and dominates. For $\kappa = 0$ we obtain the form of eq(8). However for case $\kappa = 2$ the contribution to the leptons +3 times that of the quarks ($a_l = 3$) so we obtain eq(8) with $+3$ rather than -3. Since the lepton mass eigenvalues are insensitive to this sign we obtain identical masses for either form. However there is a significant difference in the coupling to neutrinos, the $\kappa = 0$ case giving $a_\nu = -3$ while $\kappa = 0$ gives $a_\nu = 0$.

5 Neutrino masses.

5.1 Dirac mass

Since the family symmetry properties of the neutrinos are the same as those of the quarks and charged leptons, the neutrino Dirac mass matrix between the doublet neutrinos and the singlet (right-handed) neutrinos is also given by eq.(11) with $\varepsilon = \epsilon$ and $a = a_\nu$. Here the expansion parameter is the same as that for up quarks since the neutrino and up quark have the same $SU(2)_R$ charges and get their mass from the same Higgs doublet.

5.2 Majorana mass

Of course it is necessary to determine the Majorana mass matrix before one can determine the effective neutrino mass matrix via the see-saw formula. Although the family symmetry properties of the right-handed neutrinos are related to those of the charged leptons it is not possible to use this information unambiguously to determine the structure of the Majorana mass matrix. In particular it may not have the same form as is found for the Dirac matrix. The reason is twofold. Firstly the Majorana masses are generated via a new $\Delta L = 2$ lepton number violating Higgs sector and it is necessary to specify the family symmetry representation content of this sector before the Majorana mass structure is fixed. Secondly the Majorana mass matrix involves the coupling of identical fermions and so antisymmetric terms allowed in the Dirac mass matrix will not arise in the Majorana matrix. Despite this, we can make some general statements about the structure. In particular we expect an hierarchical structure for the Majorana mass matrix because the underlying family symmetry ($SU(3)$), is the same as applies to the Dirac matrix which leads to a structure ordered by the expansion parameter $\varepsilon_M = < \Phi > /M'$. If a single $\Delta L = 2$ (effective) symmetry breaking field, Φ, dominates there will no possibility of degeneracy between matrix elements. Moreover, for a large part of the parameter space, the form of the mass matrix is in practice determined. In the limit that $\varepsilon_M << \epsilon$ (which is the case if the messenger sector in the Φ sector is heavier than in the electroweak breaking sector) the mixing in the neutrino sector is dominated by the mixing coming from the Dirac mass. The most probable situation is that the vev of the $\Delta L = 2$ Higgs responsible for the dominant Majorana mass comes from a single field carrying definite family symmetry charge, which generates a mass matrix in which all three eigenvalues are unequal. The matrix can be diagonalised by small rotations of $O(\varepsilon_M)$ giving $M^{\nu}_{Majorana} = Diagonal[m_1 m_2, m_3]$. It is important to note that this mass eigenstate basis is likely to be very close to that used for the Dirac matrix, eq.(11), the mass eigenstates will be the family symmetry eigenstates up to corrections of $O(\varepsilon_M)$.

6 The light neutrino mass matrix

We are now able to determine the masses and mixing angles of the light neutrinos using eq (11) in the see-saw equation $M_{eff} = M^{\nu}_{Dirac}.M^{-1}_{Majorana}.M^{\nu}_{Dirac}$. The effective Lagrangian associated with the see-saw mass is of the form

$$\begin{aligned}\mathcal{L} \;\cong\; & \frac{\epsilon^6}{m_1}(z(\nu_\mu+\nu_\tau))^2 H_2^2 + \frac{\epsilon^4}{m_2}(a_\nu w(\nu_\mu+\nu_\tau)+\epsilon(u(\nu_\mu-\nu_\tau)+z\nu_e))^2 H_2^2 \\ & + \frac{1}{m_3}(\nu_\tau + a'_\nu \nu_\nu + \epsilon c'_\nu \nu_e)^2 H_2^2. \qquad (13)\end{aligned}$$

The condition that the right handed neutrinos of mass m_1 and m_2 respectively dominate the see-saw contribution to the heaviest and next heaviest light neutrino eigenstates masses is $\frac{\epsilon^6}{m_1} > O(\frac{\epsilon^6}{M_2})$. In this case, diagonalising eq.(13) one finds the masses of neutrino mass eigen states are given approximately by

$$M_a = \frac{\epsilon^6}{m_1} 2z^2 v^2 \qquad (14)$$

$$M_b = \frac{\epsilon^6}{m_2}(2u^2+z^2)v^2 \qquad (15)$$

$$M_c < \frac{1}{m_3} v^2 \qquad (16)$$

and the states have composition

$$\nu_a \propto \quad z(\nu_\mu+\nu_\tau) + y\varepsilon'(\nu_\mu-\nu_\tau)$$

$$\nu_b \propto \quad z\nu_e + r(\nu_\mu - \nu_\tau - y\varepsilon'(\nu_\mu+\nu_\tau)/z)$$

where

$$r = \sqrt{2}(u - a_\nu w y/z) \quad = O(1)$$

¿From this one sees that the heaviest state is maximally mixed while the next state has near maximal mixing in agreement with the atmospheric and neutrino oscillation measurements. For the case $\kappa = 0$ the masses are consistent only with the LOW solar solution but for $\kappa = 2$ the LMA is also allowed [10]. Thus, due to the see-saw mechanism, the large mixing angles readily emerge from an underlying form close to that of the quarks.

7 Summary

The new data for both the quark and lepton sectors is testing our ideas for the origin of the fermion masses and mixings and raises interesting questions :

- Is there a family symmetry generating the hierarchy? Both Abelian and non Abelian symmetries look promising.
- Is there a connection between quarks and leptons? The relations following from $SU(5)$ or $SO(10)$ (or strings) seem capable of giving quantitatively acceptable relations between masses and mixing angles in the quark and lepton sectors. Somewhat surprisingly the see-saw mechanism allows quark and lepton Dirac masses to have a similar form supporting the idea of an underlying family symmetry.
- Are there significant flavour changing and CP violating processes coming, in a supersymmetric theory, from the Yukawa sector responsible for fermion masses? Estimates suggest these effects should be very close to current bounds.

The answers to these questions will lead us a long way towards understanding the origin of mass.

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