BULK AND BRANE SUPERSYMMETRY BREAKING

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Introduction

Since the beginning, extra dimensions have been intertwined with supersymmetry.

- Strings predict supersymmetry and extra dimensions.

- Extra dimensions can explain and stabilize the hierarchy;

- Supersymmetry can stabilize and explain the hierarchy.

It is natural to consider them both!

- Supersymmetry can stabilize the extra dimensions.

- Extra dimensions can break supersymmetry and ameliorate the supersymmetric flavor problem.

This talk

In this talk I will restrict my attention to bulk-plus-brane supersymmetry in $4+1$ dimensions.

I will cover

- Flat and warped geometries.
- Unbroken and broken supersymmetry.

I will also describe a four-dimensional superfield theory that captures the essential features of the analysis.

We will see that the effective theory provides a powerful way to study supersymmetry breaking in extra dimensions.

References

There are many references.

Collaborators:

Altendorfer, Belyaev,* Delgado, Feruglio, Moroi, Nemeschansky, Poppitz, Redi, Xiong, Zhang, Zwirner

Other authors:

Arkani-Hamed, Barbieri, Chacko, Dienes, Falkowski, Gherghetta, Giudice, Gregoire, Hall, Horava, Kaloper, Kaplan, Kribs, Lalak, Luty, Marti, Meissner, Mirabelli, Murayama, Nibbelink, Nilles, Nomura, Olechowski, Peskin, Pokorski, Pomarol, Ponton, Randall, Rattazzi, Schmaltz, Smith, Sundrum, Wacker, Weiner, Witten, Zucker ....

Five-dimensional supergravity

We will start by considering five-dimensional supergravity – without a cosmological constant. The action is

$$S_{\text{bulk}} = \int d^5x e_5 \Big\{ -\frac{1}{2}R + \frac{i}{2}\widetilde{\Psi}^i_M \Gamma^{MNK} D_N \Psi_{Ki} - \frac{1}{4}F_{MN}F^{MN} - \frac{1}{6\sqrt{6}}\epsilon^{MNPQK} F_{MN}F_{PQ}B_K + \ldots \Big\}.$$

The supersymmetry transformations are

$$\delta e^A_M = i\widetilde{\mathcal{H}}^i \Gamma^A \Psi_{Mi}, \qquad \delta B_M = i\frac{\sqrt{6}}{2}\widetilde{\Psi}^i_M \mathcal{H}_i$$

$$\delta \Psi_{Mi} = 2D_M \mathcal{H}_i + \frac{1}{2\sqrt{6}}\left(\Gamma_{MNK} - 4g_{MK}\Gamma_N\right) F^{NK}\mathcal{H}_i.$$

This theory is invariant under $N = 2$ supersymmetry in five dimensions – as well as under the $SU(2)$ automorphism symmetry of the $N = 2$ algebra.

$S^1/\mathbb{Z}_2$ orbifold

We compactify the theory on an $S^1/\mathbb{Z}_2$ orbifold, obtained by identifying $z \to -z$, where $z = x^5$.

We decompose the symplectic Majorana spinors Ψ_i and $\mathcal{H}_i$ into two-component Weyl spinors:

$$\Psi_1 = -\Psi^2 = \binom{\psi_{1\alpha}}{\overline{\psi}_2^{\dot{\alpha}}}, \quad \Psi_2 = \Psi^1 = \binom{-\psi_{2\alpha}}{\overline{\psi}_1^{\dot{\alpha}}}.$$

We then choose the following parity assignments:

$$\begin{array}{rccccccc} \text{even}: & \partial_m & e_m^a & e_5^{\hat{5}} & B_5 & \eta_1 & \psi_{m1} & \psi_{52} \\ \text{odd}: & \partial_5 & e_m^{\hat{5}} & e_5^a & B_m & \eta_2 & \psi_{m2} & \psi_{51}. \end{array}$$

With these assignments, the Lagrangian and supersymmetry transformations are invariant under the $\mathbb{Z}_2$ symmetry.

Boundary conditions

We place tensionless branes, $\Sigma_{1,2}$, at the orbifold fixed points.

We require that all odd bosonic fields vanish on the branes:

$$e_m^{\hat{5}} = e_5^a = B_m = 0 \qquad \text{on } \Sigma_{1,2}.$$

However, we let odd fermionic fields jump, consistent with their equations of motion. A trick is to let

$$\eta_2 \to \varepsilon(z)\,\eta_2, \quad \psi_{m2} \to \varepsilon(z)\,\psi_{m2}, \quad \psi_{51} \to \varepsilon(z)\,\psi_{51},$$

where the redefined fields are *even* under parity. Jumping fields require brane actions for consistency.

Consistency

Consistency requires that the supersymmetry transformations of odd bosonic fields must vanish on the branes. This forces

$$\eta_2 = \alpha_{1,2}\eta_{1,2}, \qquad \psi_{m2} = \alpha_{1,2}\psi_{m1}, \qquad \psi_{51} = -\alpha^*_{1,2}\psi_{52} \qquad \text{on } \Sigma_{1,2}.$$

These jumps are consistent with the following brane action:

$$S_{\text{brane}} = -2\int d^5x e_4 \left[(\alpha_1\delta(z) - \alpha_2\delta(z-\pi R))\psi_{m1}\sigma^{mn}\psi_{n1} + h.c.\right].$$

Closure of the supersymmetry algebra requires that we change

$$\delta\psi_{52} = \delta\psi_{52}\Big|_{\text{old}} - 4\eta_2\delta(z) + 4\eta_2\delta(z-\pi R).$$

Supersymmetry

It is not hard to show that the bulk-plus-brane system is invariant under the modified supersymmetry transformations – for *any* values of α_1 and α_2.

However, there is only a Killing spinor when $\alpha_1 = \alpha_2 = \alpha$.

In that case

$$\eta_1 = \eta \qquad \eta_2 = \alpha\eta_1$$

is the Killing spinor corresponding to the unbroken supersymmetry. Note that $\eta_2 \neq 0$!

When $\alpha_1 \neq \alpha_2$, supersymmetry is spontaneously broken by the "superpotentials" on the branes.

Goldstino

The order parameter is nonlocal – it depends on branes at $z = 0$ and $z = \pi R$. How is the information transferred?

Let's carry out a Kaluza-Klein expansion:

$$\begin{aligned}
\psi^{+}(z) &= \frac{1}{\sqrt{\pi R}}\left[\psi_0^{+} + \sqrt{2}\sum_{\rho=1}^{\infty}\psi_\rho^{+}\cos(\rho z/R)\right] \\
\psi^{-}(z) &= \frac{1}{\sqrt{\pi R}}\left[\sqrt{2}\sum_{\rho=1}^{\infty}\psi_\rho^{-}\sin(\rho z/R)\right]
\end{aligned}$$

where $\psi^{+} = (\psi_{m1}, \psi_{52})$ and $\psi^{-} = (\psi_{m2}, \psi_{51})$.

The KK excitations of ψ_{51} and ψ_{52} are unphysical. They are Goldstinos associated with breaking $N = 2$ in five dimensions to $N = 2$ in four. The Goldstinos give mass to KK excitations of the gravitino.

Mass matrix

The KK expansion gives rise to the following infinite-dimensional mass matrix:

$$\mathcal{M}_{3/2} = \frac{1}{R}\left(\begin{array}{c|cc|cc|c} \alpha_- & \alpha_+ & \alpha_+ & \alpha_- & \alpha_- & \dots \\ \hline \alpha_+ & \alpha_- + 1 & \alpha_- & \alpha_+ & \alpha_+ & \dots \\ \alpha_+ & \alpha_- & \alpha_- - 1 & \alpha_+ & \alpha_+ & \dots \\ \hline \alpha_- & \alpha_+ & \alpha_+ & \alpha_- + 2 & \alpha_- & \dots \\ \alpha_- & \alpha_+ & \alpha_+ & \alpha_- & \alpha_- - 2 & \dots \\ \hline \dots & \dots & \dots & \dots & \dots & \dots \end{array}\right),$$

where $\alpha_\pm = \alpha_1 \pm \alpha_2$, in the basis $(\psi_0^1, \psi_1^+, \psi_1^-, \psi_2^+, \psi_2^-, \dots)$.

Diagonalizing the matrix, we find

$$M = \frac{1}{R}\left\{\frac{1}{\pi}\tan^{-1}\left[\frac{4\alpha_-}{(\alpha_+^2 - \alpha_-^2) + 4}\right] + \rho\right\}, \quad (\rho = 0, \pm 1, \pm 2, \dots),$$

When $\alpha_1 = \alpha_2$, $\alpha_- = 0$, the masses are not shifted, and supersymmetry is not broken.

Kaluza-Klein mediation

The infinite-dimensional mass matrix shows that the Goldstone fermion for breaking $N = 1 \to N = 0$ is a linear combination of ψ_{51}, ψ_{52}, and their KK excitations.

The KK excitations communicate the fact that the symmetry breaking is nonlocal in the fifth dimension.

In the literature this is sometimes called "radion mediation" – but it might perhaps be more accurately called Kaluza-Klein mediation of supersymmetry breaking.

The mechanism is analogous to the way supersymmetry breaking is communicated in the strongly-coupled heterotic string.

Eigenfunctions

The eigenfunctions can be found by integrating the five-dimensional equations of motion. For the case $\alpha_1 = \alpha$, $\alpha_2 = 0$, we have

$$\partial_5 \psi_{n1} + M\psi_{n2} = 0, \qquad -\partial_5 \psi_{n2} + M\psi_{n1} = 2\alpha\psi_{n1}\delta(z),$$

in terms of the original fields. The solution is

$$\psi_{n1} = \cos[M|z| - \tan^{-1}\alpha]\chi_n^{(\rho)} \qquad \psi_{n2} = \epsilon(z)\sin[M|z| - \tan^{-1}\alpha]\chi_n^{(\rho)},$$

where

$$M = \frac{\rho}{R} + \frac{\tan^{-1}\alpha}{\pi R}.$$

This agrees with the previous calculation. Note that $\psi_{m2}(\pi R) = 0$, as required by the absence of a brane action when $\alpha_2 = 0$.

Alternative eigenfunctions

Now consider the eigenfunctions

$$\psi_{n1} = \cos[\epsilon(z)(M|z| - \tan^{-1}\alpha)]\chi_n^{(\rho)}, \quad \psi_{n2} = \sin[\epsilon(z)(M|z| - \tan^{-1}\alpha)]\chi_n^{(\rho)}$$

These eigenfunctions agree with the previous eigenfunctions everywhere – except at the singular points.

The new eigenfunctions obey the following five-dimensional equations of motion:

$$\partial_5\psi_{n1} + M\psi_{n2} = 2\tan^{-1}\alpha\,\psi_{n2}\delta(z), \quad -\partial_5\psi_{n2} + M\psi_{n1} = 2\tan^{-1}\alpha\,\psi_{n1}\delta(z)$$

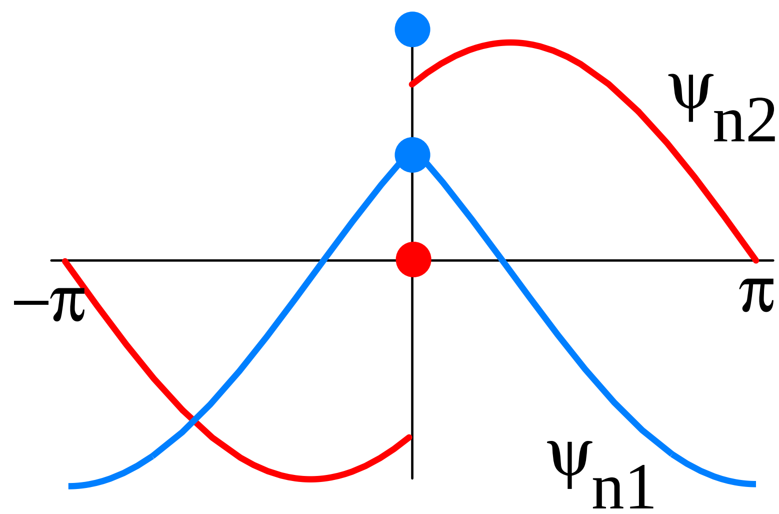
They are consistent with the brane action

$$S_{\text{brane}} = -2\alpha\int d^5x e_4\,[\psi_{m1}\sigma^{mn}\psi_{n1} + \psi_{m2}\sigma^{mn}\psi_{n2} + h.c.]\;\delta(z).$$

Singular sources

This new action is supersymmetric after modifying $\delta\psi_{51}$ *and* $\delta\psi_{52}$.

With the first action, ψ_{m1} is continuous across the brane, while with the second, $\psi_{m1}^2+\psi_{m2}^2$ is continuous. The value of ψ_{m1} on the brane differs in the two cases!



The top blue dot cannot be obtained by ordinary Fourier analysis!

More singular sources

Similar problems are known to arise in the theory of differential equations with singular terms.

We choose to avoid these issues by *defining* the brane action to be continuous.

Nevertheless, note that

$$\tan^{-1}\alpha\int dz(\psi_{m1}^2+\psi_{m2}^2)\delta(z)=\beta\int dz\ \psi_{m1}^2(1+\tan[\epsilon(z)\beta]^2)\delta(z)$$
$$=\beta\int dz\ \psi_{m1}^2[1-\epsilon(z)^2\beta^2-\frac{2}{3}\epsilon(z)^4\beta^4+...]\delta(z)$$
$$=\beta\int dz\ \psi_{m1}^2[1-\frac{1}{3}\beta^2-\frac{2}{15}\beta^4+...]\delta(z)=\alpha\int dz\ \psi_{m1}^2.$$

Here $\beta=\tan^{-1}\alpha$ and

$$\int dz\,\epsilon(z)^{2p}\,\delta(z)=\frac{1}{2p+1}\int dz\,\delta(z).$$

Generalized Scherk-Schwarz

The new eigenfunctions

$$\psi_{n1} = \cos[\epsilon(z)(M|z| - \tan^{-1}\alpha)]\chi_n^{(\rho)} \quad \psi_{n2} = \sin[\epsilon(z)(M|z| - \tan^{-1}\alpha)]\chi_n^{(\rho)}$$

are discontinuous rotations of smooth eigenfunctions:

$$\begin{pmatrix}\psi_{n1}\\ \psi_{n2}\end{pmatrix} = \begin{pmatrix}\cos[\epsilon(z)(\tan^{-1}\alpha)] & \sin[\epsilon(z)(\tan^{-1}\alpha)]\\ -\sin[\epsilon(z)(\tan^{-1}\alpha)] & \cos[\epsilon(z)(\tan^{-1}\alpha)]\end{pmatrix}\begin{pmatrix}\cos(Mz)\chi_n^{(\rho)}\\ \sin(Mz)\chi_n^{(\rho)}\end{pmatrix}.$$

This discontinuous twist is an example of a generalized Scherk-Schwarz mechanism.

In flat space, symmetry-breaking brane mass terms can be generated by discontinuous twists!

Five-dimensional AdS supergravity

Let us now consider the supersymmetric Randall-Sundrum scenario, following the same technique. We start with bulk supergravity

$$\begin{aligned} S_{\text{bulk}} &= \int d^5x e_5 \Big\{ -\frac{1}{2}R + 6\lambda^2 + \frac{i}{2}\widetilde{\Psi}^i_M \Gamma^{MNK} D_N \Psi_{Ki} - \frac{1}{4}F_{MN}F^{MN} \\ &\qquad -\frac{3}{2}\lambda\, \vec{q}\cdot\vec{\sigma}_i{}^j \widetilde{\Psi}^i_M \Sigma^{MN}\Psi_{Nj} - \frac{1}{6\sqrt{6}}\epsilon^{MNPQK}F_{MN}F_{PQ}B_K + \dots \Big\}. \end{aligned}$$

The supersymmetry transformations are

$$\begin{aligned} \delta e^A_M &= i\widetilde{\mathcal{H}}^i \Gamma^A \Psi_{Mi}, \qquad\qquad \delta B_M = i\frac{\sqrt{6}}{2}\widetilde{\Psi}^i_M \mathcal{H}_i \\ \delta \Psi_{Mi} &= 2\Big(D_M \mathcal{H}_i - i\frac{\sqrt{6}}{2}\lambda\, \vec{q}\cdot\vec{\sigma}_i{}^j B_M \mathcal{H}_j\Big) + i\lambda\, \vec{q}\cdot\vec{\sigma}_i{}^j\, \Gamma_M \mathcal{H}_j + \dots \end{aligned}$$

The cosmological constant comes from gauging a $U(1)$ subgroup of $SU(2)$, parametrized by a unit vector $\vec{q} = (q_1, q_2, q_3)$.

Parity and boundary conditions

As before, we assign parities

$$\begin{array}{rcccccccccc} \text{even}: & \partial_m & e_m^a & e_5^{\hat{5}} & B_5 & \eta_1 & \psi_{m1} & \psi_{52} & q_{1,2} & \lambda \\ \text{odd}: & \partial_5 & e_m^{\hat{5}} & e_5^a & B_m & \eta_2 & \psi_{m2} & \psi_{51} & q_3 & \end{array}$$

consistent with the action and supersymmetry transformations.

We require odd bosonic fields to vanish on the branes,

$$e_m^{\hat{5}} = e_5^a = B_m = 0 \qquad \text{on } \Sigma_{1,2}$$

and redefine the odd fermionic fields – and q_3 – as follows:

$$\eta_2 \to \varepsilon(z)\,\eta_2, \quad \psi_{m2} \to \varepsilon(z)\,\psi_{m2}, \quad \psi_{51} \to \varepsilon(z)\,\psi_{51}, \quad q_3 \to \varepsilon(z)\,q_3.$$

Brane action

We take the brane action to be

$$S_{\text{brane}} = \int d^5x e_4[-3(\lambda_1\delta(z) - \lambda_2\delta(z-\pi R)) \\ -2(\alpha_1\delta(z) - \alpha_2\delta(z-\pi R))\psi_{m1}\sigma^{mn}\psi_{n1} + h.c.],$$

where $\lambda_{1,2}$ are brane tensions. The case $\lambda_1 = \lambda_2 = \lambda$ gives a four-dimensional effective theory in flat Minkowski space.

The case $|\lambda_{1,2}| < \lambda$ gives rise to AdS_4.

As before, we redefine the supersymmetry transformations:

$$\delta\psi_{52} = \delta\psi_{52}\Big|_{\text{old}} - 4\eta_2\delta(z) + 4\eta_2\delta(z-\pi R).$$

Consistency

Consistency of the construction requires

$$\omega_{ma\hat{5}} = \varepsilon(z)\lambda_{1,2}e_{ma}, \quad \eta_2 = \alpha_{1,2}\eta_1, \quad \psi_{m2} = \alpha_{1,2}\psi_{m1}, \quad \psi_{51} = -\alpha^*_{1,2}\psi_{52}$$

on $\Sigma_{1,2}$. Variation of these conditions also demands

$$\lambda_{1,2} = -\frac{\alpha_{1,2}q^*_{12} + \alpha^*_{1,2}q_{12} + (\alpha_{1,2}\alpha^*_{1,2} - 1)q_3}{1 + \alpha_{1,2}\alpha^*_{1,2}}\,\lambda.$$

This implies that the tensions λ_1 and λ_2 are bounded by the inequality

$$|\lambda_{1,2}| \leq \lambda.$$

Supersymmetry

It is a straightforward but nontrivial exercise to show that the bulk-plus-brane action is supersymmetric.

The total variation receives three contributions: two from the bulk and one from the brane.

On the brane we need

$$\begin{aligned} \delta e_m^a &= i\left(1+\alpha_{1,2}\alpha_{1,2}^*\varepsilon^2\right)\eta_1\sigma^a\overline{\psi}_{m1} + h.c. \\ \delta\psi_{n1} &= 2\widehat{D}_n\eta_1 + i\left(\lambda\left(q_{12}^* + \alpha_{1,2}^*\varepsilon^2 q_3\right) + \lambda_1\alpha_{1,2}^*\varepsilon^2\right)\sigma_n\overline{\eta}_1 + \ldots \end{aligned}$$

together with

$$\lambda_{1,2} = -\,\frac{\alpha_{1,2}q_{12}^* + \alpha_{1,2}^*q_{12} + (\alpha_{1,2}\alpha_{1,2}^* - 1)q_3}{1+\alpha_{1,2}\alpha_{1,2}^*}\,\lambda.$$

Note that the brane supersymmetry transformations take an unusual form. The new terms are essential!

Results

In the warped case, we have seen

- The bulk actions with even and odd gravitino mass terms can be continuously connected using the vector $\vec{q}$.

- The Randall-Sundrum fine-tuning is *not* enforced by supersymmetry.

- The five-dimensional bulk-plus-brane system can be supersymmetrized if and only if the effective four-dimensional theory reduces to Minkowski or AdS_4.

- The brane superpotentials $\alpha_{1,2}$ are not free. They are determined by the brane tensions. Scherk-Schwarz?

Brane-localized SUSY breaking

How, then, can we break supersymmetry? Let's imagine that supersymmetry is broken *dynamically* on one of the branes.

At low energies, we decouple all brane matter except the Goldstone fermion.

Then we couple the brane-localized Goldstone fermion to the bulk-plus-brane system.

We have succeeded to do this when the fermion is localized on the Planck brane.

Visible brane?

Brane action

The method is as before. We take the brane action to be

$$\begin{aligned} S_{\text{brane}} &= \int d^5x e_4 \Big\{ -3\lambda_1 - 2\alpha_1 \psi_{m1}\sigma^{mn}\psi_{n1} - \frac{i}{2}\bar{\chi}\bar{\sigma}^m \widehat{D}_m \chi \\ & -(\lambda q_{12} + \alpha_1(\lambda_1 + \lambda q_3))\chi\chi - \frac{i}{2} v_1^2 \psi_{m1}\sigma^m \bar{\chi} \\ & - \frac{1}{4\sqrt{6}} v_1^2 e_5^{\hat{5}} F^{m5} \chi \sigma_m \bar{\chi} + h.c. \Big\} \delta(z). \end{aligned}$$

The new brane action induces the following changes to the boundary conditions:

$$\begin{aligned} \psi_{m2} &= \alpha_1 \psi_{m1} - \frac{i}{12} v_1^2 \sigma_m \bar{\chi} \\ \psi_{51} &= -\alpha_1^* \psi_{52} - \frac{1}{12} v_1^2 e_5^{\hat{5}} \chi \\ \omega^{a\hat{5}} &= -\frac{i}{12} v_1^2 \eta_1 \sigma^a \bar{\chi} + h.c. \end{aligned}$$

on the surface Σ_1.

Supersymmetry invariance

The Goldstone fermion has the following nonlinear transformation:

$$\delta\chi = v_1^2 \eta_1 .$$

With this transformation, the system is supersymmetric provided

$$v_1^{*4} = \lambda_1(1 + \alpha_1\alpha_1^*) + \lambda\left(\alpha_1 q_{12}^* + \alpha_1^* q_{12} + (\alpha_1\alpha_1^* - 1)q_3\right).$$

The Goldstone fermion relaxes the relation between λ_1 and λ. Now, the low energy effective theory can be Minkowski, de Sitter or anti de Sitter in four dimensions.

Note, though, that as $\lambda, \lambda_1 \to 0$, $v_1 \to 0$. The Goldstone fermion decouples as the warp factor vanishes.

Gravitino mass

How does the gravitino mass relate to the scale of supersymmetry breaking on the brane?

We set $\lambda_1 = \lambda_2 = \lambda$, so that the four-dimensional effective theory is Minkowski, and carry out the KK reduction.

We find

$$\kappa_5\, v_1^2 = 2\sqrt{6}\lambda \ \left[\frac{Y_1(ge^{\pi R\Lambda})J_1(g) - J_1(ge^{\pi R\Lambda})Y_1(g)}{Y_1(ge^{\pi R\Lambda})J_2(g) - J_1(ge^{\pi R\Lambda})Y_2(g)}\right].$$

where $g = m_{3/2}/\lambda$. In the limit $ge^{\pi R\Lambda} \ll 1$, when the KK excitations are heavy, this gives

$$m_{3/2} \sim \kappa_4 v_1^2.$$

This is consistent with our intuition for physics on the Planck brane.

Effective field theory

I will close by describing a formalism that captures the essential features of this analysis in terms of a supersymmetric effective field theory. The theory will be written in terms of flat-space $N = 1$ superfields.

We start by describing the gravity multiplet. We decouple the four-dimensional gravity fields, except for the supergravity auxiliary field M, which is part of the chiral compensator,

$$\Sigma = 1 + \theta\theta M^*.$$

We also retain the bosonic parts of the radion multiplet

$$T = R + iB_5 + \theta\theta\, F_T,$$

where R is the radius and B_5 is part of the graviphoton.

Superspace integration

We use the following superspace integration rules:

- D-terms

$$\frac{3}{2\pi}\int_{-\pi}^{\pi} d\phi \int d\theta^2 d\bar{\theta}^2\ \bar{\Sigma}\Sigma\ (T+\bar{T})\ e^{-\lambda(T+\bar{T})|\phi|}\, F(\bar{\Phi}, V, \Phi)$$

- Superpotential terms

$$\frac{3}{2\pi}\int_{-\pi}^{\pi} d\phi \int d\theta^2\ \Sigma^3\ T\ e^{-3\lambda T|\phi|}\, P(\Phi)$$

- Gauge kinetic terms

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \int d\theta^2\ T\, W W$$

In each case, the warping is described by the chiral compensator Σ.

Kähler potential

For the case of pure gravity, we carry out the integral to find

$$\frac{3}{\lambda\pi}\int d\theta^2 d\bar{\theta}^2\ \bar{\Sigma}\Sigma\ \left(1 - e^{-\pi\lambda(T+\bar{T})}\right).$$

This is of the usual supergravity form,

$$3\int d\theta^2 d\bar{\theta}^2\ \bar{\Sigma}\Sigma\ e^{-K/3},$$

where the Kähler potential K is

$$K = -3\log\left(1 - e^{-\pi\lambda(T+\bar{T})}\right),$$

up to an overall constant. Note that as $\lambda \to 0$, the action reduces to no-scale supergravity, with

$$K = -3\log(T + \bar{T}).$$

Constant superpotentials

Let us test this formalism by adding constant superpotentials to the branes at $z = 0$ and $z = \pi R$.

Therefore we add the following superpotential terms to the superspace action:

$$
\begin{aligned}
\delta S &= \frac{3}{2\pi} \int_{-\pi}^{\pi} d\phi \int d\theta^2 \; \Sigma^3 \; e^{-3\lambda T|\phi|} (\alpha_1 \delta(\phi) - \alpha_2 \delta(\phi - \pi)) + h.c. \\
&= \frac{3}{2\pi} \int d\theta^2 \; \Sigma^3 \; (\alpha_1 - \alpha_2 e^{-3\pi\lambda T}) + h.c.
\end{aligned}
$$

The constants α_1 and α_2 are the brane-localized superpotentials presented earlier.

No-scale model

Let us first consider the case with $\lambda = 0$.

Then it is easy to see that the supergravity auxiliary fields obey the following equations:

$$M = 0, \qquad\qquad F_T = -\frac{3}{2\pi}(\alpha_1 - \alpha_2).$$

Moreover, the potential is flat – with zero vacuum energy – for any value of $\alpha_{1,2}$.

Supersymmetry is broken when $\alpha_1 \neq \alpha_2$, and the radion supermultiplet mediates the supersymmetry breaking.

The no-scale model finds its natural home in extra dimensions.

Warped case

Let us repeat the exercise for $\lambda \neq 0$.

One can show that the supergravity auxiliary fields obey the following equations:

$$M = -\frac{3\alpha_1\lambda}{2}, \qquad F_T = -\frac{3}{2\pi}\left(\alpha_1 - \alpha_2 e^{-\pi\lambda R}\right).$$

The potential is

$$\mathcal{V} = \frac{9\lambda}{4\pi}\left[\frac{\alpha_1^2 - \alpha_2^2 e^{-4\pi\lambda R}}{(1 - e^{-2\pi\lambda R})^2}\right].$$

The minimum of this potential is *not* at zero energy, so this theory does not live in flat space.

We cannot add a constant superpotential in the warped case without adding other dynamics to ensure that the minimum of the potential is at zero energy.

Dynamical SUSY breaking

Now let us add a Goldstone fermion to the Planck brane.

We can describe the Goldstone fermion by a nonlinear chiral superfield, $\Theta = \theta + \chi + \ldots$

The following superpotential term contains the Goldstino kinetic energy, as well as its nonlinear completion:

$$\begin{aligned}\delta S &= \frac{3}{2\pi}\int_{-\pi}^{\pi} d\phi \int d\theta^2 \; \Sigma^3 \; e^{-3\lambda T|\phi|}(\alpha_1 + \beta_1 \Theta^2)\delta(\phi) + h.c. \\ &= \frac{3}{2\pi}\int d\theta^2 \; \Sigma^3 \; (\alpha_1 + \beta_1 \theta^2) + \ldots + h.c.\end{aligned}$$

The supergravity auxiliary fields obey the following equations:

$$M = -\frac{3\alpha_1 \lambda}{2}, \qquad F_T = -\frac{3\alpha_1}{2\pi}.$$

Dynamical SUSY breaking

The potential $\mathcal{V}$ is easy to calculate. It is identically zero when

$$\beta_1 = \frac{9\alpha_1^2}{4}\,\lambda.$$

There is a solution for every value of α_1, which corresponds to the freedom to choose the scale of supersymmetry breaking.

As expected, the Goldstone fermion decouples as $\lambda \to 0$.

We find that there is no consistent vacuum when the Goldstino lives on the visible brane.

We cannot accommodate a Goldstino on the visible brane without including additional dynamics to ensure that the minimum of the potential is at zero energy.

Example: gaugino mediation

The power of this formalism becomes evident when we move beyond pure gravity to treat additional bulk dynamics.

Consider, for example, gaugino mediation, with a single bulk $N = 2$ vector multiplet. The multiplet can be described by two $N = 1$ superfields, Φ and V.

The Lagrangian has two parts. The first contains the kinetic energy for Φ,

$$\frac{3}{\pi g_5^2}\int_{-\pi}^{\pi} d\phi \int d\theta^2 d\bar{\theta}^2\ \bar{\Sigma}\Sigma\ (T+\bar{T})^{-1}\ e^{-\lambda(T+\bar{T})|\phi|}\left(\partial_5 V - \frac{1}{\sqrt{2}}(\Phi - \bar{\Phi})\right)^2$$

The second contains the kinetic energy for the gauge field V,

$$\frac{1}{8\pi g_5^2}\int_{-\pi}^{\pi} d\phi \int d\theta^2\ T\,WW.$$

These terms describe the $N = 2$ vector multiplet, together with its coupling to the radion and the supergravity auxiliary fields.

Supersymmetry breaking

This formalism can be used to study all sorts of higher-dimensional supersymmetry breaking.

Brane masses

Scherk-Schwarz

Brane-localized dynamics

Fayet-Iliopoulos terms

Bulk/brane interactions

It provides a simple way to extract the essential physics.

Work is in progress as we speak!

Summary

In this talk I described the supersymmetric Randall-Sundrum scenario.

We saw that – in the flat case – supersymmetry can be broken by brane-localized mass terms.

In the warped case, supersymmetry can be broken by nonlinear dynamics on the Planck brane.

In each case, the physics could be understood in terms of an effective field theory.

The effective field theory offers a powerful approach to studying supersymmetry breaking in more general situations.