Supersymmetric Particle Physics from Intersecting D-Branes

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Abstract

We review recent progress that has been made in the study of particle physics implications from Type II string with intersecting D-branes. While in the past the focus was on deriving particle physics of four-dimensional N=1 supersymmetric solutions of perturbative heterotic string, recent progress has focused on Type II string theory orientifold compactifications with D-branes. We describe examples of chiral supersymmetric constructions of Type IIA orientifolds with D6-branes intersecting at general angles and highlight the specific construction of the three-family supersymmetric Standard-like Models and supersymmetric GUT models. These constructions, when lifted to M theory, correspond to the compactification of M theory on compact, singular $G_2$ holonomy spaces. We further discuss the phenomenology of these models, such as gauge and Yukawa couplings and dynamical supersymmetry breaking in the additional gauge sector of the models.

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I. INTRODUCTION

If string theory is relevant to nature, at low energies it should give rise to an effective theory containing the Standard Model. Whether string theory can live up to this promise depends on how the string vacuum describing the observable world is selected among a highly degenerate moduli space – a question that we know very little about. Nevertheless, one can use experimental constraints as guiding principles to construct semi-realistic models, and explore with judicious assumptions, the resulting physical implications, in particular to particle physics. This is the basic premise of string phenomenology – hopefully, by exploring the generic features of string derived models, we can learn some new stringy physics that are important for low energy predictions.

Until a few years ago, the construction of four-dimensional string theory solutions was carried out mainly in the framework of weakly coupled heterotic string, in which a number of semi-realistic models have been constructed and analyzed[1]. Meanwhile, model building from other string theories did not seem very promising, partly due to the no-go theorem of perturbative Type II strings. However, M theory unification made important progress in uncovering non-perturbative aspects of string theory: we now understand that these perturbative models represent only a corner of M theory – the true string vacuum may well be in a completely different regime in which the perturbative heterotic string description breaks down.

Our view of string phenomenology changed drastically with the advent of D-branes. The techniques of conformal field theory in describing D-branes and orientifold planes allow, in principle for the construction of semi-realistic string models in another calculable regime of M theory, as illustrated by the various four-dimensional $N = 1$ supersymmetric Type II orientifolds [2–13]. In these models, chiral fermions appear on the worldvolume of the D-branes since they are located at orbifold singularities in the internal space. Semi-realistic models from non-supersymmetric type IIB orientifolds of this kind have been constructed in [13], with supersymmetry breaking due to the presence of brane-antibrane configurations in the model.

An alternative to obtain chiral fermions, which has only recently been exploited in model building is to consider branes at angles. In certain configurations of intersecting D-branes, the spectrum of open strings stretched between them may contain chiral fermions, localized at the intersection[14]. The spectrum of open strings stretched between the intersecting D-branes contains chiral fermions which are localized at the intersection. This fact was employed in [15–19] (and subsequently in [20–23]) in the construction of non-supersymmetric brane world models. In particular, numerous examples of three-family Standard-like models as well as GUT models were obtained. However, the dynamics to determine the stability of non-supersymmetric models are not well understood, especially when the string scale is close to the Planck scale (since the non-supersymmetric models are subject to large quantum corrections). Typically, the models are unstable when D-branes are intersecting at angles (since supersymmetry is generically broken).

Nevertheless, recently supersymmetric orientifold models with branes at angles have been constructed [24–26], resulting in the first examples of $\mathcal{N} = 1$ supersymmetric four-dimensional models with the quasi-realistic features of the Standard Model in this context. In addition to the Standard-like Models, an example of a supersymmetric $SU(5)$ GUT
model with four families of quarks and leptons (i.e., a net number of four $10$-plets and four $\bar{5}$-plets) was presented in [25]. The original construction is based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with D6-branes wrapping specific supersymmetric three-cycles of the six-torus ($T^6 = T^2 \times T^2 \times T^2$). Subsequent phenomenological features of this class of Standard-like models was explored in [27, 28]. In [28] the calculation of the leading contributions to the Yukawa couplings was given explicitly and the implications of these couplings for the fermion mass hierarchy was discussed.

In [27] a detailed study of the gauge couplings and their renormalization group (RG) flow we studied. In particular at String scale, these couplings depend on an additional modulus parameter $\chi \equiv R_2^{(1)}/R_1^{(1)}$, where $R_{1,2}^{(i)}$ are the respective radii of the $i$-th two-torus. While the Standard-Model gauge sector does not predict realistic low-energy values of gauge couplings (primarily due to the additional Higgs and exotic fields in the massless spectrum), the additional non-Abelian gauge sector $Sp(2) \times Sp(2) \times Sp(4)$, allows for the intriguing possibility of dynamical supersymmetry breaking in this sector and the consequences for the moduli stabilization[29]. For more details on these developments see Refs. [27–29].

Recently, a new example of the supersymmetric three-family left-right symmetric model based on $T^6/\mathbb{Z}_4$ orientifold was constructed [30]. Further developments [31] involve the construction of a larger class of supersymmetric three-family Standard-like Models, based on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds, by exploring the wrapping of D6-branes along more general supersymmetric three-cycles (and implementing RR tadpole cancellation conditions). A systematic exploration of a general class of supersymmetric three-family $SU(5)$ GUT models arising from $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds with D6-branes wrapping general supersymmetric three-cycles was given in [32]. Preliminary phenomenological study of these models was also given there.

The supersymmetric orientifold models considered here correspond in the strong coupling limit to compactifications of M theory on certain singular $G_2$ manifolds. As discussed in [26], the D-brane picture provides a simple description of how chiral fermions arise from singularities of $G_2$ compactifications [24, 25, 33, 35–37]. More recently, there have been some interests in exploring the phenomenological properties (e.g., the problem of doublet-triplet splitting, threshold corrections, and proton decay) of GUT models derived from $G_2$ compactifications [38, 39]. It is therefore interesting to explore if the features suggested in [38, 39] apply to this class of orientifold models.

In the following we shall highlight the seminal features of the first example of the supersymmetric three-family Standard-like model. We refer the readers to the original paper[24, 25] for more detailed discussions.

II. SUPERSYMMETRIC MODELS FROM ORIENTIFOLDS AND INTERSECTING D6-BRANES

For concreteness, we consider an orientifold of type IIA on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. Generalization to other orbifolds would involve similar techniques, and presumably analogous final results. The orbifold actions have generators $\theta$, $\omega$ acting as $\theta: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3)$, and $\omega: (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3)$ on the complex coordinates $z_i$ of $T^6$, which is assumed to be factorizable. The orientifold action is $\Omega R$, where $\Omega$ is world-sheet parity, and $R$ acts
by \( R : (z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3) \). The model contains four kinds of O6-planes, associated to the actions of \( \Omega R, \Omega R\theta, \Omega R\omega \), \( \Omega R\theta\omega \). The cancellation of the RR crosscap tadpoles requires an introduction of \( K \) stacks of \( N_a \) D6-branes \( (a = 1, \ldots, K) \) wrapped on three-cycles (taken to be the product of 1-cycles \((n^i_a, m^i_a)\) in the \( i^{th} \) two-torus), and their images under \( \Omega R \), wrapped on cycles \((n'_a, -m'_a)\). In the case where D6-branes are chosen parallel to the O6-planes (orientifold 6-planes), the resulting model is related by T-duality to the orientifold in [3], and is non-chiral. Chirality is however achieved using D6-branes intersecting at non-trivial angles.

The cancellation of untwisted tadpoles impose constraints on the number of D6-branes and the types of 3-cycles that they wrap around. The cancellation of twisted tadpoles determines the orbifold actions on the Chan-Paton indices of the branes (which are explicitly given in [24, 25]).

The condition that the system of branes preserves \( \mathcal{N} = 1 \) supersymmetry requires \[14\] that each stack of D6-branes is related to the O6-planes by a rotation in \( SU(3) \): denoting by \( \theta_i \) the angles the D6-brane forms with the horizontal direction in the \( i^{th} \) two-torus, supersymmetry preserving configurations must satisfy \( \theta_1 + \theta_2 + \theta_3 = 0 \). This in turn impose a constraint on the wrapping numbers and the complex structure moduli \( \chi_i = R_i^2/R_i^1 \).

The rules to compute the spectrum are analogous to those in [17]. Here, we summarize the resulting chiral spectrum in Table I found in [24, 25], where

\[
I_{ab} = (n^1_an^1_b - m^1_an^1_b)(n^2_an^2_b - m^2_an^2_b)(n^3_an^3_b - m^3_an^3_b) \tag{1}
\]

<table>
<thead>
<tr>
<th>Sector</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( aa )</td>
<td>( U(N_a/2) ) vector multiplet</td>
</tr>
<tr>
<td>( aa )</td>
<td>3 Adj. chiral multiplets</td>
</tr>
<tr>
<td>( ab + ba )</td>
<td>( I_{ab} ) chiral multiplets in ((N_a/2, N_b/2)) rep.</td>
</tr>
<tr>
<td>( ab' + b'a )</td>
<td>( I_{ab'} ) chiral multiplets in ((N_a/2, N_b/2)) rep.</td>
</tr>
<tr>
<td>( aa' + a'a )</td>
<td>( -\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a,O6}) ) chiral multiplets in sym. rep. of ( U(N_a/2) )</td>
</tr>
<tr>
<td>( aa' + a'a )</td>
<td>( -\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a,O6}) ) chiral multiplets in antisym. rep. of ( U(N_a/2) )</td>
</tr>
</tbody>
</table>

**TABLE I:** General spectrum on D6-branes at generic angles (namely, not parallel to any O6-plane in all three tori). The spectrum is valid for tilted tori. The models may contain additional non-chiral pieces in the \( aa' \) sector and in \( ab, ab' \) sectors with zero intersection, if the relevant branes overlap.

**A. Standard-Like Model**

Here we present the first example leading to a three-family Standard-like Model massless spectrum. The D6-brane configuration is provided in table II, and satisfies the tadpole
cancellation conditions. The configuration is supersymmetric for \( \chi_1 : \chi_2 : \chi_3 = 1 : 3 : 2 \).

<table>
<thead>
<tr>
<th>Type</th>
<th>( N_a )</th>
<th>( (n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, \tilde{m}_a^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>8</td>
<td>( (0, 1) \times (0, -1) \times (2, \tilde{0}) )</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>2</td>
<td>( (1, 0) \times (1, 0) \times (2, \tilde{0}) )</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>4</td>
<td>( (1, 0) \times (1, -1) \times (1, \frac{3}{2}) )</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>2</td>
<td>( (1, 0) \times (0, 1) \times (0, -\tilde{1}) )</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>6+2</td>
<td>( (1, -1) \times (1, 0) \times (1, \frac{1}{2}) )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>4</td>
<td>( (0, 1) \times (1, 0) \times (0, -\tilde{1}) )</td>
</tr>
</tbody>
</table>

TABLE II: D6-brane configuration for the three-family model.

The resulting spectrum is given in table III, where the last column provides the charges under a particular anomaly-free \( U(1) \) linear combination which plays the role of hypercharge. The spectrum of chiral multiplets, regarding their quantum numbers under the Standard Model group \( SU(3) \times SU(2) \times U(1)_Y \), corresponds to three quark-lepton generations, plus a number of vector-like Higgs doubles, as well as an anomaly-free set of chiral exotic matter. This last set of states is chiral under the Standard Model group, so it cannot be made massive until electroweak symmetry breaking. Hence the model suffers from the presence of light exotics which most likely render it unrealistic. Our main point in presenting it is, however, to illustrate the possibility of building semirealistic models in our setup. It is conceivable that one can construct more realistic models with this approach such that these phenomenological problems are absent.

**B. Phenomenology of the Standard-like Model**

The properties of the perturbative spectrum are discussed in more detail in [27], including the properties of the multiple Higgs doublets, the three regular families, the fourth exotic family, and alternative assignments. The properties of the additional gauge interactions and the possibilities for breaking them at the electroweak or intermediate scales are discussed in [27, 29]. The model does not have the conventional form of gauge unification because each group factor is associated with a different set of branes. However, the string-scale couplings are predicted in terms of the ratio of the Planck to string scales and a geometric factor. The low energy MSSM couplings are too small due to the multiple Higgs fields and exotic matter, while the quasi-hidden sector groups are asymptotically free. The implications of these results for the spectrum are discussed in detail in [27]. In particular, the fractionally charged exotic states presumably disappear from the low energy spectrum due to hidden sector charge confinement, to be replaced by composite states with the appropriate quantum numbers to form the left-handed components of an exotic fourth family.

The Yukawa couplings are due to the world-sheet instantons associated with the action of the string worldsheet stretching among the intersection points where the corresponding chiral matter fields are located [16]. The leading contribution to the Yukawa couplings
TABLE III: Chiral Spectrum of the open string sector in the three-family model. The non-Abelian gauge group is $SU(3) \times SU(2) \times USp(2) \times USp(2) \times USp(4)$. Some vector-like sectors have not been included for the sake of clarity.

is therefore proportional to $\exp(-A/\alpha')$ where $A$ is the smallest area of the triangle associated with the corresponding brane intersections and $\alpha'$ is the string tension. While the complete calculation of the Yukawa couplings requires the techniques of computing string amplitudes involving twisted fields of the conformal field theory describing the open strings, we will approach the study systematically only by studying the leading order contributions to these couplings. Within this context the Yukawa couplings were calculated in [28]. Implications for the fermion mass hierarchy of the model are also discussed in [28].

C. GUT Model

Here we present first the first supersymmetric $SU(5)$ GUT model with four families. The D6-brane configuration is

$$
N_a = (n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, m_a^3)
$$

| Sector | Non-Abelian Reps. | $Q_3$ | $Q_1$ | $Q_2$ | $Q_5$ | $Q_8'$ | $Q_Y$
|--------|-------------------|-------|-------|-------|-------|--------|-------|
| $A_1B_1$ | $3 \times 2 \times (1, 2, 1, 1, 1)$ | 0 | 0 | -1 | ±1 | 0 | ±1/2
| $A_1B_1$ | $3 \times 2 \times (1, 2, 1, 1, 1)$ | 0 | 0 | -1 | 0 | ±1 | ±1/2
| $A_1C_1$ | $2 \times (3, 1, 1, 1, 1)$ | -1 | 0 | 0 | ±1 | 0 | 1/3, ±2/3
| $A_1C_1$ | $2 \times (3, 1, 1, 1, 1)$ | -1 | 0 | 0 | ±1 | 0 | ±1/3, ±2/3
| $B_1C_1$ | $(3, 2, 1, 1, 1)$ | 1 | 0 | -1 | 0 | 0 | 1/3
| $B_1C_1$ | $(1, 2, 1, 1, 1)$ | 0 | 1 | -1 | 0 | 0 | -1/3
| $B_1C_2$ | $(1, 2, 1, 1, 1)$ | 0 | 0 | 1 | 0 | 0 | 0
| $B_2C_1$ | $(3, 1, 2, 1, 1)$ | 1 | 0 | 0 | 0 | 0 | 1/3
| $B_2C_1$ | $(1, 1, 2, 1, 1)$ | 0 | 1 | 0 | 0 | 0 | -1/3
| $B_1C'_1$ | $2 \times (3, 2, 1, 1, 1)$ | 1 | 0 | 1 | 0 | 0 | 1/3
| $B_1C'_1$ | $2 \times (1, 2, 1, 1, 1)$ | 0 | 1 | 1 | 0 | 0 | -1/3
| $B_1B'_1$ | $2 \times (1, 1, 1, 1, 1)$ | 0 | 0 | -2 | 0 | 0 | 0
| $B_1B'_1$ | $2 \times (1, 3, 1, 1, 1)$ | 0 | 0 | 2 | 0 | 0 | 0
which is supersymmetric for \( \arctan \chi_1 - \arctan \chi_2 + \arctan(\chi_3/2) = 0 \). We consider that the first set of 16 branes is split in two parallel stacks of 10 and 6. The resulting spectrum is

\[
U(5) \times U(3) \times USp(16) \\
3(24 + 1, 1, 1) + 3(1, 8 + 1, 1) + 3(1, 1, 119 + 1) \\
4(10, 1, 1) + (5, 1, 16) + 4(5, 3, 1) + (1, 3, 16) + 4(1, 3, 1) \tag{2}
\]

The model is a four-family \( SU(5) \) GUT, with additional gauge groups and matter content. Notice that turning on suitable vev’s for the adjoint multiplets the model corresponds to splitting the \( U(5) \) branes. This provides a geometric interpretation of the GUT Higgsing to the Standard Model group upon splitting \( U(5) \to U(3) \times U(2) \times U(1) \).

III. RELATION TO COMPACTIFICATION OF M THEORY ON \( G_2 \) HOLONOMY SPACES

M theory compactification on a manifold \( X \) with \( G_2 \) holonomy gives rise to an \( \mathcal{N} = 1 \) theory in four dimensions. If \( X \) is smooth, the low energy theory is relatively uninteresting since it contains (in addition to \( \mathcal{N} = 1 \) supergravity) only Abelian vector multiplets and neutral chiral multiplets.\[?\] However, non-Abelian gauge symmetries and chiral fermions can arise when the manifold \( X \) is singular. Isolated conical singularities of \( G_2 \) manifolds have been studied recently from different points of view [24, 25, 33–37]. We now discuss how some of these results can be understood within the orientifold setup.

A useful approach to building \( G_2 \) holonomy spaces is to construct type IIA configurations preserving four supercharges and lifting them to M theory. However, not all four-dimensional \( \mathcal{N} = 1 \) supersymmetric vacua from M theory correspond to \( G_2 \) holonomy compactifications, since the M theory lifts may may contain additional sources, i.e. M-branes or G-fluxes, other than a pure gravitational background. Hence one needs to start with IIA configurations containing D6-branes, O6-planes (and/or RR 1-form backgrounds, which are absent in our setup), only. When lifted to M theory, a collection of \( N \) D6-branes becomes a multi-centered Taub-NUT space [41], whereas an O6-plane becomes an Atiyah-Hitchin manifold [42]. Hence, IIA configurations involving these ingredients, and preserving four supercharges, when lifted to M theory correspond to a purely geometrical background, \( i.e. \), of 11 dimensional space-time compactified on a \( G_2 \) holonomy space. In this respect, the models considered here correspond to M theory compactified on \( G_2 \) holonomy space which give rise to non-Abelian gauge symmetries and chiral fermions. The origin of the non-Abelian gauge symmetries is well-known: gauge bosons arises from the massless M2-brane states wrapped in the collapsed 2-cycles in the multi-Taub-NUT lift of overlapping IIA D6-branes [43]. In the following we remark on the appearance of chiral fermions from the M theory viewpoint.

In configurations where the RR 7-form charges are locally canceled (namely, 2 D6-branes and their 2 images are located on top of each O6-plane), the M theory lift is remarkably simple. The M theory circle is constant over the base space \( B_6 \), leading to a total variety \( (B_6 \times S^1)/\mathbb{Z}_2 \), where the \( \mathbb{Z}_2 \) flips the coordinate parameterizing the M theory circle, and acts on \( B_6 \) as an antiholomorphic involution (hence changing the holomorphic
3-form to its conjugate). This is the type of configurations considered in [44–46] and the resulting models are non-chiral.

In models with D6-branes at angles, chiral fermions arise. In fact, the type IIA description with intersecting D6-branes allows to identify the nature of the singularities of the $G_2$ holonomy space which lead to chiral fermions. The following analysis also makes contact with [37]. Away from the intersections of IIA D6-branes and/or O6-planes, the IIA configuration corresponds to D6-branes and O6-planes wrapped on (disjoint) smooth supersymmetric 3-cycles, which we denote generically by $Q$. The corresponding $G_2$ holonomy space hence corresponds to fibering a suitable Hyperkähler four-manifold over each component of $Q$. That is, an $A$-type ALE singularity for $N$ overlapping D6-branes, and a $D$-type ALE space for D6-branes on top of O6-planes (with the Atiyah-Hitchin manifold for no D6-brane, and its double covering for two D6-branes etc. Intersections of objects in type IIA therefore lift to co-dimension 7-singularities, which are isolated up to orbifold singularities. It is evident from the IIA picture that the chiral fermions are localized at these singularities.

The structure of these singularities has been studies in [37]. One starts by considering the (possibly partial) smoothing of a Hyperkähler ADE singularity to a milder singular space, parameterized by a triplet of resolution parameters (D-terms or moment maps in the Hyperkähler construction of the space). The kind of 7-dimensional singularities of interest are obtained by considering a 3-dimensional base parameterizing the resolution parameters, on which one fibers the corresponding resolved Hyperkähler space. The geometry is said to be the unfolding of the higher singularity into the lower one. This construction guarantees the total geometry admits a $G_2$ holonomy metric. To determine the matter content arising from the singularity, one decomposes the adjoint representation of the A-D-E group associated to the higher singularity with respect to that of the lower. One obtains chiral fermions with quantum numbers in the corresponding coset, and multiplicity given by an index which for an isolates singularity is one.

It is easy to realize this construction arises in the M theory lift of the models presented in the previous section. For example, at points where two stacks of $N$ D6-branes and $M$ D6-branes intersect, the M theory lift corresponds to a singularity of the $G_2$ holonomy space that represents the unfolding of an $A_{M+N-1}$ singularity into a 4-manifold with an $A_M$ and an $A_{N-1}$ singularity. By the decomposition of the adjoint representation of $A_{M+N-1}$, we expect the charged matter to be in the bi-fundamental representation of $SU(N) \times SU(M)$ gauge group, in agreement with the IIA picture. A different kind of intersection arises when $N$ D6-branes intersect with an O6-plane, and consequently with the $N$ D6-brane images. The M theory lift corresponds to the unfolding of a $D_N$ type singularity into an $A_{N-1}$ singularity. The decomposition of the adjoint representation predicts the appearance of chiral fermions in the antisymmetric representation of $SU(N)$, in agreement with the IIA picture. In fact this is the origin of the 10 representations in our previous $SU(5)$ model.

We would also like to note that the generic class of models described here may exhibit some interesting phenomena, e.g., the existence of non-perturbative equivalences among seemingly different models, which nonetheless share the same M theory lift, in analogy with [46]. On the other hand the type IIA transitions in which intersecting D6-branes recombine (which are T-dual of small instanton transitions) would have interesting M
theory descriptions, in which the topology of the $G_2$ holonomy space changes. It would be interesting to explore possible connections of such process with [34]. We hope that our explicit constructions may provide a useful laboratory to probe new ideas in the studies of manifolds with $G_2$ holonomy.

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On the other hand, smooth special holonomy spaces provide suitable backgrounds for constructing regular fractional brane configurations as viable gravity duals of strongly coupled field theories. For a review, see [40] and references therein.