Introduction to the AdS/CFT Correspondence

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Abstract

In this talk we give a brief introduction to the AdS/CFT correspondence, describe some tests, and mention some recent developments.

1 Introduction

The AdS/CFT correspondence is one of the most significant results that string theory has produced. It refers to the existence of amazing dualities between theories with gravity and theories without gravity, and is also sometimes referred to as the gauge theory-gravity correspondence. The prototype example of such a correspondence, as originally conjectured by Maldacena [1], is the exact equivalence between type IIB string theory compactified on $AdS_5 \times S^5$, and four-dimensional $N = 4$ supersymmetric Yang-Mills theory. The abbreviation AdS_5 refers to an anti-de Sitter space in five dimensions, S^5 refers to a five-dimensional sphere. Anti-de Sitter spaces are maximally symmetric solutions of the Einstein equations with a negative cosmological constant. The large symmetry group of 5d anti-de Sitter space matches precisely with the group of conformal symmetries of the $N = 4$ super Yang-Mills theory, which for a long time has been known to be conformally invariant. The term AdS/CFT correspondence has its origin in this particular example, CFT being an abbreviation for conformal field theory. Since then, many other examples of gauge theory/gravity dualities have been found, including ones where string theory is not compactified on an anti-de Sitter space and where the dual field theory is not conformal. Nevertheless, all these dualities are often still referred to as examples of the AdS/CFT correspondence. For more background information and more details, see the various reviews [2, 3, 4, 5, 6, 7, 8, 9, 10, 11].

At first sound, it is quite startling that a duality between a theory with gravity and a theory without gravity could exist. But what is a consistent theory of gravity anyway? String theory provides a consistent framework to compute finite quantum corrections to classical general relativity, but the full non-perturbative structure of string theory is not very well understood. The AdS/CFT correspondence relates a theory with gravity in d dimensions to a local field theory without gravity in $d - 1$ dimensions. If true, it implies that actual quantum degrees of freedom of gravity

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cannot resemble the degrees of freedom of a local field theory defined on the same space. That this has to be true follows among other things from the existence of black holes, as we briefly review in the next section. If the degrees of freedom in gravity would be local, one would imagine that one can have arbitrarily large volumes with fixed energy-density. However, we know that already in classical gravity this is not true: a fixed energy-density in a sufficiently large volume will collapse into a black hole.

In fact, hints for the existence of gauge theory/gravity duality have been around for quite some time, most notably in the case of three dimensions[1]. Gravity in three dimensions can, modulo various subtleties, be described by a Chern–Simons theory [12, 13]. Chern–Simons theory is a topological field theory and when put on manifold with a boundary reduces to a $1+1$ dimensional field theory on the boundary [14, 15]. For compact gauge groups, a precise relation between Hilbert spaces and correlation functions can be established. For non-compact gauge groups such as the ones relevant for $2+1$ dimensional gravity, there still is an equivalence at the level of the actions, but the precise map between Hilbert spaces and correlations functions is not quite as well understood. Nevertheless, the duality between Chern–Simons theory and two-dimensional conformal field theory has many features in common with the AdS/CFT duality.

Is is very hard to directly prove the equivalence between type IIB string theory on $AdS_5 \times S^5$, and four-dimensional $N = 4$ super Yang-Mills theory. For one, as alluded to above, we do not have a good definition of non-perturbative type IIB string theory. Even at string tree level, we do not (yet) know how to solve the theory completely. From this perspective, perhaps a more appropriate perspective is to view $N = 4$ super Yang-Mills theory as the definition of non-perturbative type IIB string theory on the $AdS_5 \times S^5$ background. This implies in particular that the non-perturbative quantum degrees of freedom of string theory do not resemble those it seems to contain at low energies.

A weaker form of the AdS/CFT correspondence is obtained by restricting to low-energies on the string theory side. At low-energies, type IIB string theory on $AdS_5 \times S^5$ reduces to type IIB supergravity on $AdS_5 \times S^5$. The corresponding limit on the gauge theory side is one where both N and $g^2_{YM}N$ become large, where N is the rank of the $U(N)$ gauge group of the $N = 4$ supersymmetric gauge theory (not to be confused with the N appearing in $N = 4$), and g^2_{YM} is the gauge coupling constant. The equivalence between type IIB supergravity on $AdS_5 \times S^5$ and $N = 4$ gauge theory in the large N, large $g^2_{YM}N$ limit has been very well tested by now.

[1]In three of less dimensions, gravity has no local propagating degrees of freedom, so perhaps it is not a good example; still, it will turn out that this case has many similarities with the story in dimensions above three.

2 Large N and holography

The AdS/CFT correspondence is related to two deep ideas in physics.

The first of these is the idea that large N gauge theory is equivalent to a string theory [16]. The perturbative expansion of a large N gauge theory in $1/N$ and $g_{YM}^2 N$ has the form

$$Z = \sum_{g \geq 0} N^{2-2g} f_g(\lambda) \tag{1}$$

where $\lambda \equiv g_{YM}^2 N$ is the so-called 't Hooft coupling. This is similar to the loop expansion in string theory

$$Z = \sum_{g \geq 0} g_s^{2g-2} Z_g, \tag{2}$$

with the string coupling g_s equal to $1/N$. Through some peculiar and not completely understood mechanism, Feynman diagrams of the gauge theory are turned into surfaces that represent interacting strings (but see [17]). Apparently, this is precisely what happens in the AdS/CFT correspondence.

The second is the idea of holography [18, 19]. This idea has its origin in the study of the thermodynamics of black holes. It was shown by Bekenstein and Hawking [20] that black holes can be viewed as thermodynamical systems with a temperature and an entropy. The temperature is directly related to the black body radiation emitted by the black hole, whereas the entropy is given by $S = A/4G$, with G the Newton constant and A the area of the horizon of the black hole. With these definitions, Einstein's equations of general relativity are consistent with the laws of thermodynamics. Since in statistical physics entropy is a measure for the number of degrees of freedom of a theory, it is rather surprising to see that the entropy of a black hole is proportional to the area of the horizon. If gravity would behave like a local field theory, one would have expected an entropy proportional to the volume. A consistent picture is reached if gravity in d dimensions is somehow equivalent to a local field theory in $d-1$ dimensions. Both have an entropy proportional to the area in d dimensions, which is the same as the volume in $d-1$ dimensions. The analogy of this situation to that of an hologram, which stores all information of a 3d image in a 2d picture, led to the name holography. The AdS/CFT correspondence is holographic, because it states that quantum gravity in five dimensions (forgetting the compact five sphere) is equivalent to a local field theory in four dimensions.

3 Anti-de Sitter space

To describe the correspondence in some more detail, we first need to describe the geometry of anti-de Sitter space in some more detail. Five-dimensional anti-de Sitter space can be described as the five-dimensional manifold

$$-X_0^2 - X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 = L^2$$

embedded in a six-dimensional space with metric

$$ds^2 = -dX_0^2 - dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2 + dX_5^2.$$

It can be roughly thought of as a product of four-dimensional Minkowski space times an extra radial coordinate. The metric on Minkowski space is however multiplied by an exponential function of the radial coordinate, and anti-de Sitter space is therefore an example of a warped space: in a suitable local coordinate system,

$$ds^2 = L^2(dr^2 + e^{2r}(\eta_{\mu\nu}dx^\mu dx^\nu)). \tag{3}$$

The parameter L is just a scale factor. The limit where the radial coordinate goes to infinity and the exponential factor blows up is called the boundary of anti-de Sitter space. This boundary is the place where the dual field theory lives. One can indeed verify that string theory excitations in anti-de Sitter space extend all the way to the boundary [21]. In this way one obtains a map from string theory states to states in the field theory living on the boundary. We see that e^r sets the scale of the Minkowski part of the metric, and it turns out that e^r can quite literally also be viewed as a scale of the dual field theory (see e.g. [22]).

4 Correlation functions

The AdS/CFT correspondence in the form in which it was proposed in [1] did not yet provide a detailed map between AdS and CFT quantities. Such a map was given in [23, 24] and makes the correspondence much more explicit. To describe it, we first consider the AdS side, and in particular, we consider a free field with mass m propagating in anti-de Sitter space. The field equation

$$(\Box + m^2)\phi = 0 \tag{4}$$

has two linearly independent solutions that behave respectively as

$$e^{-\Delta r}, \qquad e^{(\Delta-4)r} \tag{5}$$

as $r \to \infty$, where

$$\Delta(\Delta - 4) = m^2. \tag{6}$$

Consider now a solution of the supergravity equations of motion with the boundary condition that the fields behave near $r = \infty$ as

$$\phi_i(r, x^\mu) \sim \phi_i^0(r, x^\mu)e^{(\Delta-4)r}. \tag{7}$$

Then the map between AdS and CFT quantities is given by

$$\exp(-\Gamma_{\text{sugra}}(\phi_i)) = \left\langle \exp\left(\int d^4x\, \phi_i^0 \mathcal{O}_i\right)\right\rangle \tag{8}$$

where the left-hand side is the supergravity action evaluated on the classical solution given by ϕ_i, and the right-hand side is a generating function for correlation functions in super Yang-Mills theory. Actually, we should really use the full string theory partition function subject to the relevant boundary conditions on the left-hand side, to which the supergravity approximation is only the saddle-point approximation. For many applications the above formula suffices though. We also see that there should be an operator $\mathcal{O}_i$ in Yang-Mills theory for every field ϕ_i in AdS. With some further work, one can show that this operator needs to have conformal dimension Δ_i. This yields a non-trivial prediction which we come back to in section 7.1. Finally, note that we restricted to scalar fields in (8). The full AdS/CFT correspondence should of course involve all the AdS degrees of freedom, not just the scalar ones.

5 Mapping between parameters

In order to illustrate the fact that the AdS/CFT duality is an example of a strong-weak coupling duality, we give the relations between the parameters of both theories.

String theory on $AdS_5 \times S^5$ has a dimensionless string coupling constant g_s, which measures the string interaction strength relevant for string splitting and joining, a dimensionful string length l_s, which sets the size of fluctuations of the string world-sheet, and another dimensionful parameter L, the curvature radius of AdS_5 and S^5 that appears in (3).

Four-dimensional $N = 4$ super Yang-Mills theory with gauge group $U(N)$ has, besides the rank N of the gauge group, a dimensionless coupling constant g_{YM}^2.

The identification of the parameters reads

$$g_s = g_{YM}^2, \qquad (L/l_s)^4 = 4\pi g_{YM}^2 N = 4\pi\lambda. \tag{9}$$

By comparing the expansions (1) and (2), we now see that the AdS/CFT correspondence is indeed an example of a weak/strong coupling duality. Depending on the choice of parameters, either AdS or the CFT provides a weakly coupled description of the system, but never both at the same time. Gauge theory is a good description for small $g_{YM}^2 N$ and small g_{YM}^2, whereas string theory is good for large $g_{YM}^2 N$ and small g_{YM}^2. Therefore, the AdS/CFT correspondence can be applied in two directions. We can use string theory to learn about gauge theory, and we can use gauge theory to learn about string theory.

6 Derivation of the AdS/CFT correspondence

The derivation of the AdS/CFT correspondence given in [1] crucially involves the notion of D-branes. D-branes are certain extended objects in string theory, that were introduced by Polchinski in [25]. They are labeled by the number of dimensions of the object, so that a D0 brane is like a particle, a D1 brane is like a string, a D2 brane is like a membrane, etc. There are two ways to think about D-branes. On the one hand, they are solitonic solutions of the equations of motion of low-energy closed string theory. On the other hand, they are objects in open string theory with the property that open strings can end on them. Open strings have a finite tension, and their center of mass cannot be taken arbitrarily far away from the D-brane. As a consequence, the degrees of freedom of the open string can effectively only propagate in a direction parallel to the brane: one says that they are confined to the brane, or that they live on the brane. The open string spectrum can be reproduced directly from the soliton in the closed string description via a collective coordinate quantization.

As a very crude analogy, one can think about two ways to describe a monopole. On the one hand, one can think of it as the 't Hooft-Polyakov monopole, in which case it is an extended soliton solution of the Yang-Mills-Higgs equations of motion. On the other hand, one can view a monopole as a point particle, on which magnetic field lines can end. Both descriptions have their advantages, as do the open and closed string descriptions of D-branes.

To derive the AdS/CFT correspondence, one starts with a stack of D3-branes. This has a description both in terms of open and closed strings. Next, one takes a suitable low-energy limit of the system, which involves taking $l_s \to 0$. The open string description reduces to $N = 4$ super Yang-Mills theory, whereas the closed string description reduces to string theory on $AdS_5 \times S^5$. Thus, the AdS/CFT duality arises as a consequence of the duality between open and closed strings. This duality is most easily visualized by thinking of a cylinder. It can be viewed either as the world-sheet of a closed string moving along an interval, and equivalently as an open string moving along a circle.

7 Tests of the AdS/CFT correspondence

7.1 spectrum of operators

In order for (8) to hold, there should for every gauge-invariant operator in the gauge theory exist a corresponding closed string field on $AdS_5 \times S^5$, whose mass is related to the scaling dimension of the operator according to (6).

It is difficult to test this statement in full generality, because we don't know the spectrum of super Yang-Mills theory at strong coupling, nor do we know the string spectrum at strong coupling. However, we do know the spectrum of a subset of the operators of super Yang-Mills theory at strong coupling, namely the so-called BPS operators.

BPS operators $\mathcal{O}$ have the property that the corresponding state $|\psi\rangle = \mathcal{O}|0\rangle$ is annihilated by some hermitian supersymmetry generator Q, which satisfies $Q^2 = \Delta + J$, with J some $U(1)$ generator[2]. If a state $|\psi\rangle$ is annihilated by Q, it obeys

$$0 = |Q|\psi\rangle|^2 = (\Delta + J)||\psi\rangle|^2 \tag{10}$$

and therefore $\Delta = -J$. The eigenvalues of J are quantized, and therefore as long as there is no phase transition at finite coupling constant the weak and strong coupling values of Δ have to be identical.

In ten-dimensional string theory, there are massless and massive fields. The dimensions of the operators corresponding to massive fields scale as $(L/l_s) \sim (g_{YM}^2 N)^{1/4}$, and vary continuously with the Yang-Mills coupling constant. Such operators can therefore not be BPS, and it is difficult to compare them to operators in the gauge theory.

Massless fields, on the other hand, have scaling dimensions that are independent of $g_{YM}^2 N$, and these should therefore correspond to BPS operators. It has been verified [23] that there is indeed a precise match between massless fields in string theory, and BPS operators in the gauge theory. A first necessary condition for such a matching to be possible is that the global symmetries match. The space $AdS_5 \times S^5$ has isometry group $SO(2,4) \times SO(6)$, whereas $N = 4$ super Yang-Mills theory is invariant under the conformal group $SO(2,4)$ and the R-symmetry group $SO(6)$. The bosonic global symmetries therefore indeed agree. In addition, there are fermionic symmetries that also agree, and together with the bosonic symmetries these form the supergroup $PSU(2,2|4)$. One can verify directly that the massless fields in string theory fall in the same multiplets of $PSU(2,2|4)$ as the BPS operators of super Yang-Mills theory.

7.2 correlation functions

Correlation functions have been considered in great detail, see e.g. [11]. The results are quite encouraging, although one has to face the same problem as when comparing the spectrum of operators, namely one can only compare quantities that can be computed in the field theory at strong coupling. This restricts the set of correlation functions that can be compared to the ones that satisfy non-renormalization theorems in the field theory, so that the strong coupling answer is a straightforward extrapolation of the weak coupling answer. Such correlation functions have been compared to the string theory answer, so far always with success. In addition, the AdS/CFT

[2] The reason that Δ appears is that the Hamiltonian of a conformal field theory has a continuous spectrum when quantized on R^4, but on a cylinder $S^3 \times R$ it has a discrete spectrum, with eigenvalues given by Δ.

correspondence has suggested new non-renormalization theorems, which remain to be proven in field theory, for extremal correlation functions of the form

$$\langle \mathcal{O}_{\Delta}(x)\mathcal{O}_{\Delta_1}(x_1)\dots\mathcal{O}_{\Delta_n}(x_n)\rangle \tag{11}$$

where $\Delta = \sum_i \Delta_i$.

7.3 Wilson loops

Another interesting quantity to compare is the vacuum expectation value of a Wilson loop,

$$\langle \mathrm{Tr}(\mathrm{Pexp}\oint_C A_\mu dx^\mu)\rangle. \tag{12}$$

A Wilson loop in field theory can be expanded in terms of local operators, and using (8) one could in principle use this to determine which computation in supergravity (or string theory) one would have to do to compute this same expectation value. However, it turns out that the string theory calculation has a direct geometric interpretation [26, 27]. A Wilson loop is described by a string world-sheet in AdS, which extends all the way to the boundary of AdS, and approaches there the curve C. A classically stable string world-sheet is a minimal area surface, so that the computation of the Wilson loop vacuum expectation value reduces to a minimal area problem. For a rectangular Wilson loop, one can solve the minimal area problem and in particular find the quark-quark energy as a function of their separation r,

$$E = -\frac{4\pi^2(2g_{YM}^2 N)^{1/2}}{\Gamma(\frac{1}{4})^4 r}. \tag{13}$$

Interestingly, the 't Hooft coupling $g_{YM}^2 N$ appears with a fractional power, and therefore this result cannot remain valid at weak coupling, where the answer will be qualitatively different. The result in (13) is an example of a non-trivial prediction of the AdS/CFT correspondence.

It is amusing to notice that the fundamental string in AdS is the same as the QCD string of Yang-Mills theory. In some sense we have come a full circle towards the original motivation for string theory, namely as an effective theory for the strong interactions.

7.4 finite temperature

So far the field theory, $N = 4$ super Yang-Mills theory, was a theory at zero temperature, and it was obtained in the derivation of the AdS/CFT correspondence as the low energy limit of the degrees of freedom associated to a stack of D3-branes. It turns out that this setup can be generalized to non-zero temperature by employing so-called near extremal D3-branes. In particular, we can compute the free energy as a function of temperature, using either the field theory or the dual gravitational description. The results one obtains are

$$F_{\text{sugra}} = -\frac{\pi^2}{8} N^2 V T^4, \qquad F_{\text{SYM}} = -\frac{\pi^2}{6} N^2 V T^4. \tag{14}$$

There is no disagreement, since the first result is valid at strong coupling in the gauge theory, the second at weak coupling. As the coupling varies, one expects a smooth transition between these two results (for a discussion, see e.g. [10]), but the possibility of a phase transition at a finite value of the coupling has not been ruled out completely.

More generally, we can directly consider a finite temperature version of the AdS/CFT correspondence, by replacing the boundary geometry R^4 in (3) by $S^3 \times S^1$, where the S^1 represents periodic imaginary time so that the system is indeed at finite temperature [28]. The five-dimensional geometry that solves the string equations of motion and has this boundary is no longer anti-de Sitter space, but a space with a different metric. Actually, there are two different five manifolds with conformal boundary $S^3 \times S^1$. One is a finite temperature version of Euclidean AdS, the other describes a Schwarzschild black hole in AdS.

The finite temperature version of AdS dominates the supergravity path integral at low temperatures. In this geometry, the expectation value of the Wilson loop vanishes, and the center of the gauge group is unbroken, as in a confining phase.

The second solution has a nonzero expectation value for the Wilson loop, and the center is broken, as in a deconfining phase.

Altogether this provides a tantalizing geometric picture of the confinement/deconfinement transition: it is mapped to topology changing process in gravity!

7.5 glue balls and the string tension

So far, successful tests of AdS/CFT relied heavily on supersymmetry in order to compare answers at weak and strong coupling. Of course, for more realistic applications, it is desirable to consider cases with less or no supersymmetry. One way to break supersymmetry is to take supersymmetric Yang-Mills at finite temperature, and to take the $T \to \infty$ limit. Naively, the field theory reduces to some non-supersymmetric extension of three-dimensional QCD. To test this, one can try to compute quantities in this version of AdS/CFT and compare those to lattice results in three-dimensional QCD. In particular, one can compute glueball masses using field equations in AdS, and string tensions using the Wilson loop.

There is a considerable disagreement with lattice results. The string theory in question has many additional states with a mass of the order of $T \sim \Lambda_{\text{QCD}}$, which is also the mass scale of the glueballs. At present, no controlled way to remove the cutoff and reach a weak coupling limit has been found.

The string that appears in the AdS/CFT correspondence yields results that differ from the strong coupling lattice results, and this in turn differs from the continuum lattice results. For a detailed overview, see [29]. The latter two are separated by a phase transition, but whether there are any such phase transitions in AdS/CFT remains an open problem.

8 Other solutions

Besides the prototype example of the AdS/CFT correspondence, namely the duality between string theory on $AdS_5 \times S^5$ and $N = 4$ supersymmetric Yang-Mills theory, many other AdS/CFT dualities have been found. These involve p-dimensional AdS_p spaces for various values of p, and are dual to suitable conformal field theories. For instance, type IIB string theory compactified on $AdS_3 \times S^3 \times T^4$ is dual to a certain $1+1$ dimensional conformal field theory [1]. This CFT is relatively well-understood, and it is this CFT that featured in the first counting of microstates of a black hole by Strominger and Vafa [30].

AdS/CFT dualities with less supersymmetry can be found by dividing by a discrete group. For example, if G is a discrete subgroup of $SO(6)$, we can consider $AdS_5 \times S^5/G$, which is dual to a gauge theory with less supersymmetry. This gauge theory is typically of 'quiver type,' 'moose type,' or 'deconstruction type' [31].

9 Interpretation of the extra dimension

Other examples of the AdS/CFT duality no longer involve an anti-de Sitter space. For instance, one can deform the gauge theory by a relevant operator, and in some cases one can solve explicitly for the corresponding supergravity solutions. These gravity solutions are then dual to renormalization group flows that flow between a UV and an IR fixed point. The UV fixed point is the original undeformed gauge theory, the IR fixed point is the low energy limit of the deformed CFT. For such flows one can prove a c-theorem: c, the central charge, is a quantity that appears in the conformal anomaly of the gauge theory, and one can prove that it always decreases along renormalization group flow trajectories. In the gravitational description, c is directly related to the metric of the supergravity solution.

We have actually been somewhat careless in setting up the correspondence as in eqn (8). To do things properly, one should choose a cutoff $r = r_0$, evaluate the supergravity quantities on the truncated space where $r \leq r_0$, and finally take the cutoff r_0 to infinity. In this process, the supergravity action diverges. It can be rendered finite by subtracting local counterterms, exactly as in the standard renormalization of field theories. For a review, see [32]. This illustrates once more the statement that the fifth dimension can be viewed as an energy scale.

In fact, one of the questions that the AdS/CFT immediately raises and perhaps we should have considered right at the beginning, is that of the interpretation of the extra fifth dimension in the field theory. As suggested above, it is closely related to the energy scale in the dual field theory. From the 5d gravitational point of view, low-energy processes in field theory stay close to the boundary of AdS, whereas high-energy processes penetrate deeper in the interior [22]. One can even show that the invariance under 5d general coordinate transformations implies the Callan-Symanzik renormalization group equations in the field theory [33, 34]. Thus, from the 5d point of view, the renormalization group is on an equal footing with 4d Poincaré invariance.

10 AdS/CFT with a cutoff

What happens if we do not remove the cutoff r_0, but instead keep it finite? On AdS, many fields have nonnormalizable modes that correspond to parameters and coupling constants in the dual field theory. Once we have a finite cutoff, all these fields become normalizable, and they therefore correspond to dynamical degrees of freedom in the dual description. In particular, the dual theory will no longer be a field theory, but it will become a field theory coupled to dynamical gravity. Whether one uses this description, or the higher dimensional one that involves truncated AdS, depends on the choices of parameters. Normally the most weakly coupled one will be the most useful.

This version of the AdS/CFT correspondence is very relevant for brane world scenarios and generic string compactifications. The latter typically contain branes, and whether one uses the actual brane description or the dual supergravity description depends on the number of branes, their coupling constants, and the curvature of the ambient geometry. Unfortunately we have no time to venture further in this interesting direction.

11 High energy scattering/deep inelastic scattering

At first sight, the AdS/CFT correspondence, or any duality between string theory and gauge theory, seems at odds with the known fact that the scattering of glueballs at high energies is hard, whereas string scattering at high energies is soft, due to their extended nature. The resolution sits in the fact that AdS is a warped space. When an object moves away from the boundary of AdS, its size is exponentially reduced. Very high energy processes in the gauge theory are described by strings which propagate a long distance from the boundary of AdS before they interact. By that time, the size of the strings has been exponentially reduced, and this compensates for the softness of string interactions to make it into a hard process in the gauge theory [35]. Besides such effects, which are due to the geometric warping of AdS, other gauge theory processes crucially involve strong gravity physics like black hole formation [36]. It is also possible to study the physics of deep inelastic scattering and the parton model from the AdS/CFT point of view [37].

12 Towards a QCD string?

A more involved version of the AdS/CFT correspondence is the one given in [38]. It was discovered by studying branes stuck in singularities in string theory. The gauge theory that appears is an $N = 1$ theory in four dimensions with gauge group $SU(N) \times SU(N + M)$. There are two chiral superfields A_i in the $(N, \overline{N + M})$ representation of the gauge group, and two chiral superfields B_i in the $(\overline{N}, N + M)$ representation. In addition, there is a nontrivial superpotential of the form $W \sim \epsilon^{ij}\epsilon^{kl}\mathrm{tr}(A_i B_k A_j B_l)$.

This field theory has a remarkable property: it has running gauge couplings, but does not become free at either low or high energies. The gauge coupling becomes strong either way. Strongly coupled $N = 1$ theories in four dimensions often admit a dual weakly coupled description, a duality known as Seiberg duality [39]. The same is true here: both at low energies and at high energies there exist dual descriptions. However, these dual descriptions have the same problem: they are not weakly coupled at either low or high energies. Again, they admit suitable dual descriptions. The full picture that emerges is that of an infinite "cascade" of gauge theories, that continues indefinitely at high energies, with an ever increasing rank of the gauge group, but terminates at low energies once e.g. the rank of one of the gauge groups becomes one. At that point, the gauge theory becomes confining. Strictly speaking we need an infinite amount of fine tuning of irrelevant operators to obtain this infinite cascade, but quite remarkable, the dual description of this gauge theory quite naturally sees the same cascade. The ranks N and M of the gauge group become non-trivial functions of the radial coordinate of the dual AdS-like geometry. This also confirms once more the interpretation of the extra fifth dimension in the AdS/CFT correspondence as an energy scale in the field theory.

The AdS-like geometry that is dual to this infinite cascade has several nice features. String theory on this background exhibits (i) confinement, (ii) glueballs and baryons with a mass scale that emerges through dimensional transmutation, exactly as in the gauge theory, (iii) gluino condensates that break the $\mathbf{Z}_{2M}$ chiral symmetry to $\mathbf{Z}_2$, and (iv) domain walls separating different vacua.

The gauge theory at low energies reduces to a pure $N = 1$ supersymmetric Yang-Mills theory. Does the dual geometry therefore provide a dual string theory for pure supersymmetric Yang-Mills theory, the long sought for QCD string? Not really, because it has new degrees of freedom beyond those of the field theory that appear at

Λ_{QCD}, as we already mentioned in the section 7.5. This is a generic problem in trying to find weakly coupled string theory descriptions of gauge theories. To decouple the additional degrees of freedom, we need to make the curvature of the AdS-like geometry large, while keeping the string coupling g_s small. String theory in a strongly curved background is described by a strongly coupled $1+1$ dimensional field theory. The structure of the sigma models relevant for the AdS/CFT correspondence is not very well understood, but there has been progress in this direction recently (see [40] and references therein) , and the prospect of finding a string theory dual of QCD remains an exciting possibility.

13 Other string effects in gauge theories: large quantum numbers and pp-waves

Instead of trying to find a precise string theory dual description of pure $N=1$ supersymmetric Yang-Mills theory, it is also interesting to look for more qualitative stringy behavior in gauge theories.

One place to find such behavior is to look at states with a large scaling dimension proportional to N, the rank of the gauge group. Many gauge theories have baryons with such scaling dimensions, and it turns out that they are not described by strings but by branes in the dual geometrical description [41]. Thus, it is also possible to discover branes in gauge theory.

A related example is to consider operators with a large spin s, like for example $\mathrm{tr}(\Phi D_{\mu_1} \ldots D_{\mu_s} \Phi)$, where Φ is some field that transforms in the adjoint representation of $U(N)$. Such operators correspond to folded rotating closed strings in the dual geometrical description. One can compute the scaling dimension of these operators both in the field theory and in the dual geometrical description. This confirms the equivalence between the two, as one finds in both cases that it behaves like $s + \log s$ [42].

A more complete way to recover string theory from a gauge theory has been described in [43]. The idea is to take a particular scaling limit of the AdS/CFT correspondence. This scaling limit, when applied to the AdS geometry, yields a different geometry known as a "pp-wave". In fact, many geometries admit scaling limits in which they reduce to pp-waves, as originally shown by Penrose [44]. String theory on the pp-wave, in the absence of string interactions, can be exactly solved, and in particular the free string spectrum can be obtained.

On the field theory side, the same limit can be taken. In this limit only a subset of the operators of the full $N=4$ super Yang-Mills theory survive, namely those for which the scaling dimension Δ and a certain global $U(1)$ quantum number J have the property that $\Delta + J$ scales as $N^{1/2}$, while $\Delta - J$ is kept finite, as one takes $N \to \infty$.

Interestingly, this set of operators is in one-to-one correspondence with the set of free string states. This is the first example where a complete string spectrum has been obtained from a gauge theory, albeit in a special scaling limit.

The ground state of the string theory (in light-cone quantization) is described in the gauge theory by the operator $\mathrm{tr}(Z^J)$, where Z is a complex scalar field in the adjoint representation of the gauge group with charge $J = +1$ under the distinguished global $U(1)$ symmetry. The simplest excited states of the string are operators of the form $\sum_i a_i \mathrm{tr}(Z^i \Phi Z^{J-i})$ where the a_i are phases. The string appears from this point of view as composed of "string bits." The string bits are the operators Z, and the string is composed of a string of J bits. Exciting the string amounts to introducing impurities like Φ that are distributed with phases (i.e. a discrete momentum) along the chain of Z's.

A similar discretized picture of string theory can be obtained in several other gauge theories as well, see e.g. [45]. It is also presently being investigated whether one can correctly recover string interactions, or even the full string field theory, from the gauge theory [46]. The emerging picture is that there is a complete agreement, for infinitesimal 't Hooft coupling. However, the only quantities for which the Penrose limit exists are correlation functions in the field theory where one set of operators is taken to $t = -\infty$, the 'in' states, while the other set is taken to $t = +\infty$, the 'out' states. Such correlation functions map to string theory S-matrix elements between corresponding string theory in and out states.

14 Outlook

The AdS/CFT correspondence is still a very lively research area, with many open problems. Clearly, it would be great if one could identify the dual string theories directly for more realistic gauge theories, like $N = 1$ QCD, and other asymptotically free gauge theories. Also, one would like to find holographic dual descriptions of flat space, and of time-dependent backgrounds such as our universe. In addition, it would be very helpful if one could solve string theory on AdS at tree level, in particular at strong coupling, which should correspond to free super Yang-Mills theory at large N.

But one of the most important, difficult and unsolved problems in the AdS/CFT correspondence is to reconstruct 5d local gravitational physics directly from the dual 4d field theory point of view. In particular, we would like to know in what way the local gravitational description breaks down. Does such a breakdown occur in a local way at the Planck length, are in a non-local way at much larger length scales? The AdS/CFT correspondence seems to prefer the second answer, which is also the answer that may provide a resolution to the black hole information paradox. This paradox is based on the fact that semiclassically, everything that falls into a black hole is converted into purely thermal radiation, with no memory of the object that fell in except for its mass and perhaps a few other quantum numbers. Such a process contradicts the usual rules of quantum mechanics, and we can either give up on quantum mechanics or give up on the semiclassical approximation to quantum gravity; AdS/CFT prefers the latter.

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