$N=1$ MIRRORSYMMETRY

P. MAYR

* STRING EFFECTIVE LOW ENERGY ACTION:

$N=1$ 4D SUPERGRAVITY

* DEFINED (AT 2∂) BY 3 "FUNCTIONS"

  - $K(\Phi, \bar{\Phi})$ KÄHLER POTENTIAL
  - $W(\Phi)$ SUPERPOTENTIAL
  - $f(\Phi)$ GAUGE KINETIC FCT

  $\left.\right\}$ HOLOMORPHIC IN CMP'S Φ

* FOR A CLASS OF "OPEN-CLOSED TYPE II STRINGS" THE HOLOMORPHIC DATA W & f CAN BE COMPUTED EXACTLY FROM FUNDAMENTAL STRING THEORY

* VERY SPECIAL $W(\Phi), f(\Phi), \ldots$

$$\begin{matrix} W(\Phi) \\ f(\Phi) \end{matrix} \longleftrightarrow \text{PARTITION FUNCTIONS } \mathcal{F}_{g,h} \text{ OF A 2D TFT}$$

↳ "$N=1$ SPECIAL GEOMETRY"

Open-Closed Type II Strings

* Type II in 10 D: $N=2$ Supersymmetry

- RR sector: Gauge potentials $A^{(p)}$ $\begin{cases} p \text{ odd} & \text{IIA} \\ p \text{ even} & \text{IIB} \end{cases}$

- Couple to D-branes

$$\int_{WV} A^{(p+1)}$$

Dp-brane:
p Space, 1 Time Dim

* Compactif. to 4D

$\longrightarrow$ $N=2$, $D=4$

X: "Calabi-Yau" Manifold (Ricci-flat Kähler MF

* There are two closely related modifications that break (i.e.) $N=2$ SUSY

- $C^{(d)}$: d-cycle in X $H_d(X, \mathbb{Z})$

BG Flux

- $\int_{C^{(p)}} F^{(d)} = N_K$

BG Branes

- D-brane wrapping $C^{(p)}$ & filling ST, $\# = M_K$

* So a Type II open-closed compactification is specified by

$$(X\,;N_k, M_k)$$

Flux — Branes

	Flux	Branes
IIA	C^{even}	C^{odd}
IIB	C^{odd}	C^{even}

* More generally: also NS fluxes & branes

* S_{eff} depends on VEV's of scalar fields in CMP's

"Moduli" $\phi \longleftrightarrow$ Geometry of X and branes

*** Complex def. type**

- Volumes of 3-manifolds
- Volume form $\Omega^{3,0}$

CS — z_a: Complex structure of X

OS — $\hat{z}_\alpha$: "Position" of (D-)brane on $C^{(3)}$

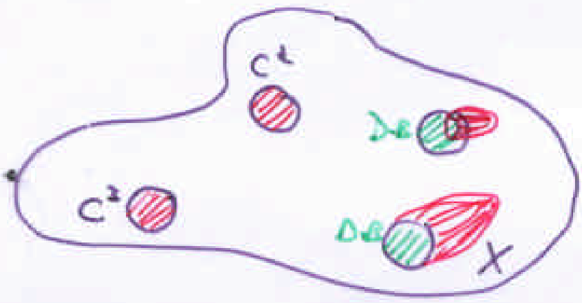
*** Kähler type**

- Volume of 2-manifolds
- Volume form J Kähler form

t_a: Kähler structure of X

$\hat{t}_\alpha$: "Position" of (D-)brane on $C^{(2)}$

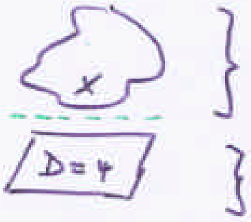
$\hookrightarrow$ Space $\mathcal{M}$ of VEV's of CMP's



E Eff. Space-Time Action From Topol. String's

* We would like to know S_{eff}, in particular $W(\phi), f(\phi)$ for (X, N_K, M_K) in dependece of $(z_a, z_\alpha; t_a, t_\alpha)$

* ST is defined by 2D CFT on the worldsheet Σ



Internal CFT of $\hat{c}=3$ $\longleftrightarrow$ X + Branes

* 4D effective action is computed from CFT corr. fcts

$$\underset{ST}{\sim\sim} = \langle CFT \rangle \approx \langle 4D \rangle \cdot \boxed{\langle \text{INT } \hat{c}=3 \rangle} \to W(\phi)$$

* In general, the internal part is hard to compute exactly:

— String loop exp — W_S corrections

* For a special set of space-time amplitudes the internal corr. fct is computed by a simpler, topological 2D theory. Some important cases:

- Witten
- Bershadsky, Cecotti, Ooguri, Vafa
- Antoniadis, Gava, Narain, Taylor
- Ooguri, Vafa

<u>N=2</u>: Holomorphic prepot $\mathcal{F}(A) \longleftrightarrow \mathcal{F}_{0,0}$

$\mathcal{F}_{g,h}$: Top. PF at genus g, with h boundaries

<u>N=1</u>: Superpotential $W \longleftrightarrow \mathcal{F}_{0,0}, \mathcal{F}_{0,1}$ 

Gauge kinetic fct $f \longleftrightarrow \mathcal{F}_{0,2}$ 

✗ . . .

B A-MODEL

Witten

* COMPATIBLE WITH D-BRANE / $C^{(3)}$ $\xrightarrow{\text{here}}$ IIA

* TFT CORRELATOR GETS CONTRIBUTIONS FROM

HOLOMORPHIC MAPS $\Sigma \longrightarrow X$ Σ: EUCLIDEAN WS

- CONSTANT MAPS:

- WORLD-SHEET INST:

RSF C $\sim e^{-\mathrm{Vol}\, C}$

$\hookrightarrow$ GEOMETRIC ANSWER: $\boxed{\langle \text{A-MODEL} \rangle = \text{class.} + \sum n_k e^{-\mathrm{Vol}\, C_k}}$

n_k: "NUMBER" OF RSF WITH FIXED VOL. IN X

* DEPENDS ONLY ON KÄHLER TYPE MODULI

* INSTANTON EXP. CAN BE COMPUTED BY "COUNTING" MAPS

- Kontsevi
- Katz, Li
- Graber, Zaslow

! WS INSTANTONS $\longleftrightarrow$ SPACE-TIME INSTANTONS

- COUPLING CONSTANT IN RR SECTOR IS NOT g_{string}

$$S_{4D} = \int e^{\kappa\varphi} (NS) + (\mathrm{Vol}) \cdot (RR)$$

- Ex: 4D RR GAUGE THEORY $\longrightarrow$ $\frac{1}{g_Y^2} = \mathrm{Vol}(C_2)$

* TOPOL. A-MODEL COMPUTES SPACE-TIME INSTANTON EXPANSION

B-MODEL

Witten

* COMPATIBLE WITH D-BRANE ON C^{EVEN}

  $\longrightarrow$ IN THIS CONTEXT: IIB

* $\langle$B-MODEL$\rangle$ GETS CONTRIBUTIONS FROM CONSTANT MAPS

  ↳ CLASSICAL RESULT = EXACT

* FOR GENUS 0 ($\longleftrightarrow$ $W(\theta)$) AND BRANES ON $C^{(2)}$, THE ANSWER IS COMPUTED BY INTEGRALS OF $\Omega^{3,0}$

$$f(z_i, \hat{z}_i) = \int_{M_3} \Omega$$

M_3 $\xrightarrow{\text{FLUX}}$ 3-CYCLE IN X

M_3 $\xrightarrow{\text{BRANE}}$ 3-CHAIN IN X WITH BOUNDARY $\partial\Gamma = C^{(2)}$

- Taylor, Vafa
- P.M.

- Aganagic, Vafa
- Witten

3 Physical Interpretation & Mirror Symmetry

IIA / X

N_K: Fluxes on C^{even}

n_K: Branes on $C^{(3)}$

A-model

$$W \to c_{\text{lass}} + \sum n_K e^{-\text{Vol}\, C_K}$$

$$= W(t_a, \hat{t}_\alpha)$$

\+ Weak coupling expansion
↳ Physical interpretation

− Computable, but tedious

IIB / $\tilde{X}$

N_K: Fluxes on $C^{(3)}$

n_K: Branes on $C^{(2)}$

B-model

$$W \to \int \Omega^{3,0}$$

$$= W(z_a, \hat{z}_\alpha)$$

"Decoupling hypothesis"
Douglas

\+ Exact result

− No obvious weak coupling interpretation

MIRROR SYMMETRY

$IIA/X \overset{!}{\simeq} IIB/\tilde{X}$

$X, \tilde{X}$: MIRROR MANIFOL[D]

("T-DUALITY" SYZ)

+ BRANES

- Mori, Vafa
- Mori, Iqbal, Vafa
- Aganagic, Vafa

$\mathcal{N}=1$ MIRROR MAP

$z_a(t_a, \tilde{t}_\alpha)$

- Aganagic, Klemm, Vafa
- PM

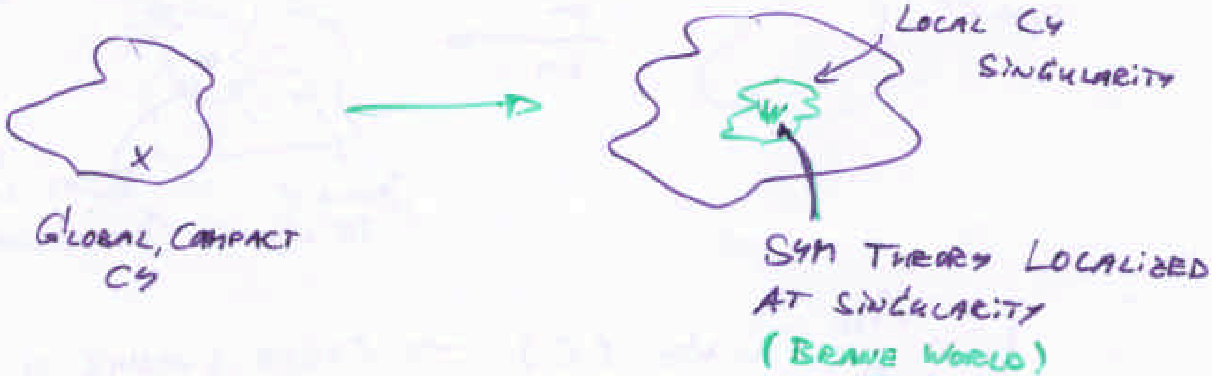
$\hookrightarrow W(z) \longrightarrow W(z(t)) = \sum e^{-t}$

EXACT B-MODEL RESULT

WEAK COUPLING EXPANSION

$\boxed{2}$ Exact $W=1$ Superpotentials For $N=1$ SYM + Gravity

* FT Embedded In Type II Compactifications Can Be Studied From Local CY Singularities



- FT Limit : $\frac{\Lambda_{FT}}{M_{PL}} \sim \mathrm{Vol}(C) \ll 1$

  ↳ Vanishing Cycles

* Local Singularities Are Simpler.

One Can Design Singularities That Support A Large Class Of $N=2$ $D=4$ SYM Theories And Use Mirror Symmetry To Compute The Exact $N=2$ Prepotential $\mathcal{F}(A)$

"Geometric Engineering"

* To Study $N=1$ SYM Theories

↳ Local Singularities + Flux

* FLUX $\longrightarrow$ MASS TERMS $m\,\Phi^2_{adj}$ — Taylor, Vaf…

— DV

* $N=1$ SUPERPOTENTIAL

$$W = N_K \cdot \Pi_K \qquad \Pi_K = \int \Omega^{3,0}$$

$\angle$ $N=1$ VACUA WITH

— CONFINEMENT BY MONOPOLE CONDENSATION — Seiberg Witten

— GAUGINO CONDENSATE

* AN EXTREMLY POWERFUL METHOD TO DESIGN LOCAL SINGULARITIES WITH FLUXES IS

LARGE N DUALITY: — Vafa

FLUX GEOMETRIES BY A (DYNAMICAL) TRANSITION FROM D-BRANE GAUGE THEORIES

$\angle$ - LARGE CLASS OF EXACT $N=1$ RESULTS — Cachazo, Intriligator, Fiol, Vafa, Kato

- GENERAL PERTURBATIONS $\Sigma\, a_K \Phi^K_{adj}$

- SEIBERG DUALITY

* STRING THEORY CORRECTIONS

$W_{N=1} = W_{FT} + \varepsilon^2 W_{GT} + \dots$ — P.M.

GAUGINO CONDENSATE + GRAVITY $\longrightarrow$ $N=1 \rightarrow N=0$

— COMPUTABLE N.P. SUSY-BREAKING

— LEADING COSM. CONSTANT = ZERO

— SMALL STRING SCALE IS FAVORED

☐ S_{eff} DEPENDS ON VEV'S OF SCALAR FIELDS ϕ IN 4D;

$\phi \longleftrightarrow$ GEOMETRY OF X + BRANES

☐ COMPLEX DEFORMATIONS

- VOLUMES OF 3-MANIFOLDS CS, z_a COMPLEX STR.

* IN STRING THEORY, THE "FUNCTIONS" $W(\phi)$, $f(\phi)$ HAVE A MORE FUNDAMENTAL MEANING AS TOPOLOGICAL PARTITION FCTS OF THE WORLD-SHEET THEORY

* THIS IMPLIES VERY SPECIAL PROPERTIES...

* WHAT IS THE GENERAL STRUCTURE THAT DISTINGUISHES THE OPEN-CLOSED STRING-EFF. ACTION FROM GENERIC N=1 SUGRA?

N=1 SPECIAL GEOMETRY (HOL. HALF)

Lunde P.M.

- IN THESE OPEN-CLOSED TYPE II COMPACTIFICATIONS, THE GENERAL FORM OF THE SUPERPOTENTIAL IS

$$W = \sum_{\Sigma} L_{\Sigma} W_{\Sigma}(\phi)$$

W_Σ : SET OF HOL. POTENTIALS IN THE CFP'S

L_Σ : FLUX AND BRANES $\rightarrow \mathbb{Z}^N \simeq H_3(X, Z;$

- THE POTENTIALS W_Σ ARE IN A SENSE GENERALIZATIONS OF THE HOL. PREPOTENTIAL $\mathcal{F}$ OF N=2 SPECIAL GEOMETRY

- TFT: $W_\Sigma \longleftrightarrow$ CHIRAL RING

  - RING OF 2D SUPERFIELDS $\Phi_A^{(q)}$

$$\Phi_A^{(1)} \cdot \Phi_B^{(1)} = C_{AB}{}^{\Sigma} \, \Phi_\Sigma^{(2)}$$

  - RING STRUCTURE "CONSTANTS"

$$C_{AB}{}^{\Sigma}(t) = \partial_A \partial_B W_\Sigma(t)$$

- TOP. FLAT CONNECTION ∇_A ON FAMILY OF TFT'S PARAMETRIZED BY $N=1$ MODULI FIELDS

$$(\nabla_A - C_A)\, W_\varepsilon^k = 0$$

- THIS LEADS TO A SYSTEM OF LINEAR DIFFERENTIAL EQUATIONS THAT DETERMINE THE EXACT INSTANTON EXPANSION OF $W_{N=1}$

$$\mathcal{D}\, W_\varepsilon = 0$$

- SOLUTIONS: - EXACT POTENTIALS $W_\varepsilon(z)$
  - $N=1$ MIRROR MAP $z(t)$

$$\boxed{W(t) = \sum n_k\, e^{-t}}$$

* THE GENERAL PROPERTIES OF 2D TFT *
SHOULD LEAD TO MANY MORE INTERESTING
CONSTRAINTS ON THE $N=1$ EFFECTIVE
STRING SUPERGRAVITY, SUCH AS E.G.
RECURSION RELATIONS FOR AN ∞ #
OF HIGHER DERIVATIVE COUPLINGS

↳ TO BE CONTINUED...

* • tt^* EQUATIONS (CECOTTI & VAFA)
• HOLOMORPHIC ANOMALIES (BERSHADSKY, CECOTTI, OOGURI, VAFA)

- OPEN-CLOSED TYPE II COMPACTIFICATION

  CY X + FLUXES & BRANES (N_κ, n_κ)

- 4D SCALAR FIELDS $\longleftrightarrow$ GEOMETRY OF X AND BRANES

  – $(z_a, \hat{z}_a)$ CS TYPE

  – $(t_a, \hat{t}_a)$ KÄHLER TYPE

- $W(\phi)$, $f(\phi)$ $\longleftrightarrow$ $\langle TFT \rangle$

  – A-MODEL : INSTANTON EXP. $f(t_a, \hat{t}_a)$

  – B-MODEL : EXACT RESULT $f(z_a, \hat{z}_a)$

- MIRROR SYMMETRY:

$$\frac{\text{A-MODEL}}{X} \cong \frac{\text{B-MODEL}}{\tilde{X}}$$

+ FLUX + BRANES   + FLUX + BRANES