$N=1$ MIRRORSYMMETRY

P. MAYR

* STRING EFFECTIVE LOW ENERGY ACTION:

$N=1$ 4D SUPERGRAVITY

* DEFINED (AT 2∂) BY 3 "FUNCTIONS"

  - $K(\Phi,\bar{\Phi})$ KÄHLER POTENTIAL
  - $W(\Phi)$ SUPERPOTENTIAL
  - $f(\Phi)$ GAUGE KINETIC FCT

  $\left.\right\}$ HOLOMORPHIC IN CMP'S Φ

* FOR A CLASS OF "OPEN-CLOSED TYPE II STRINGS" THE HOLOMORPHIC DATA W & f CAN BE COMPUTED EXACTLY FROM FUNDAMENTAL STRING THEORY

* VERY SPECIAL $W(\Phi), f(\Phi), \ldots$

$$\begin{matrix} W(\Phi) \\ f(\Phi) \end{matrix} \longleftrightarrow \begin{matrix} \text{PARTITION FUNCTIONS } F_{g,h} \\ \text{OF A 2D TFT} \end{matrix}$$

↳ "$N=1$ SPECIAL GEOMETRY"

Open-Closed Type II Strings

* Type II in 10 D: $N=2$ Supersymmetry

- RR sector: Gauge Potentials $A^{(p)}$ $\begin{cases} p \text{ Odd} & \text{IIA} \\ p \text{ Even} & \text{IIB} \end{cases}$

- Couple to D-branes

$$\int_{WV} A^{(p+1)}$$

Dp-brane:
p Space
1 Time
Dim

* Compactif. to 4D

$\longrightarrow$ $N=2$, $D=4$

X: "Calabi-Yau" Manifold
(Ricci-Flat Kähler MF

* There are two closely related modifications that break (i.e.) $N=2$ Susy

- $C^{(d)}$: d-cycle in X $\qquad H_d(X, \mathbb{Z})$

BG Flux

- $\int_{C^{(p)}} F^{(d)} = N_k$

BG Branes

- D-brane wrapping $C^{(p)}$ & filling ST, $\# = M_k$

* SO A TYPE II OPEN-CLOSED COMPACTIFICATION IS SPECIFIED BY

$$(X ; N_k, M_k)$$

	FLUX	BRANES
IIA	C^{EVEN}	C^{ODD}
IIB	C^{ODD}	C^{EVEN}

* MORE GENERALLY: ALSO NS FLUXES & BRANES

* S_{EFF} DEPENDS ON VEV'S OF SCALAR FIELDS IN CMP'S

"MODULI" $\phi \longleftrightarrow$ GEOMETRY OF X AND BRANES

* COMPLEX DEF. TYPE

- VOLUMES OF 3-MANIFOLDS
- VOLUME FORM $\Omega^{3,0}$

CS — z_a: COMPLEX STRUCTURE OF X

OS — $\hat{z}_\alpha$: "POSITION" OF (D-)BRANE ON $C^{(3)}$

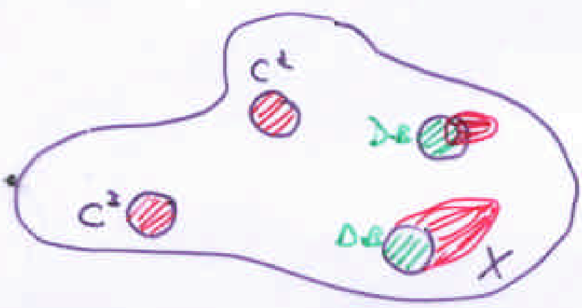
* KÄHLER TYPE

- VOLUME OF 2-MANIFOLDS
- VOLUME FORM J KÄHLER FORM

t_a: KÄHLER STRUCTURE OF

$\hat{t}_\alpha$: "POSITION" OF (D-)BRANE ON $C^{(3)}$

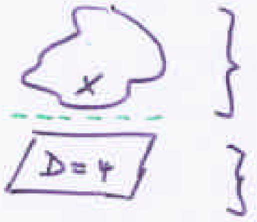
$\hookrightarrow$ SPACE $\mathcal{M}$ OF VEV'S OF CMP'S



⊡ Eff. Space-Time Action From Topol. String's

* We would like to know S_{eff}, in particular $W(\phi), f(\phi)$ for (X, N_K, M_K) in dependece of $(z_a, z_\alpha ; t_a, t_\alpha)$

* ST is defined by 2D CFT on the worldsheet Σ



Internal CFT of $\hat{c}=3$ $\longleftrightarrow$ X + Branes

* 4D effective action is computed from CFT Corr. Fcts

$$\underset{ST}{\sim\!\!\sim} = \langle CFT \rangle \approx \langle 4D \rangle \cdot \boxed{\langle \text{INT } \hat{c}=3 \rangle} \to W(\phi)$$

* In general, the internal part is hard to compute exactly:

— String loop exp — W_S corrections

* For a special set of space-time amplitudes the internal corr. fct is computed by a simpler, topological 2D theory. Some important cases:

- Witten
- Bershad
Cecotti
Ooguri
Vafa

__N=2__: Holomorphic prepot $\mathcal{F}(A) \longleftrightarrow \mathcal{F}_{0,0}$

- Antoni
Gava
Narain
Taylor

$\mathcal{F}_{g,h}$: Top. PF at genus g, with h boundaries

- Ooguri
Vafa

__N=1:__ Super potential $W \longleftrightarrow \mathcal{F}_{0,0}, \mathcal{F}_{0,1}$

Gauge kinetic fct $f \longleftrightarrow \mathcal{F}_{0,2}$

✗ . . .

B A-MODEL

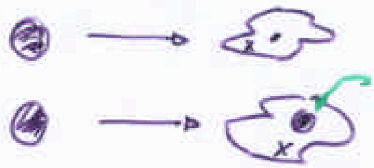
Witten

* COMPATIBLE WITH D-BRANE / $C^{(2)}$ $\xrightarrow{\text{here}}$ IIA

* TFT CORRELATOR GETS CONTRIBUTIONS FROM

HOLOMORPHIC MAPS $\Sigma \longrightarrow X$ Σ: EUCLIDEAN WS

- CONSTANT MAPS:
- WORLD-SHEET INST:



RSF C $\sim e^{-\text{Vol}\,C}$

$\hookrightarrow$ GEOMETRIC ANSWER:

$$\boxed{\langle \text{A-model} \rangle = \text{class.} + \sum n_k\, e^{-\text{Vol}\, C_k}}$$

n_k: "NUMBER" OF RSF WITH FIXED VOL. IN X

* DEPENDS ONLY ON KÄHLER TYPE MODULI

* INSTANTON EXP. CAN BE COMPUTED BY "COUNTING" MAPS

- Kontsevi
- Katz, Li
- Graber, Zaslow

! WS INSTANTONS $\longleftrightarrow$ SPACE-TIME INSTANTONS

- COUPLING CONSTANT IN RR SECTOR IS NOT g_{string}

$$S_{4D} = \int e^{\kappa\phi}(NS) + (\text{Vol}) \cdot (RR)$$

- Ex: 4D RR GAUGE THEORY $\longrightarrow$ $\frac{1}{g_{YM}^2} = \text{Vol}(C_2)$

* TOPOL. A-MODEL COMPUTES <u>SPACE-TIME</u> INSTANTON EXPANSION

B-model

Witten

* Compatible with D-brane on C^{even}

  $\longrightarrow$ In this context: IIB

* $\langle$B-model$\rangle$ gets contributions from constant maps

  $\hookrightarrow$ Classical result = <u>Exact</u>

* For genus 0 ($\leftrightarrow W(0)$) and branes on $C^{(2)}$, the answer is computed by integrals of $\Omega^{3,0}$

$$f(z_a,\hat{z}_a) = \int_{M_3} \Omega$$

$$M_3 \begin{cases} \xrightarrow{\text{Flux}} \text{3-cycle in } X \\ \xrightarrow{\text{Brane}} \text{3-chain in } X \text{ with boundary } \partial\Gamma = C^{(2)} \end{cases}$$

- Taylor, Vafa
- P.M.

- Aganagic, Vafa
- Witten

3 Physical Interpretation & Mirror Symmetry

IIA / X

N_K: Fluxes on C^{even}

n_K: Branes on $C^{(3)}$

A-model

$$W \to c_{lass} + \sum n_K \, e^{-Vol\, C_K}$$

$$= W(t_a, \hat{t}_K)$$

+ Weak Coupling Expansion
  ↳ Physical interpretation

− Computable, But Tedious

IIB / $\tilde{X}$

N_K: Fluxes on $C^{(3)}$

n_K: Branes on $C^{(2)}$

B-model

$$W \to \int \Omega^{3,0}$$

$$= W(z_a, \hat{z}_K)$$

"Decoupling Hypothesis"
Douglas

+ Exact Result

− No Obvious Weak Coupling interpretation

MIRROR SYMMETRY

$$IIA/X \overset{!}{\simeq} IIB/\tilde{X}$$

$X, \tilde{X}$: MIRROR MANIFOL[D]
("T-DUALITY" SYZ)

+ BRANES

- Mari, Vafa
- Mari, Iqbal, Vafa
- Aganagic, Vafa

$\mathcal{N}=1$ MIRROR MAP

$$z_a(t_a, \bar{t}_\alpha)$$

- Aganagic, Klemm, Vafa
- AM

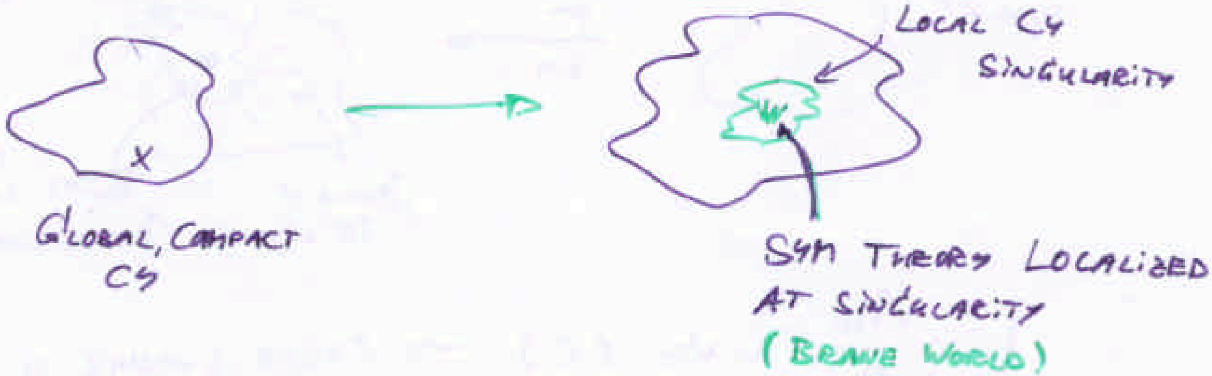
$$\Rightarrow W(z) \longrightarrow W(z(t)) = \sum e^{-t}$$

EXACT B-MODEL RESULT

WEAK COUPLING EXPANSION

4) EXACT W=1 SUPERPOTENTIALS FOR N=1 SYM + GRAVITY

* FT EMBEDDED IN TYPE II COMPACTIFICATIONS CAN BE STUDIED FROM LOCAL CY SINGULARITIES



- FT LIMIT : $\frac{\Lambda_{FT}}{M_{PL}} \sim \mathrm{VOL}(C) \ll 1$

  $\hookrightarrow$ VANISHING CYCLES

* LOCAL SINGULARITIES ARE SIMPLER.
ONE CAN DESIGN SINGULARITIES THAT SUPPORT A LARGE CLASS OF N=2 D=4 SYM THEORIES AND USE MIRROR SYMMETRY TO COMPUTE THE EXACT N=2 PREPOTENTIAL $\mathcal{F}(A)$

"GEOMETRIC ENGINEERING"

* TO STUDY N=1 SYM THEORIES

$\hookrightarrow$ LOCAL SINGULARITIES + FLUX

* FLUX $\rightarrow$ MASS TERMS $m\,\Phi^2_{adj}$ — Taylor, Vafa — Dm

* $N=1$ SUPERPOTENTIAL

$$W = N_K \cdot \Pi_K \qquad \Pi_K = \int \Omega^{3,0}$$

$\hookrightarrow$ $N=1$ VACUA WITH

- CONFINEMENT BY MONOPOLE CONDENSATION — Seiberg Witten
- GAUGINO CONDENSATE

* AN EXTREMLY POWERFUL METHOD TO DESIGN LOCAL SINGULARITIES WITH FLUXES IS

LARGE N DUALITY: — Vafa

FLUX GEOMETRIES BY A (DYNAMICAL) TRANSITION FROM D-BRANE GAUGE THEORIES

$\hookrightarrow$ - LARGE CLASS OF EXACT $N=1$ RESULTS — Cachazo, Intriligator, Fiol, Vafa, Katz

- GENERAL PERTURBATIONS $\Sigma a_K \Phi^K_{adj}$
- SEIBERG DUALITY

$\boxed{+}$ STRING THEORY CORRECTIONS

$$W_{N=1} = W_{FT} + \varepsilon^2 W_{GT} + \ldots$$ P.M.

GAUGINO CONDENSATE + GRAVITY $\longrightarrow$ $N=1 \rightarrow N=0$

- COMPUTABLE N.P. SUSY-BREAKING
- LEADING COSM. CONSTANT = ZERO
- SMALL STRING SCALE IS FAVORED

S_{eff} DEPENDS ON VEV'S OF SCALAR FIELDS Φ IN $\mathcal{M}$'s

$\Phi \longleftrightarrow$ GEOMETRY OF X + BRANES

COMPLEX DEFORMATIONS

* IN STRING THEORY, THE "FUNCTIONS" $W(\Phi)$, $f(\Phi)$, ... HAVE A MORE FUNDAMENTAL MEANING AS TOPOLOGICAL PARTITION FCTS OF THE WORLD-SHEET THEORY

– VOLUMES OF 3-MANIFOLDS CS

z_a COMPLEX STR.

$\mathcal{J}C^{2,0}$

* THIS IMPLIES VERY SPECIAL PROPERTIES...

* WHAT IS THE GENERAL STRUCTURE THAT DISTINGUISHES THE OPEN-CLOSED STRING-EFF. ACTION FROM GENERIC N=1 SUGRA?

N=1 Special Geometry (Hol. Half)

Lande P.M.

- In these open-closed Type II compactifications, the general form of the superpotential is

$$W = \sum_{\Sigma} L_{\Sigma} W_{\Sigma}(\phi)$$

W_{Σ} : Set of hol. potentials in the CFT's

L_{Σ} : Flux and branes $\rightarrow \mathbb{Z}^N \simeq H_3(X, Z;$

- The potentials W_{Σ} are in a sense generalizations of the hol. prepotential $\mathcal{F}$ of N=2 special geometry

- TFT: $W_{\Sigma} \longleftrightarrow$ Chiral Ring

  - Ring of 2D superfields $\Phi_A^{(q)}$

$$\Phi_A^{(1)} \cdot \Phi_B^{(1)} = C_{AB}{}^{\Sigma} \, \Phi_{\Sigma}^{(2)}$$

  - Ring structure "constants"

$$C_{AB}{}^{\Sigma}(t) = \partial_A \partial_B W_{\Sigma}(t)$$

- TOP. FLAT CONNECTION ∇_A ON FAMILY OF TFT'S PARAMETRIZED BY $N=1$ MODULI FIELDS

$$(\nabla_A - C_A)\, W_\Sigma^K = 0$$

- THIS LEADS TO A SYSTEM OF LINEAR DIFFERENTIAL EQUATIONS THAT DETERMINE THE EXACT INSTANTON EXPANSION OF $W_{N=1}$

$$\mathcal{D}\, W_\Sigma = 0$$

- SOLUTIONS: - EXACT POTENTIALS $W_\Sigma(z)$
  - $N=1$ MIRROR MAP $z(t)$

$$\boxed{W(t) = \sum n_k e^{-t}}$$

* THE GENERAL PROPERTIES OF 2D TFT*
SHOULD LEAD TO MANY MORE INTERESTING
CONSTRAINTS ON THE $N=1$ EFFECTIVE
STRING SUPERGRAVITY, SUCH AS E.G.
RECURSION RELATIONS FOR AN ∞ #
OF HIGHER DERIVATIVE COUPLINGS

↳ TO BE CONTINUED...

* • tt^* EQUATIONS (CECOTTI & VAFA)
• HOLOMORPHIC ANOMALIES (BERSHADSKY, CECOTTI, OOGURI, VAFA)

- OPEN-CLOSED TYPE II COMPACTIFICATION

  CY X + FLUXES & BRANES $(N_K, \mathcal{M}_K)$

- 4D SCALAR FIELDS $\longleftrightarrow$ GEOMETRY OF X AND BRANES

  - $(z_a, \bar{z}_a)$ CS TYPE
  - $(t_a, \hat{t}_a)$ KÄHLER TYPE

- $W(\phi), f(\phi)$ $\longleftrightarrow$ $\langle TFT \rangle$

  - A-MODEL : INSTANTON EXP. $f(t_a, \hat{t}_a)$
  - B-MODEL : EXACT RESULT $f(z_a, \hat{z}_a)$

- MIRROR SYMMETRY :

$$\frac{\text{A-MODEL}}{X} \;\cong\; \frac{\text{B-MODEL}}{\tilde{X}}$$

+ FLUX + BRANES $\qquad$ + FLUX + BRANES