Aspects of N=1 Mirror Symmetry

Peter Mayr

CERN Theory Division, CH-1211 Geneva 23, Switzerland

In this talk we discuss some recent developments that make it possible to compute exact, instanton corrected couplings for a large class of $\mathcal{N} = 1$ supersymmetric four-dimensional string compactifications. In particular the non-perturbative superpotential $W(\phi)$ can be determined in these theories and the longstanding issue of supersymmetry breaking and vacuum selection can be studied explicitly. The relevant four-dimensional string backgrounds are so-called open-closed type II strings compactifications on Calabi–Yau manifolds X with certain background fluxes and background branes. Specifically the branes wrap non-trivial cycles in X and simultaneously fill space-time. The intention of this note is to give an overview over the setup, ideas and results for the interested non-expert, while we refer to the references at the end for further details.

The four-dimensional string effective low energy action is an $\mathcal{N} = 1$ supergravity theory coupled to Yang-Mills and matter described by chiral superfields ϕ. The standard supergravity action is determined by two functions. The first is a real function $G(\phi, \bar{\phi})$ of the chiral superfields

$$G(\phi, \bar{\phi}) = K(\phi, \bar{\phi}) + \ln(W(\phi)) + \ln(\bar{W}(\bar{\phi})). \tag{1}$$

Here K is the Kähler potential determining the kinetic terms by $g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$, where we use subscripts for derivatives with respect to the chiral superfields, e.g. $\partial_i = \partial/\partial\phi_i$. Moreover, the holomorphic function $W(\phi)$ is the $\mathcal{N} = 1$ superpotential.

There is another holomorphic function $f(\phi)$ that determines the kinetic terms of the Yang-Mills fields[1], contained in the F-term

$$\mathcal{L}_{YM} = \frac{1}{4} \int d^2\theta \, f(\phi) \, \mathrm{tr}\, W_\alpha^2, \tag{2}$$

where W_α is the chiral spinor superfield with the gaugino as the lowest component.

More precisely, the "functions" G and f represent sections of two bundles over the $\mathcal{N} = 1$ parameter space $\mathcal{M}$ defined as the space of vev's for the chiral superfields ϕ. We will loosely refer to this space as the $\mathcal{N} = 1$ moduli space although the superpotential $W(\phi)$ will, in general, fix some of the vev's of the "moduli" ϕ. However, in the type II

[1] For simplicity we discuss the case of a single simple group.

backgrounds considered below, the superpotential is often entirely of a non-perturbative origin, and thus (at least a subset of) the scalar field vev's will be indeed moduli of the perturbative theory.

In the following we will sketch how the holomorphic data $W(\phi)$ and $f(\phi)$ can be computed exactly for these type II backgrounds from the underlying 2d theory on the string world-sheet. This result includes an in general infinite series of instanton corrections. The relevant instanton corrections are described by non-perturbative configurations in the 2d world-sheet theory, the so-called world-sheet instantons. These are extended Euclidean string world-sheets that wrap 2-cycles C of volume V in the compactification manifold X.

Importantly, the world-sheet instanton corrections to the effective action describe often the effect of genuine space-time instantons in the four-dimensional theory. The reason is the familiar fact that the space-time coupling constants of the type II fields from the Ramond-Ramond (RR) sector are determined by the geometric moduli of the compactification manifold, and not by the dilaton (which governs the string loop expansion). In the simplest case of a class of string theory embeddings of a standard Yang-Mills gauge group in the RR sector, the four-dimensional gauge coupling g is given by

$$g^{-2} = Vol(\tilde{C}), \tag{3}$$

where $\tilde{C}$ is a fixed holomorphic 2-cycle in the compactification manifold X. A world-sheet instanton that wraps $\tilde{C}$ once, comes with a weight factor $\exp(-Vol(\tilde{C}))$. By eq.(3), this coincides with the weight factor e^{-1/g^2} of a space-time instanton in the gauge theory.

In a concrete setup it is usually possible to understand the connection between world-sheet and space-time physics in detail. In fact many of the exact superpotentials, derived from world-sheet instantons as discussed below, have a space-time interpretation in terms of gaugino condensation in an asymptotically free Yang-Mills factor. As will be further discussed below, these non-perturbative superpotentials for the Yang-Mills theory coupled to gravity break supersymmetry dynamically, in accordance with previous discussions of supersymmetry breaking by gaugino condensation [28].

The methods described in the following thus lead to phenomenologically interesting, four-dimensional $\mathcal{N} = 1$ string compactifications with non-perturbative supersymmetry breaking, which is *exactly computable* from the fundamental string theory. Moreover, the string effective theory is not just a generic $\mathcal{N} = 1$ supergravity; instead it has miraculous additional features inherited from the underlying string theory. It should be mentioned that, at the same time, the open-closed type II backgrounds comprise a rather large class of four-dimensional string theories which can accommodate the degrees of freedom of the standard model (as usual, the tricky part is rather to obtain just the standard model plus gravity, with the couplings to additional sectors suppressed at low energy and consistent with experiments). Thus these open-closed type II strings combine specific predictions, interesting physics and computability, properties that render them to a very promising candidate for a systematic, top-bottom string phenomenology.

Let us start by describing in more detail what is meant by an open-closed type

II compactification. As is well-known, the bosonic degrees of freedom of the type II string in ten dimensions include, apart from the metric $g_{\mu\nu}$, the NS B-field $B_{\mu\nu}$ and the dilaton φ, a set of gauge potentials $A^{p+1}_{\mu_1\cdots\mu_{p+1}}$ from the RR sector. Here the superscript $p+1$ denotes the degree of the form A^{p+1}, taking even or odd values in the two versions of the type II string called IIA and IIB, respectively. The sources for the RR gauge potentials are the D-branes which couple, amongst others, via the minimal world-volume coupling

$$c_{p+1}\int_{WV} A^{p+1},$$

where c_{p+1} are some dimensionful constants. In addition there are 5-branes in the NS sector of the type II strings that represent sources for the dual of the NS 2-form B.

To obtain a four-dimensional string vacuum, one may compactify the type II string on a six-dimensional manifold X. If X is a Calabi–Yau manifold, then the resulting theory has $\mathcal{N}=2$ supersymmetry in four dimensions. There are two closely related modifications of this compactification that break supersymmetry in a way that may be described by an effective $\mathcal{N}=1$ supergravity theory with non-trivial superpotential. The first is a modification of the closed string sector, by adding background fluxes of the field strengths F^{p+2} of the gauge fields on non-trivial cycles in X. The second is the addition of branes that wrap non-trivial cycles in X and simultaneously fill space-time. In particular the presence of background D-branes adds an open string sector, hence the nomenclature open-closed type II strings.

As is apparent from the above, the open-closed type II background will be coarsely specified by a set of flux and brane numbers associated with the non-trivial homology cycles in X. Restricting to the RR sector, these are the integers

$$N_k=\frac{1}{2\pi}\int_{C_k^{(p+2)}} F^{p+2}, \qquad M_k=\#\text{ of branes on }C_k^{(p+1)}. \tag{4}$$

Here k is some label for the basis of homology cycles in $H_*(X)$. Similarly, adding NS fluxes and branes would be described by another set of integers. In the following we restrict to RR fluxes and D-branes for simplicity.

Note that the RR fluxes and branes are defined on cycles of different dimensions in X. E.g. in the type IIA string, where p is even, the flux numbers N_k are assigned to elements of the even-dimensional homology $H^{even}(X)=H^0\oplus H^2\oplus H^4\oplus H^6$. The brane numbers M_k are assigned to the odd-dimensional homology, which is in fact only non-trivial in dimension three for a Calabi–Yau 3-fold. Therefore they describe D6-branes wrapped on 3-cycles in X. In the type IIB string, where p is odd, the roles of the even and odd dimensional cycles for fluxes and branes are exchanged.

Thus an open-closed string compactification with RR backgrounds is coarsely specified by the data

$$(X;N_k,M_k)$$

defining the Calabi–Yau manifold as well as the flux and brane numbers[2]. More specifically the open-closed type II background $(X;N_k,M_k)$ depend also on several continuous parameters representing vev's of chiral superfields ϕ in the four-dimensional theory.

[2] In general, the background fluxes and branes will lead to a back-reaction of the type II

They specify the shape of the geometry of X and in addition the brane configurations within it. We are interested in the ϕ dependence of the effective supergravity theory for a given type II background $(X; N_k, M_k)$.

There is an important peculiarity of the scalar field sector of these open-closed type II backgrounds that will be crucial to the following. Essentially the moduli fields split into two sets of complex structure type and Kähler type, respectively. These two sets are largely decoupled from each other in the effective theory[3]: One set determines the holomorphic F-terms, while the other set enters only the D-terms. In general there is also a weak dependence of the F–terms on the second set of moduli; this dependence is however of a universal type and can be recovered from relatively simple arguments.

With a restriction on the internal dimension of the brane, discussed momentarily, all of the moduli fields have an simple interpretation in terms of the type II brane geometry. In particular the above split into two sectors is rather simple: the first set, denoted by z_A in the following, measures the volumes of minimal 3-manifolds within X and is of a complex structure type. That is, the volume form is given by the holomorphic $(3,0)$ form Ω on X. The second set, denoted by t_A in the following, measures the volumes of holomorphic 2-manifolds within X, with the volume form given by the Kähler form J.

Moreover each type of moduli gets contributions from the closed and the open string sector. Moduli in the closed string sector measure the volume of cycles Γ without boundaries in X and describe the geometry of X. Moduli in the open string sector measure the volumes of submanifolds $\hat{\Gamma}$ with boundaries $\partial\hat{\Gamma}$ lying on the D-brane. Deforming the D-brane wrapping within X changes the volume of the submanifold $\hat{\Gamma}$ and it is in this way that the four-dimensional scalar fields parametrize the geometry of the brane.

Let us illustrate this with an example of a type IIA string configuration, where the closed string fluxes take non-trivial values on even-dimensional cycles and in addition there can be a D6-brane wrapped on a 3-cycle L in X. The holomorphic superpotential $W(t_A)$ for this type IIA background depends only on the Kähler type of moduli. They are defined by the volumes of 2-manifolds $\Gamma_A \subset X$ as

closed string:	$\mathrm{Im}\, t_a = \int_{\Gamma_a} J$	$a = 1, ..., h^2(X)$	(Kähler structure of X)
open string:	$\mathrm{Im}\, t_\beta = \int_{\hat{\Gamma}_\beta} J$	$\beta = 1, ..., h^1(L)$	(“position” of D6 brane on $L \subset X$)

Here Γ_a is a basis for the 2-cycle homology $H_2(X)$ and moreover $\hat{\Gamma}$ is a basis of 2-cycles with boundaries on the D6-brane. In the simplest case the latter are a basis of discs ending on a basis of non-trivial 1-cycles on the D-brane world-volume L. This is illustrated in Fig.1.

---

geometry such that X is no longer Calabi–Yau. However, this back-reaction does not enter the computation of the holomorphic functions $W(\phi)$ and $f(\phi)$ in the supergravity action [32] and can be neglected for this purpose.

[3] This is known as the “decoupling hypothesis” of ref.[8].

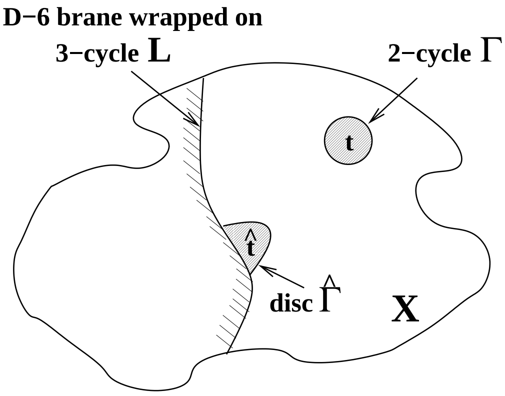

Fig 1.a Open and closed string Kähler moduli in the type IIA compactification. The imaginary part of a closed string modulus t measures the volume of a holomorphic 2-cycle Γ in the compactification manifold X. The imaginary part of a closed string modulus $\hat{t}$ measures the volume of a holomorphic 2-manifold $\hat{\Gamma}$ with non-trivial boundary on the world-volume L of the wrapped D6-brane.

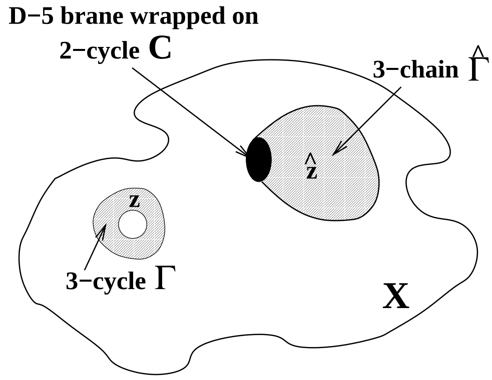

Fig 1.b Open and closed string complex structure moduli in the type IIB compactification. The imaginary part of a closed string modulus z measures the volume of a holomorphic 3-cycle Γ in the compactification manifold X. The imaginary part of a closed string modulus $\hat{z}$ measures the volume of a holomorphic 3-manifold $\hat{\Gamma}$ with non-trivial boundary on the world-volume C of the wrapped D5-brane.

There are also moduli for gauge fields on the 2-manifolds Γ_a and $\hat{\Gamma}_\beta$, namely the B field in the closed string sector and the gauge field A on the D-brane world-volume in the open string sector. They promote the above volume moduli to complex scalar fields in four dimensions.

A similar discussion holds for the type IIB string, where the superpotential $W(z_A)$ depends on the complex structure type of moduli. Again there is a contribution from the closed string sector that measures volumes of closed 3-manifolds in X and specifies the complex structure of X. Again moduli from the open string sector arise from the volumes of 3-manifolds with boundary on the wrapped D-brane, if we restrict to D5-branes wrapped on 2-cycles within X. This is the aforementioned condition that we impose on the dimension of the branes in the type IIB theory. It is expected that one may generalize this framework also to the more general case, which would involve branes wrapped entirely on X and with non-trivial gauge field backgrounds on the world volume. However the precise form of such a generalization is not known at the present time.

To compute a term in the effective string action for an open-closed type II background as described above from the fundamental string theory, one computes a 2d correlation function in the conformal field theory (CFT) on the string world-sheet.

The general relation may be sketched as

$$\langle S.T.\rangle_{eff} = \sum_{\Sigma} \langle CFT \rangle_{\Sigma}, \tag{5}$$

where the l.h.s. is a specific effective vertex in the space-time theory and the r.h.s is a sum of conformal field theory correlators associated with it, defined on string world-sheets Σ. In general, the right hand side is hard to compute as it gets contributions from an infinite number of world-sheet topologies, representing the expansion in string loops. In addition there are also quantum effects on the world-sheet Σ of the string.

However, it has been known since quite some time that special F-terms in the effective space-time theory get contributions only at a fixed topology of the string world-sheet [7][6][29]. These are the terms

$$\int d^4x \int d^2\theta\, \mathcal{F}_{g,h}(\phi) \mathcal{W}^{2g}(h\,N)(W_\alpha^2)^{h-1} \tag{6}$$

where $\mathcal{W}$ is the superfield containing the graviphoton field strength and W_α is the spinor superfield for the world-volume gauge fields on the D-brane. Moreover g is the number of handles and h is the number of boundaries of the string world-sheet Σ. In particular, $(g, h) = (0, 1)$ corresponds to the $\mathcal{N} = 1$ superpotential and $(g, h) = (0, 2)$ to the gauge kinetic function

$$W(\phi) = \mathcal{F}_{0,1}, \qquad f(\phi) = \mathcal{F}_{0,2}.$$

Moreover, the moduli dependent functions $\mathcal{F}_{g,h}(\phi)$ are the partition functions of a topological version of the type II string. This requires an explanation. In the CFT correlation functions, corresponding to the very special amplitudes (6) by the general relation (5), only a tiny subset of the string states contributes. In fact these states are the degrees of freedom of a "topologically twisted" world-sheet theory, whose dynamics is by far simpler then that of the original 2d theory for the world-sheet of the physical type II string. Thus the amplitudes of this topological string, which can be computed exactly by the methods discussed below, are identical to the amplitudes of the physical string theory (only) for the subset of correlation functions entering the space-time couplings (6).

Of course we will not have the time here to discuss the computation of the effective action in detail, except for some key points. The perhaps most important aspect, which makes contact with the title of this talk, is a symmetry between different type IIA and type IIB backgrounds, called mirror symmetry. Essentially, mirror symmetry is a sophisticated version of T-duality. In the present context, it identifies the type IIA string, compactified on the open-closed string background $(X; N_k, M_k)$ with a type IIB theory compactified on a different background $(X'; N'_k, M'_k)$. Although mirror symmetry is believed to be a non-perturbative symmetry, it matches the perturbative expansions on both sides (as T-duality is a perturbative concept). The mapping between X and its so-called mirror manifold X', as well as between the fluxes and branes on the two manifolds, is known.[4].

The important rôle of mirror symmetry is due to the fact that the two equivalent type II backgrounds give two very different descriptions of the same physical system. The type IIA theory comes with a natural weak coupling expansion which, usually, makes it easy to identify the microscopic degrees of freedom and a perturbatively defined effective theory for the space-time physics. The weak coupling limit is the limit in which the volumes of the holomorphic 2-manifolds, measured by the Kähler type moduli t_A, get large and thus the instanton corrections are exponentially suppressed. The holomorphic couplings in the type IIB theory, on the other hand, do not come with an obvious weak coupling expansion. The crucial difference to the type IIA theory is that there are no world-sheet instantons at all and a single, classical CFT correlator gives the exact result for the holomorphic amplitudes (6). On one hand, the lack of instanton effects makes it hard to identify a weak coupling description of the space-time physics in terms of a known, perturbatively defined effective theory. On the good side, it is the key point for the exact computability of the effective action for the open-closed type II background: after all, a classical string computation is not that difficult.

Putting the two mirror descriptions together, however, leads to a powerful framework for both, understanding *and* computing the space-time physics. In practice, one may first identify the type IIA background for an interesting space-time theory, say a string theory embedding of $SU(N)$ Yang-Mills theory, by taking the genuine weak coupling limit in which instanton effects are suppressed. Mirror symmetry allows to map the string background to an equivalent type IIB compactification, where the exact couplings (6) may be obtained by a classical computation. In some sense the rôle of the two descriptions may be compared to the similar situation in asymptotically free Yang-Mills theory: in the UV, weak coupling regime, the perturbative description in terms of weakly coupled gauge bosons is valid. While in the confined IR region, the relevant degrees of freedom are gauge singlets and an underlying, perturbative gauge theory description is not evident. Of course the latter phase would correspond to the type IIB description in the above sense; once again, its remarkable property is that in this case, it can be computed exactly from the classical string theory.

Let us illustrate the above discussion at the hand of the $\mathcal{N} = 1$ superpotential, which is also the best understood case. The open-closed type IIB background we consider is defined by the Calabi–Yau manifold X, RR background fluxes on the odd-dimensional cohomology $H^3(X)$ and D5-branes wrapping 2-cycles C_ν inside X. The background fluxes lead to the following geometric expression for the superpotential [14,30,26]:

$$W_{cl}(z_a) = \int \Omega \wedge H \;=\; \sum_\alpha N_\alpha \cdot \Pi^\alpha(z_a). \tag{7}$$

The parameters N_α in (7) specify the integer 3-form fluxes on X, as in (4). This superpotential depends on the vev's of the closed string moduli z_a, which represent

[4] For a modern perspective of the mirror map on type II strings and D-branes, as well as for references to the earlier literature, see [15,16].

scalars in $\mathcal{N}=1$ chiral multiplets. As mentioned earlier, these moduli measure the volumes of the odd homology of X, which is encoded in the period vector Π^α of the holomorphic $(3,0)$-form Ω on X:

$$\Pi^\alpha = \int_{\Gamma^\alpha} \Omega(z_a), \qquad \Gamma^\alpha \in H_3(X, \mathbf{Z}).$$

The parameters N_α in (7) specify the integer 3-form fluxes on X.

On the other hand, the background D-branes wrapped on C_ν lead to an additional superpotential [3,34,18]:

$$W_{op}(z_a, \hat{z}_\beta) \;=\; N_\nu \cdot \int_{\hat{\Gamma}^\nu} \Omega(z_a) \;=\; \sum_\nu N_\nu \cdot \Pi^\nu(z_a, \hat{z}_\beta)\,. \tag{8}$$

that depends on the open string moduli z_β in addition to the closed string moduli z_a. As explained earlier, these open string moduli measure the volumes of 3-manifolds $\hat{\Gamma}^\nu$ with boundaries lying on the wrapped D-brane on C_ν.

The combined superpotential

$$W_{\mathcal{N}=1} \;=\; W_{cl}(z_a) + W_{op}(z_a, \hat{z}_\beta) \;\equiv\; \sum_\Sigma N_\Sigma \,\cdot\, \Pi^\Sigma(z_A), \tag{9}$$

has a nice geometric interpretation in terms of defining a Hodge structure on a certain cohomology group [25,21,22]. Specifically this group may be defined as the dual of the relative homology group $H_3(X, \cup C_\nu)$, which is a homology group defined modulo boundaries on the D-branes. It combines the 3-cycles Γ^α associated with the closed string moduli and the 3-chains $\hat{\Gamma}^\nu$ associated with the open string moduli.

The interpretation in terms of a Hodge structure on the relative cohomology leads to a quick determination of the instanton expansion of the exact superpotential for two reasons. Firstly, the Hodge structure defines a natural weak coupling expansion in terms of new coordinates t_A, defined by a special set of integrals

$$t_A(z_B) = \int_{\tilde{\Gamma}_A} \Omega\,. \tag{10}$$

The relation (10) between the weak coupling coordinates t_A and the original complex structure type moduli z_A is called the $\mathcal{N}=1$ mirror map [25]. In fact the coordinates t_A are precisely the weak coupling coordinates for the perturbative expansion in terms of world-sheet instantons of the type IIA model mirror, as defined in [29,3,4].

The second simplification arising from the mathematical framework of a Hodge structure is that it gives a system of linear differential equations for all the integrals of Ω appearing in (9) and (10). The solution to these equations, and thus the instanton expansion of the superpotential, can be easily computed by standard methods.

The fact that genuine space-time instanton expansions in four-dimensional $\mathcal{N}=1$ string backgrounds can be computed exactly in such a simple way appears like an unexpected miracle. Technically the main simplification arose from the fact that the space-time couplings (6) are computed by the simpler, topological version of the world-sheet theory. Is there a more physical interpretation of this unexpected simplification?

An explanation of this kind may be well originating from the high symmetry of string theory, especially a group of "duality symmetries" of the effective space-time action, under which the theory transforms in an interesting way. If such a duality group exists, then the moduli dependent functions in the effective action are of a very special, "automorphic" type, as they must transform properly under the duality transformations. E.g. for the duality group $SL(2,\mathbf{Z})$, associated with reparametrizations of the complex structure τ of a 2-torus, the transformation behavior, together with the asymptotic behavior at special points, uniquely fixes a holomorphic function $f(\tau)$. Similarly the existence of a more general, non-trivial duality group in the four-dimensional open-closed string theories considered here, would naturally lead to substantial restrictions on the moduli dependence of the effective action.

In fact there is a beautiful interpretation of the holomorphic partition functions $\mathcal{F}_{g,h}$ which uncovers at least part of their special structure [29]. By interpreting the same partition functions as the result of certain one-loop amplitudes in the strong-coupling limit of type IIA, described by M-theory, one arrives at the prediction that these functions have miraculous integrality properties. E.g., in the case of the D-brane superpotential the prediction is that its instanton expansion is of the form

$$W(t_A) \;=\; \sum_{\{n_C\}} N_{\{n_C\}} \sum_{k=1}^{\infty} \frac{1}{k^2} \Big(\prod_C e^{2\pi i k n_C \mathrm{Vol}(C)} \Big), \tag{11}$$

where the label C runs over all the holomorphic curves in X and the sum is over the wrapping numbers n_C of the world-sheets around these curves C. The remarkable fact is that the coefficients $N_{\{n_C\}}$ of the instanton expansion count the "number" of possible wrappings in X with given numbers $\{n_C\}$ and are *integers*. This is certainly a highly distinguished property of the effective string superpotential, as compared to the general expression expected from a generic supergravity theory. Such integrality properties are familiar from automorphic functions of certain discrete groups, making contact to the above comments on the duality symmetries of the theory.

We conclude this overview with a list of references where some results and arguments can be looked up.

The space-time interpretation of the flux-induced superpotential in terms of gaugino condensation in an embedded Yang-Mills theory has been studied in [30][26]. In particular it has been shown that the superpotential breaks supersymmetry *only* in the theory coupled to gravity, making contact to claims in the earlier literature [28]. An extremely powerful framework to "engineer" flux configurations associated with more general superpotentials in a large class of Yang-Mills theories has been described in [32]. One of the ideas is that a weakly coupled D-brane configuration, describing the UV regime of an asymptotically free Yang-Mills theory embedded into string theory, flows in the IR to a strongly coupled phase described by a different geometry, where the branes have been replaced by the fluxes. Thus this transition gives a beautiful geometric realization of confinement of the D-brane gauge fields that have disappeared in the flux phase. In this way a large class of more general flux superpotentials for a large class of asymptotically free Yang-Mills theories have been studied in [9,11].

The computation of the D-brane superpotentials (8) has been pioneered in [3], based on the original work [33] on topological open strings. This was also studied in [31,8,18,4,25,12,17,5,21,22,1,10]. The B-model description of the combined flux and brane superpotential in terms of a mixed Hodge structure on the relative cohomology group is given in [25,21,22].

It is worth mentioning, that the holomorphic partition functions $\mathcal{F}_{g,h}$ are also computable in the type IIA theory, by summing up the corrections from world-sheet instantons "by hand". This may be somewhat tedious in practice, but can be done systematically for non-compact toric varieties. This has been done for a class of non-compact D-brane geometries in [23,19,13,27].

Yet another, extremely powerful approach to determine the partition functions $\mathcal{F}_{g,h}$ in the A-model, is the computation in the Chern-Simons theory [33] that describes the internal world-volume theory of the D6-brane in the type IIA background [2,10,24]. Most remarkably, this computation leads to closed expressions for all world-sheet topologies.

There are many more interesting works on various aspects which may be found in the references to the references and have been omitted here for lack of space.

Acknowledgments: This work was supported by the Deutsche Forschungsgemeinschaft.

References

[1] B. Acharya, M. Aganagic, K. Hori, and C. Vafa, hep-th/0202208.
[2] M. Aganagic, M. Marino, and C. Vafa, hep-th/0206164.
[3] M. Aganagic and C. Vafa, hep-th/0012041.
[4] M. Aganagic, A. Klemm, and C. Vafa, Z. Naturforsch. A**57** 1 (2002), hep-th/0105045.
[5] M. Aganagic and C. Vafa, hep-th/0105225.
[6] I. Antoniadis, E. Gava, K.S. Narain, and T. Taylor, Nucl. Phys. **B413** (1994) 162, hep-th/9307158.
[7] M. Bershadsky, S. Cecotti, H. Ooguri, and C. Vafa, Commun. Math. Phys. **165** 311 (1994), hep-th/9309140.
[8] I. Brunner, M. R. Douglas, A. E. Lawrence, and C. Römelsberger, JHEP**0008** 015 (2000), hep-th/9906200.
[9] F. Cachazo, K. A. Intriligator, and C. Vafa, Nucl. Phys. B**603** 3 (2001), hep-th/0103067; F. Cachazo, S. Katz, and C. Vafa, hep-th/0108120; F. Cachazo, B. Fiol, K. A. Intriligator, S. Katz, and C. Vafa, Nucl. Phys. B**628** 3 (2002), hep-th/0110028.
[10] D. E. Diaconescu, B. Florea, and A. Grassi, hep-th/0205234, hep-th/0206163.
[11] J. D. Edelstein, K. Oh, and R. Tatar, JHEP **0105**, 009 (2001), hep-th/0104037.
[12] S. Govindarajan, T. Jayaraman, and T. Sarkar, hep-th/0108234.
[13] T. Graber and E. Zaslow, hep-th/0109075.
[14] S. Gukov, Nucl. Phys. B **574**, 169 (2000), hep-th/9911011.
[15] K. Hori and C. Vafa, hep-th/0002222.
[16] K. Hori, A. Iqbal, and C. Vafa, hep-th/0005247.

[17] A. Iqbal and A. K. Kashani-Poor, hep-th/0109214.
[18] S. Kachru, S. Katz, A. E. Lawrence, and J. McGreevy, Phys. Rev. D**62** 026001 (2000), hep-th/9912151; Phys. Rev. D**62** 126005 (2000), hep-th/0006047.
[19] S. Katz and C. C. Liu, Adv. Theor. Math. Phys. **5**, 1 (2002) math.ag/0103074.
[20] J. M. Labastida and M. Marino, Commun. Math. Phys. **217**, 423 (2001), hep-th/0004196; J. M. Labastida, M. Marino, and C. Vafa, JHEP **0011**, 007 (2000), hep-th/0010102; J. M. Labastida and M. Marino, math.qa/0104180.
[21] W. Lerche and P. Mayr, hep-th/0111113.
[22] W. Lerche, P. Mayr, and N. Warner, hep-th/0207259; hep-th/0208039.
[23] J. Li and Y. S. Song, Adv. Theor. Math. Phys. **5**, 67 (2002) hep-th/0103100.
[24] M. Marino and C. Vafa, hep-th/0108064.
[25] P. Mayr, Adv. Theor. Math. Phys. **5**, 213 (2001), hep-th/0108229.
[26] P. Mayr, Nucl. Phys. B**593** 99 (2001), hep-th/0003198.
[27] P. Mayr, hep-th/0203237.
[28] H.P. Nilles, Int. J. Mod. Phys. **A5** (1990) 4199 and references therein.
[29] H. Ooguri and C. Vafa, Nucl. Phys. B **577**, 419 (2000), hep-th/9912123.
[30] T. R. Taylor and C. Vafa, Phys. Lett. B**474** 130 (2000), hep-th/9912152.
[31] C. Vafa, hep-th/9804131.
[32] C. Vafa, J. Math. Phys. **42**, 2798 (2001), hep-th/0008142.
[33] E. Witten, hep-th/9207094.
[34] E. Witten, Nucl. Phys. B**507** 658 (1997), hep-th/9706109.