Open Strings & Non-commutative Geometry

SUSY 02

DESY, Hamburg

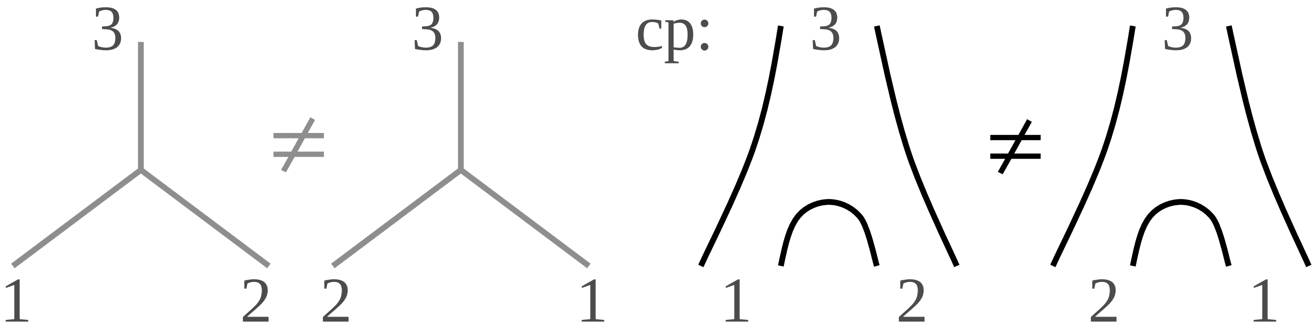
V. Schomerus
AEI Golm b. Potsdam

Introduction: Strings and NCG

Actions in classical field theory involve linear space of fields with

- bi-linear form ↔ kinetic/mass term
- (ass.) product ↔ interaction terms.

$$S = S_0 + \frac{m^2}{2}\langle \phi, \phi \rangle + \frac{g}{3!}\langle \phi, \phi * \phi \rangle$$



Non-commutative gauge theories are expected to describe behavior of massless open string modes in certain limiting regimes.

on D-branes

Plan of Talk and Some Results

Examples of NC gauge theories

Example 1: NC Yang-Mills **ext.**

Example 2: Fuzzy Gauge th. **int.**

(Matrix models)

NC gauge theory from strings

Example 1: Flat space with B≠0

Example 2: Cpact. curved space

No space-time NC field theories !

Some remarks on classical vacua
& interpretation in string theory .

(for example II)

Example 1: Moyal-Weyl Product

$f, g : \mathbb{R}^D \to \mathbb{C}$

skew sym.

$\downarrow$

$$f * g(x) = f(x)\; e^{\frac{i}{2}\Theta^{\mu\nu}\overleftarrow{\partial}_{\mu}\overrightarrow{\partial}_{\nu}}\; g(x)$$

$$[x^\rho \overset{*}{,} x^\sigma] = x^\rho * x^\sigma - x^\sigma * x^\rho \;=$$

$$x^\rho x^\sigma + \frac{i}{2}\Theta^{\rho\sigma} - x^\sigma x^\rho - \frac{i}{2}\Theta^{\sigma\rho} = i\Theta^{\rho\sigma}$$

This product arises through quantization of linear Poisson bracket:

$$\{x^\mu, x^\nu\} = \Theta^{\mu\nu}$$

↔ electrons in strong magnetic field B = Θ^{-1}

<u>Note:</u> ∂ implem. by commutator :

$$[x^\mu \overset{*}{,} f(x)] = i\Theta^{\mu\nu}\partial_\nu f(x)$$

Example 1:Non-commutative YM

$$S(A) \sim \int d^D x \, tr \, F_{\mu\nu} * F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu \overset{*}{,} A_\nu]$$

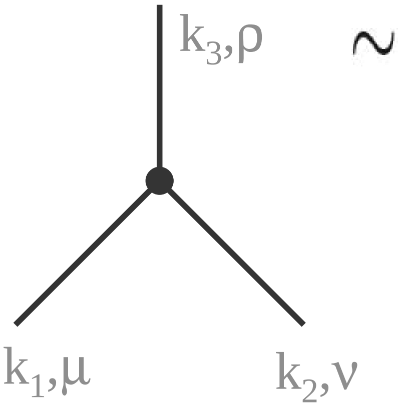
$$\uparrow \qquad\qquad\qquad\qquad \uparrow$$

$A_\nu = Mat_N(Fun(\mathbb{R}^D))$

- appears even when N = 1!
- non-local int.

Invariant under gauge transform. :

$$A \xrightarrow{\lambda} L_a\lambda - i[A_a \overset{*}{,} \lambda]$$



$$\sim (k^{12}_\mu G_{\nu\rho} + k^{23}_\nu G_{\rho\mu} + k^{31}_\rho G_{\mu\nu}) \times$$

$$\times \sin(\Theta^{\sigma\eta} k^1_\sigma k^2_\eta) \; \delta(k_1 + k_2 - k_3)$$

[Krajewski, Wulkenhaar]

Example 2: Spheres & Matrices

Space Mat_M of M×M matrices has non-commutative matrix product.

$$\mathrm{Fun}(S^2) \ni \underset{\overset{\|}{1}}{Y_0^0}, \underset{\overset{\|}{x^a}}{Y_a^1}, \ldots, \underset{\overset{\uparrow}{\text{spherical harm.}}}{Y_m^L}, \ldots$$

$$\mathrm{Mat}_2 \ni 1_2, \sigma^a \leftarrow \text{Pauli matrices}$$

$$\mathrm{Mat}_M \ni \mathbf{1}_M, \underset{\overset{\|}{(t^a)_M}}{x^a}, \ldots, \underset{\text{trunc. KK spec.}}{\mathrm{Y}_m^{M-1}}$$

Matrix algebras obtained by quantization of 2-sphere with linear PB

$$\{x^a, x^b\} = f^{ab}{}_c \, x^c = \Theta^{ab}(x)$$

Example 2: Fuzzy Gauge Theory

Matrix models

$$S^{\mathbf{N[M]}}_{\text{YM}}(A) = \frac{1}{4} tr F_{ab} F^{ab}$$

$A_a \in Mat_N(Mat_M)$

↓

$$F_{ab} = L_a A_b + L_b A_a + i[A_a, A_b] + \epsilon_{ab}{}^{c} A_c$$

↑

infinitesimal rotation: $L_a A = i[x_a, A]$

$$S^{\mathbf{N[M]}}_{\text{CS}}(A) = \frac{1}{2} tr(\epsilon^{abc} CS_{abc})$$

mass term

↓

$$CS_{abc} = L_a A_b A_c + \frac{2i}{3} A_a A_b A_c + \frac{\epsilon_{bc}{}^{d}}{2} A_a A_d$$

Such lattice like theories can preserve continuous symmetries and SUSY (here: the SU(2) rotations)

[Hoppe] [Madore] [Grosse et al.] [Watamura^2]

Branes in Flat Space and *-Product

$$S_P \sim \overset{\substack{\text{string tension}\\ \downarrow}}{\frac{1}{\alpha'}} \int_\Sigma d^2x(\overset{\substack{\text{sym.}\\ \downarrow}}{g_{\mu\nu}} + \alpha' \overset{\substack{\text{skew sym.}\\ \downarrow}}{B_{\mu\nu}})\partial_i X^\mu \partial^i X^\nu$$

$$\Uparrow$$

$$\sim \ldots + \int_{\partial\Sigma} dx_0 B_{\mu\nu} \underset{\substack{\uparrow\\ \text{boundary term}}}{X^\mu} \partial_0 X^\nu$$

Decoupling limit: $\alpha' \to 0$ **no string osc.**

$g/\alpha' \to 0$ $(g/(\alpha')^2$ fixed$)$ **no dissipation**

String endpoint coordinates are quantized ⇒ $*$ - product [Douglas, Hull] [Chu, Ho] [VS]

Dynamics of massless open string modes described by NC YM S_{YM}

$G_{OS} \sim (\alpha')^2 B\, g^{-1} B$ [Seiberg,Witten]

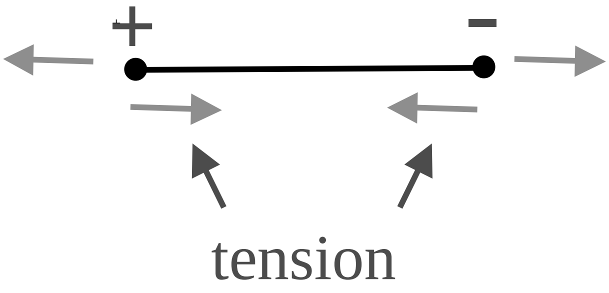
Electric fields and NCOS Theory

If $E_\mu = B_{0\mu} \neq 0$ there is problem with the decoupling limit: $(g/\alpha' \to 0)$

Ex:$_{(1+1)}$ $$\det\begin{pmatrix} -g & \alpha' E \\ -\alpha' E & g \end{pmatrix} = -g^2 + (\alpha' E)^2 \Rightarrow$$

Background unstable for $g/\alpha' < E$

$\Rightarrow \alpha'_{\mathrm{eff}} \neq \alpha'$



Limit: $\alpha' \to 0$

$g/\alpha' \overset{>}{\to} E$

$(\alpha'_{\mathrm{eff}}$ fixed)

For $E \sim E_c = g/\alpha'$ we remain with pure open string th. on NC space-time (NCOS). $\Theta \sim \alpha'_{\mathrm{eff}}$

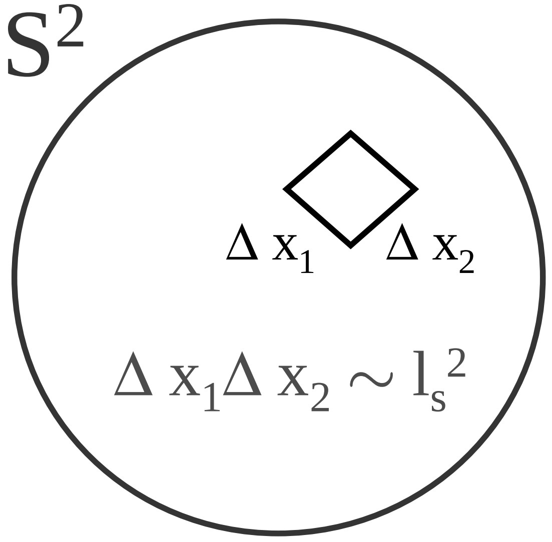
[Gopakumar et al.]
[Seiberg et al.] ‘00

Branes in a Curved Background S^3

$$\underset{\substack{\uparrow\\ \text{curvature}}}{\mathcal{R}_{\mu\nu}} - \frac{1}{4}\underset{\substack{\|\\ \partial_{[\mu}B_{\sigma\rho]}}}{H_{\mu\sigma\rho}}H_{\nu}{}^{\sigma\rho} + o(\alpha') = 0$$

$\mathcal{R} \neq 0 \Rightarrow B \neq 0$

Stable branes wrap $S^2 \subset S^3$ with a non-constant B-field (bal. tension)



[Bachas et al.]
[Alekseev,VS]

Quantized S^2 has finite # of states.

$\Rightarrow$ Matrix geometry !

Dynamics of massless open string modes described by $S = S_{YM} - S_{CS}$

[Alekseev, Recknagel, VS]

mass terms cancel

Dynamics: Some Classical Vacua

[Nekrasov, Schwarz] …   [Gopakumar et al.]

↓

Toy ex.:$_2$ $V' = m^2 \phi + g/2\ \phi * \phi = 0$

$\Leftrightarrow \pi * \pi = \pi$ $\phi = -2m^2/g\ \ \pi$

$$\pi_n = 2(-1)^n L_n(2r^2) e^{-r^2}$$

▲ Laguerre pol. $r^2 = x_1^2 + x_2^2$

Generalizes to arbitrary shape V !

Equations of motion for NC gauge theory are ‘algebraic’:

I: θ invertible

$A_0 = 0$ gauge

$$[B_m, [B^m, B^n] - i\Theta^{mn}(B)] = 0$$

$$I : B^\mu = x^\mu + \Theta^{\mu\nu} A_\nu \quad , \quad \Theta^{\mu\nu}(B) = \Theta^{\mu\nu}$$

$$II : B^a = x^a + A^a \quad , \quad \Theta^{ab}(B) = f_c^{ab} B^c$$

Interpretation of Solutions (Ex II)

[Alekseev, Recknagel, VS]

Consider N D0 branes on S^3: $S_{N[0]}$

$$[A_a,[A^a,A^b]-if^{ab}{}_{c}A^c]=0$$

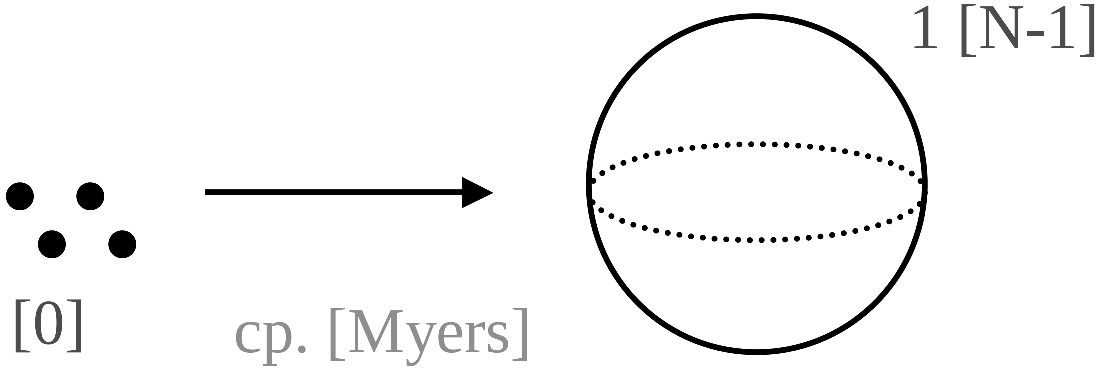
$A_a \in \mathrm{Mat}_N(\mathbb{C})$

$x^a = D^0(t^a) = 0$

Solved by $A_a = \Lambda_a$ an irrep of su(2).

$$S_{N[0]}(\Lambda+A) \sim S_{N[0]}(\Lambda) + S_{1[N-1]}(A)$$

$= \Delta m < 0$



In NCG gauge groups treated for space time (no clear destinction).

Conclusions and Outlook

Two different types of NC gauge theories arise from String Theory.

Branes in flat bgrd w $B \neq 0$ → NC Yang Mills on MW-def R^D

Branes on S^3 → Fuzzy gauge th

There are solution generating techniques that allow to construct interesting classical vacua. → Brane dynamics

see e.g. [Harvey 01]

Matrix models for a wide class of exactly solvable backgrounds have been constructed [Fredenhagen$^{\text{jr}}$, VS] ⇒ Many new condensation processes on branes in strongly curved bgrds.