DECONSTRUCTION,

$G_2$ Holonomy,

AND DOUBLET - TRIPLET SPLITTING....
I'll work in the context of SU(5) SUSY GUTS and their close cousins.

Higgs Bosons $2 \oplus \bar{2}$ of $SU(2) \times U(1)$

$2 \oplus 3 = 5$

$\bar{2} \oplus \bar{3} = 5$

To get the electroweak hierarchy plus a long-lived proton, we need to give GUT masses to the $3 \oplus \bar{3}$ - while keeping the $2 \oplus \bar{2}$ light.
That is also what we need to get the right SUSY-GUT formula for gauge couplings.

But why are the $2, \bar{2}$ light if the $3, \bar{3}$ have GUT masses?

An obvious possibility is to find a global symmetry of the low energy world that allows the $3-\bar{3}$ mass (i.e. a superpotential $\int d^2 \theta \ 3-\bar{3}$) and forbids the $2-\bar{2}$ mass.
However, this is impossible in four-dimensional $SU(5)$ theories with only finitely many fields (Goodman & Lew 1985).

Most general global symmetry acts on the 5 as

$$e^{i \phi} \left( \begin{array}{cc} e^{2i\alpha} & e^{2i\alpha} \\ e^{2i\alpha} & e^{-3i\alpha} \\ \end{array} \right)$$

Global symmetry that commutes with $SU(5)$

$SU(5)$ gauge transformation that commutes with standard model
It likewise acts on the $\tilde{5}$ as

$$e^{-i\phi} \left[ \begin{array}{ccc} e^{-2i\alpha} & & \\ & e^{-2i\alpha} & \\ & & e^{3i\alpha} \end{array} \right]$$

$\phi$ can be anything

Same $\alpha$ as for the $5$

The $3-\bar{3}$ and $2-\bar{2}$ masses both transform as $e^{i(\phi-\bar{\phi})}$

So one is allowed if and only if the other is
CAN WE AVOID THIS RESULT
BY ADDING MORE FIELDS?

LET'S TRY:

WE START WITH A $5 \oplus \bar{5}$
WITH $\phi, \bar{\phi}$ CHOSEN SO THE $2\oplus \bar{2}$
ARE LIGHT. THE TROUBLE IS THAT
THE $3\oplus \bar{3}$ ARE ALSO LIGHT.

TO ELIMINATE THEM WE ADD A
NEW $5' \oplus \bar{5'}$ WITH QUANTUM
NUMBERS UNDER THE GLOBAL
SYMMETRY SUCH THAT
$3 \bar{3}'$ AND $3' \bar{3}$ MASSES ARE ALLOWED
THIS ELIMINATES THE TRIPLETS

BUT NOW THE $a', \bar{a}'$ MAKE TROUBLE AS $a \bar{a}'$ AND $a' \bar{a}$ MASSES ARE POSSIBLE, SO THAT WE NO LONGER HAVE NATURALLY LIGHT HIGGS BOSONS. TO AVOID THIS, WE NEED TO ELIMINATE THE $a' \bar{a}'$ BY ADDING A NEW $5'' \bar{5}''$ WITH $2' \bar{2}''$ AND $2'' \bar{2}'$ MASSES... BUT NOW WE HAVE AN UNWANTED $3'' \bar{3}''$ SO WE NEED TO ADD A $5''' \bar{5}'''$ TO ELIMINATE IT...
As long as we have only finitely many four-dimensional SU(5) multiplets, we'll always run into trouble at the end of the chain... but all is well if we are willing to use infinitely many 505 pairs, of increasing masses.
This infinite collection of fields can have a natural interpretation as Kaluza-Klein harmonics in a higher-dimensional theory.

GUT symmetry breaking by Wilson lines + topological tricks so the 5, 5 transform differently under global symmetry...
(AS IN THE ORIGINAL)

(CALABI-YAU CONSTRUCTIONS)

Candelas, Horowitz,
Strominger & W 1984

THOUGH IN THAT CASE

GLOBAL SYMMETRIES
WERE NOT USED)
FOR CALABLI-YAU COMPACTIFICATION,
THIS APPROACH WAS WORKED OUT
IN THE MID-80's ... A CHARACTERISTIC
PROPERTY IS THAT THERE IS NO ENERGY
SCALE AT WHICH THE MODEL IS A
FOUR-DIMENSIONAL GUT ... 

THE USUAL GUT RELATIONSHIPS PREDICT
FOR 
FERMIOS QUANTUM NUMBERS 
GAUGE COUPLING UNIFICATION
HOLD JUST AS IN FOUR-DIMENSIONAL
GUT's, BUT THERE ARE SOME DEPARTURES
For one thing, coupling unification is more robust than in 4-D/11D GUTs, where it can be spoiled by terms of order $M_{GUT}/M_{string}$

$$T \phi^n E_u F_u$$

$n = 1/2$

With symmetry breaking by Wilson loops $\phi$ is replaced by $A_{\alpha}, \alpha > 4$

And anything like $T \phi F_{\alpha \beta} F_{\alpha u F_u}$

As $F_{\alpha \beta} = 0$
This argument works to all orders in \textsc{mgut/mstring} but actually the result is exact, as shown by a conformal field theory argument.

(This does not depend on supersymmetry)

(Wendew 1986)

Another difference: magnetic monopoles

4-dim'l field theory

\[ S^2 \]

Higher dimensions

\[ S^2 \times K \]

\[ \text{find quantum of magnetic charge is} \quad \frac{2\pi}{e \cdot n} \]
THIS RESULT WOULD BAFFLE A 4+1-DIMENSIONAL FIELD THEORIST.
WHERE ARE THE PARTICLES OF ELECTRIC CHARGE $\frac{e}{n}$?

IN HETEROPTIC STRING THEORY THEY EXIST: WRAPPED STRINGS
electric charge $\frac{e}{n}$
NOTES

THE USUAL CHARGE QUANTIZATION (AND GUT-LIKE STRUCTURE) DOES, OF COURSE, HOLD FOR PARTICLES OF ORDINARY MASS
Apart from the heterotic string on a Calabi-Yau, another way to get $N=1$ SUSY in four dimensions and a model somewhat like the standard model is to compactify $M$-theory from eleven to four dimensions on a manifold of $G_2$ holonomy

\[ \mathbb{R}^4 \times X \]

$X$ = a seven-manifold of $G_2$ holonomy

$G_2$ = the smallest "exceptional" Lie group and the only one that can be a holonomy group
This avenue wasn't pursued 20 years ago because in supergravity it doesn't work... that is, if X is smooth—needed for supergravity to be valid—one gets an abelian gauge group only and no chiral fermions.

But with the modern understanding of how to generate gauge groups and chiral matter from singularities one can do much better....
$X$ is a seven-manifold of $G_2$ holonomy, and $Q$ is a three-manifold (the "normal" space to $Q$ is singular) on which propagate $SU(5)$ (or $SO(10)$, or $E_6$) gauge fields.
Let us just look at $Q$:

$S_1$ and $S_2$ are two circles (containing further singularities) on which chiral superfields are supported.
IT TURNS OUT THAT IF WE PLACE ONE HIGGS - THE 5 - ON $S_1$ AND THE OTHER - THE $\bar{5}$ - ON $S_2$, THEN A CERTAIN DISCRETE SYMMETRY (THAT ROTATES $S_1$ OR $S_2$) TRANSFORMS THE HIGGSES IN A WAY THAT CANNOT BE ACHIEVED IN A FOUR-DIM'L SU(5) THEORY
Namely

$$5: \begin{pmatrix} 3 \\ 2 \end{pmatrix} \rightarrow e^{i\phi} \begin{pmatrix} e^{2i\pi x} & e^{2i\pi x} \\ e^{2i\pi x} & e^{-2i\pi x} \end{pmatrix}$$

As expected

But (for example)

$$\bar{5}: \begin{pmatrix} 3 \\ 2 \end{pmatrix} \rightarrow e^{i\phi} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

i.e. Just a global symmetry on $5$

But a global symmetry plus a gauge transformation on the $\bar{5}$

Clearly, we can pick $\phi, \bar{\phi}, \phi$ to allow a $3-\bar{3}$ mass term and forbid one for $2-\bar{2}$
I won't explain the details of (13) the construction, since it turns out that this aspect can be imitated (via "deconstruction" — Arkani-Hamed, Cohen, Geze & Hill, Pokorski, Wang).

By a four-dimensional theory in which the gauge group is not SU(5) but SU(5) × SU(5).

In an extreme version of lattice gauge theory, each circle is replaced by one point and the rest is ignored.
NOW WE BREAK \( SU(5) \times SU(5) \)

TO \( SU(3) \times SU(2) \times U(1) \times \Gamma \)

\( \text{embedded diagonally in } SU(5) \times SU(5) \)

WHERE \( \Gamma \) IS A FINITE SUBGROUP

OF THE HYPERCHARGE GROUP OF THE

SECOND \( SU(5) \) TIMES A GLOBAL SYMMETRY

IF HIGGS BOSONS ARE NOT \( 5 \oplus 5 \) OF \( SU(5) \),

BUT

\( (5,1) \oplus (1,5) \) OF \( SU(5) \times SU(5) \)

THEN \( \Gamma \) IS THE SORT OF DISCRETE

SYMMETRY WE WANT - THAT CAN ALLOW

3 \( \bar{3} \) AND FORBID \( 2 \bar{2} \)
IN THIS WAY WE MAKE A FOUR-DIMENSIONAL MODEL THAT REPRODUCES SOME RESULTS OF THE M-THEORY MODEL, BUT NOT OTHERS.

THE MOST STRIKING DIFFERENCE IS PERHAPS IN ELECTRIC CHARGE CONSERVATION—THE DECONSTRUCTED, FOUR-DIMENSIONAL MODEL HAS CONVENTIONAL QUANTIZATION OF ELECTRIC CHARGE, AND THE M-THEORY MODEL DOES NOT.
TO ENSURE THAT ALL QUARKS AND LEPTONS CAN GET MASSES, IT IS MOST
STRAIGHTFORWARD TO TAKE THEM TO BE THREE COPIES OF

\[(\bar{5}, 1) \oplus (10, 1)\]

SO BELOW THE GUT SCALE, THE SPECTRUM IS SO FAR

\[(\bar{5}, 1) \oplus (1, \bar{5}) \oplus 3 (\bar{5}, 1) \oplus 3 (10, 1)\]

\[\underbrace{\text{HIGGS}} \quad \underbrace{\text{QUARKS AND LEPTONS}}\]

IF WE TOOK QUARKS AND LEPTONS TO BE, SAY, \[(\bar{5}, 1) \oplus (1, 10),\] WE'D GET PECULIAR QUANTUM NUMBERS UNDER 5 AND WE'D HAVE TROUBLE GIVING MASSES TO ALL QUARKS & LEPTONS
THERE IS A LOT OF FREEDOM IN PRECISELY SPECIFYING THE GLOBAL SYMMETRY GROUP - COMING FROM THE CHOICE OF THE HYPERCHARGE SUBGROUP AND HOW IT MIXES WITH GLOBAL SYMMETRIES OF THE DIFFERENT MULTIPLETS.

WE CAN PICK THE DISCRETE SYMMETRY TO

* ALLOW $\mathbf{3} \cdot \overline{3}$, FORBID $\mathbf{2} \cdot \overline{2}$
* ALLOW QUARK, LEPTON, AND NEUTRINO MASSES
* FORBID PROTON DECAY BY OPERATORS OF DIMENSION FOUR OR FIVE

BUT OF COURSE WE DON'T HAVE A PURELY THEORETICAL REASON FOR THESE CHOICES.
Now let's return to the spectrum, which so far was (below GUT scale)

\[(\overline{5}, 1) \oplus (1, 5) \oplus 3((\overline{5}, 1) \oplus (10, 1))\]

\[\text{HIGGS} \quad \text{QUARKS, LEPTONS}\]

This spectrum is anomalous; to cancel anomalies, we might add

\[(5, 1) \oplus (1, \overline{5}) \text{ or } (10, 1) \oplus (1, \overline{10})\]

How do we interpret these fields?

It is tempting to consider them as "messenger fields" of gauge-mediated supersymmetry breaking.
LET US CALL THESE MESSENGERS $\phi$, $\bar{\phi}$

$\Phi, \bar{\Phi}$

$(5,1) (1, \bar{5})$

IN GAUGE-MEDIATED SUSY BREAKING, ONE USUALLY COUPLES THE MESSENGERS TO A CHIRAL SUPERFIELD $S$ OF THE SUPERSYMMETRY BREAKING SECTOR

$$\int d^2 \Theta \, S \phi \bar{\phi}$$

THEN CERTAIN ONE AND TWO-LOOP DIAGRAMS GIVE MASSES TO GAUGINOS AND SQUARKS, SLEPTONS
VIRTUE: NATURAL QUARK, SLEPTON
DEGENERACY $\Rightarrow$ AVOID FLAVOR CHANGING PROCESSES
AT LOW ENERGIES, UNDER THE STANDARD MODEL,
\[ \Phi = 2 \otimes 3 , \quad \bar{\Phi} = \bar{2} \otimes \bar{3} \]

AND WE MIGHT INTRODUCE SEPARATE S-FIELDS FOR THE 
2 \cdot \bar{2} AND 3 \cdot \bar{3}:

\[ \int d^2 \theta \left( S \cdot 2 \cdot \bar{2} + S' \cdot 3 \cdot \bar{3} \right) \]

IN AN SU(15) MODEL, ONE COULD ARRIVE AT THIS STRUCTURE BY COUPLING \( \Phi, \bar{\Phi} \) TO BOTH A SINGLET AND AADJOINT OF SU(15)

\( \rightarrow \) COMPONENTS BEING S, S'
In the present context, we have a more precise rationale for this, just as for the Higgs bosons, so for the messengers $\tilde{2} \tilde{2}$ transforms differently from $3 \tilde{3}$ under the global symmetry, so $\tilde{s}, \tilde{s}'$ must be different.

Minimal in gauge mediation with only one $S$ field, the ratio $\frac{\langle f_s \rangle}{\langle s \rangle}$ determines masses in the observed sector and one gets unique predictions for the spectrum.
Here, we have the two parameters

\[ \Lambda_2 = \frac{\langle F_S \rangle}{\langle S \rangle} \quad \text{and} \quad \Lambda_3 = \frac{\langle F_S' \rangle}{\langle S' \rangle} \]

So the spectrum isn't unique; it depends on their dimensionless ratio \( r = \frac{\Lambda_2}{\Lambda_3} \).

\( \Lambda_2 \) contributes to masses of \( SU(2) \) nonsinglet\$s and \( \Lambda_3 \) to those of \( SU(3) \) nonsinglet\$s.\]
so, for example, by increasing \( \overrightarrow{\tau} \) one increases the mass of left-handed sleptons and charginos relative to right-handed sleptons.

for example, in run 1 at fermilab there was a much-discussed \( e^+e^- \rightarrow \gamma \gamma + \text{missing } E_T \) event that might be a signal of gauge mediated susy breaking, but not if one assumes the spectrum of the minimal model.
Here we have a rationale for departing from the spectrum of the minimal model ... it is possible to get a better "fit" to the $e^+e^-\gamma\gamma + \not{E}_T$ event (and absence of certain other events) than the minimal model (S. Thomas and E.W.)

In the process, one can also make squarks lighter and the required $\mu$ parameter less ... alleviating the SUSY fine-tuning problem....
ONE CAN ALSO, FOR EXAMPLE,
MAKE GLUINOS MUCH LIGHTER
THAN OTHER SPARTICLES,

SOMewhat AS IN YESTERDAY'S
"PARTIAL GAUGE MEDIATION"
Depending on whether one gets $e^+e^- \rightarrow \not{\ell}T$ from left- or right-handed slepton production, one will or will not also predict $e^+ \not{\ell} \not{\ell} \rightarrow e^+ \not{\ell} \not{\ell}$ when one slepton is a neutrino.

If one includes R-parity violation, one can also (Allanach et al.) try to explain the $\mu^+ \not{\ell} \not{\ell}$ excess seen at Run 1 at Fermilab. (According to Allanach's talk, one can't quite make a good fit to both.)
HOPEFULLY THE LUMINOSITY
AT FERMILAB WILL SOON INCREASE
AND WE'LL LEARN WHICH
HINTS FROM RUN 1 ARE REAL.