More dimensions

or

more symmetry?

(Latticied/deconstructed extra dim)

Arkani-Hamed Hill
Cohen SP
Georgi Wang

(see also Witten's talk and several talks in parallel sessions)

Large ($R > 1/M_{Pl}$) extra dim
widely explored (see talks at this conference)

Ambitious

Low scale string theories

Modest (but easier)

Effective K-K field theories in $(4+n)$ dim
(n compact)

(also) gauge/matter fields in the bulk

only gravity in the bulk
(cut-off to SM on a brane)

Let's focus on: non-abelian gauge theories in $(4+n)$ dim ($n=1$ for most of the discussion)

Non-abelian gauge theories in $(4+n)$ dim

- compactification radius R

(a fundamental scale ?)

- dimensionful coupling $g_5 = [M]^{-n/2}$

$\Rightarrow$ non-renormalizable

- physical cut-off scale Λ (need UV completion above Λ)

The physics in 4d (for the sake of comparison with deconstruction)

a tower of K-K modes $M_n = \frac{\pi n}{R}$ $(n = 0, 1, \ldots)$
(5d)

and their interactions (defined by momentum conservation in the 5th dim)

$$g_5 \equiv \frac{g_0}{\sqrt{\Lambda}} \quad , \quad g_4 = \frac{g_0}{\sqrt{\Lambda R}}$$

Effectively, $n < N$, $M_N < \Lambda \Rightarrow N = R\Lambda$

Benefits (see talks at this conference)

Classical level

- richer discrete symmetries
- Scherk - Schwarz mechanism of (super)symmetry breaking
- wave functions localized in the 5th dim

⋮

applied to a number of physical problems:
- doublet - triplet splitting
- fermion masses
- supersymmetry breaking
- proton decay

⋮

Quantum level

[(in) consistency of the theory
how to reconcile 5d gauge invariance with cut-off Λ]

hope for new solutions to the hierarchy problem

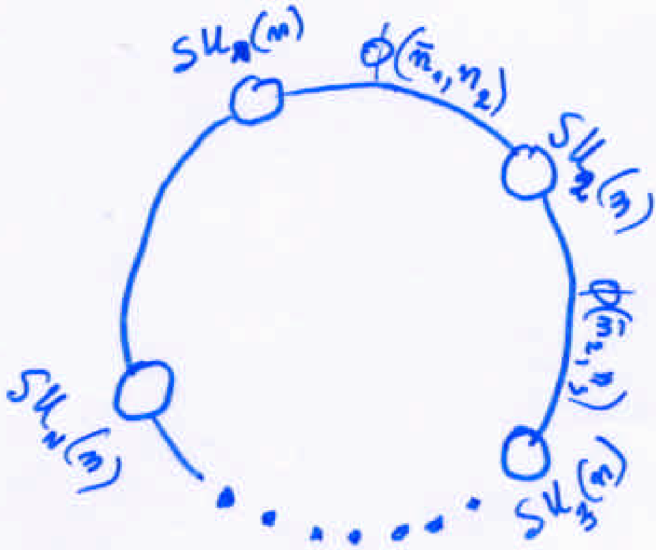
$$M_W = M_W(R, \Lambda)$$

weak dependence on Λ ?

(toy models with no quadratic divergences at 1-loop ?)
But FI terms!

The observation (Arkani-Hamed, Cohen, Georgi / Hill, SP, Wang):

there exists a 4d gauge invariant <u>and</u> <u>renormalizable</u> field theory which in its IR mimics the physics of $SU(n)$ gauge theory in 5d, truncated at $N = \Lambda R$ K-K modes



Same gauge couplings g_4

$SU(n)^N$ $\qquad \phi_i(\bar{n}_i, n_{i+1}) \qquad i = 1, N$

$\phi_N(\bar{n}_N, n_1)$

Assume potential for ϕ_i gives VEV $\phi_i = v\,\mathbb{1}$

Then $\qquad SU(n)^N \to SU(n)_{\text{diagonal}}$

$$\mathcal{L} \supset \sum_i (D_\mu \phi_i)^+ D^\mu \phi_i \supset$$

$$g_4^2 \sum_i \phi_i^+ \phi_i \left(A_i^{a\mu} - A_{i-1}^{a\mu}\right)^2$$

$$\to g_4^2 v^2 \sum_i \left(A_i^{a\mu} - A_{i-1}^{a\mu}\right)^2$$

$$M_n = 2\sqrt{2}\, g_4 v \sin\frac{\pi n}{N} \qquad 0 \leq n \leq N-1$$

For small $\frac{\pi n}{N}$: $M_n \simeq \frac{\pi n}{N} 2\sqrt{2}\, g_4 v$

Identifying $R = N a$, $a = 1/2\sqrt{2}\, g_4 v$

$$M_n \sim \frac{n}{R}$$

Also identical interactions, if $g_4 = g_0$

$$g_0, R, \Lambda \to g_4, N, v$$

$$\boxed{\begin{array}{l} \Lambda \equiv g_4 v, \quad g_0 \equiv g_4, \quad R = \dfrac{N}{g_4 v} \\ N = \Lambda R \end{array}}$$

The physics below the scale v of spontaneous gauge symmetry breaking is the same as that of 5d theory with cut-off

Non-linear σ - model approximation to the full 4d theory which includes also $V(\phi_i)$

$$\Phi_i \Rightarrow v \exp\left(i \phi_i^a T^a / v\right)$$

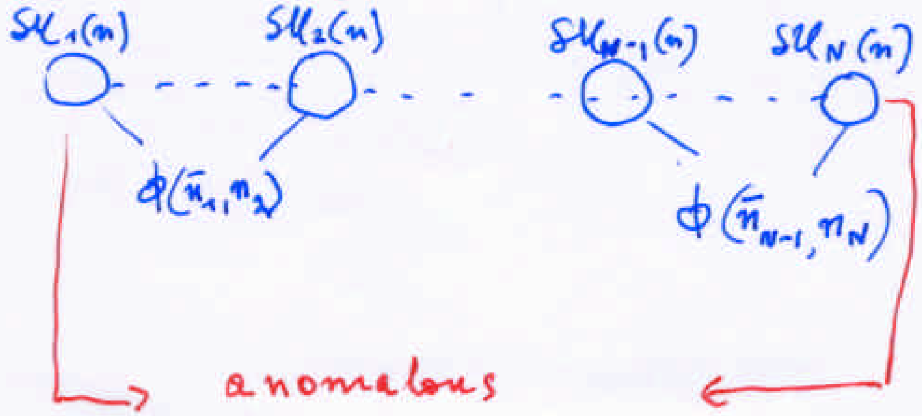
where ϕ_i^a , $i = 1 \dots N$ are Goldstone bosons of the spontaneously broken full (chiral) symmetry of $V(\Phi_i)$

$$\Phi \to L \Phi R^\dagger \qquad SU(n)^N \times SU(n)^N \to SU^N(n)$$

$N-1$ GB eaten up by gauge bosons; one remains in the physical spectrum ($\equiv A_5^{(0)}$);

{ There remain $2N$ singlets + N adjoint with $M \sim \mathcal{O}(v)$)

The <u>nature</u> of the correspondence between 4 d and 5 d theories even more evident for S_1/Z_2



Need to add $\phi(n_1)$, $\phi(\bar{n}_N)$ to cancell anomalies, and make them $\sim O(v)$ (then such fields do not have any extra dim interpretation)

<u>Remark</u>

Interpretation as lattice action

Supersymmetric extension

Csaki, Erlich, Grojean, Kribs

Raise scalar fields in previous model to 4D $N=1$ chiral multiplets and introduce V_j, $j=1\ldots N$ vector superfields associated to the gauge group $SU(n)_j$.

$\Phi_i(\varphi_i, \psi_i)$ $\qquad$ $V_j(A^a_j, \lambda^a_j)$

$\downarrow$

bi-fundamentals

After $SU(n)^N \to SU(n)_D$

4D $\mathcal{N}=2$ massless vector

$$\begin{pmatrix} & A^\mu & \\ \lambda & & \psi \\ & \varphi & \end{pmatrix} \quad \begin{array}{l} 2 \text{ components} \\ 4 \text{ components} \\ 2 \text{ components} \end{array}$$

4D $\mathcal{N}=2$ massive vector

$$\begin{pmatrix} & A^\mu & \\ \lambda & & \psi \\ & \varphi & \end{pmatrix} \quad \begin{array}{l} 3 \text{ components} \\ 4 \text{ components} \\ 1 \text{ component} \end{array}$$

5D $\mathcal{N}=1$ vector multiplet

$$\begin{pmatrix} A^\mu \\ \psi \\ \phi \end{pmatrix} \quad \begin{array}{l} 3 \text{ components} \\ 4 \text{ components} \\ 1 \text{ component} \end{array}$$

Works for hypermultiplets as well.

4D $\mathcal{N}=1$ $\quad SU(n)^N \to$ 4D $\mathcal{N}=2$ $SU(n)_D$

<u>Another extension</u>

Non-perturbative equivalence of the two theories

Csaki, Erlich, Khoze, Poppitz, Shadmi, Shirman

One can take it as

- a gauge invariant and renormalizable regularization of $d > 4$ dimensional gauge theory by reduction to 4D

But also

- a class of 4D theories that implement many seemingly extra dimensional ideas, even for small N (very coarse lattices), ~~txxxx~~ the class of 4D theories includes also models with no obvious extra dimensional correspondence

Link to extra dimensions inspiring but at the end irrelevant?

"Deconstruction" $\Rightarrow$ a model building tool for theories in 4D, with "extra dimensional" benefits

In fact, theories with replicated gauge groups and bi-fundamental matter have been discussed already earlier, e.g.

$SU(5) \times SU(5)$ (to solve GUT's problems) Barbieri, Dvali, Strumia (1994)

quiver theories (Link to string theory) Douglas, Moore 1996

in the context of the search ~~of~~ for ~~...~~ CFT's and AdS_5/CFT correspondence

Link to ~~deconstruction~~ extra dimensions (latticized) and their "benefits" may suggest new use of such theories

Grand Unification, proton decay, fermion masses etc, in theories with replicated gauge groups (use of richer discrete symmetries, "localization" of matter with respect of some factors of the full gauge groups)

Witten
Kribs (parallel session)

- $SU(5) \times SU(5)$ Witten
  Kribs
  (see also Barbieri, Dvali, Strumia 1994)

- $SU(5) \times SU(5) \times \ldots \times SU(3) \times SU(2) \times U(1)$
  Csaki, Kribs, Terning
  Cheng, Matchev, Wang

"Quantum" applications

- running of the coupling constants
  power law in $(4+n)$ $\rightarrow$
  logarithmic (with changing β coefficients) in 4D

- "Weak" solution to the hierarchy problem
  (bottom-up approach)

  Postpone "sensitive" (Λ^2) UV dependence of the electroweak symmetry breaking mechanism to a scale $\gtrsim O(10\,\mathrm{TeV})$
  (using perturbative physics)

  In practice, δm^2_H 1-loop finite
  (or at most $\ln \Lambda$)

(non-supersymmetric models)

Higgs boson as pseudo-Goldstone boson
(correspondence to a Higgs boson
as gauge boson in extra dim)

$\left(\int_0^{2\pi R} dy\, A_5\right)^2 \equiv \left(A_5^{(0)}\right)^2$

Arkani-Hamed, Cohen, Georgi
— " — , Gregoire, Wacker
Arkani-Hamed, Cohen, Katz, Nelson
— " — ... Gregoire, Wacker
Gregoire, Wacker

Cheng, Hill, Wang
He, Hill, Tait

Models based on deconstruction have the virtue of a large collection of symmetries protecting the Higgs mass and a restricting set of symmetry breaking effects.

Simplest (unrealistic) example: finite m_H, "independent" of UV cut-off to non-linear σ model, $\Lambda = 4\pi v$

Deconstruction

(renormalizable theory)

Higgs boson as pseudo - Goldstone boson of the "chiral" $SU(n)^N \times SU(n)^N$ symmetry in the scalar sector

$$\phi_i \to U_i \phi V_i^+$$

The symmetry is broken spontaneously but also explicitly, by gauge interactions $SU(n)^N$

$$\phi_i \to U_i \phi U_{i+1}^+$$

Gauge symmetry breaks $SU(n)^N$ but protects the Higgs mass!

Gauge invariant operator which contribute to the Goldstone boson mass

$$\left[Tr\, \phi_1 \phi_2 \dots \phi_N \right]^2$$

For $N > 2$ its dim > 4 and no counterterms!

Realistic models can be constructed

$$\Lambda = 4\pi v$$

First KK: $m_1 = \dfrac{g_4^2 v^2}{N^2}$

$$m_H^2 = g_4^4 v^2 / (16 \pi N^3) =$$

$$= \frac{g_D^2}{16\pi^2} m_1^2$$

↳ plays the role of cut-off

Compare e.g. to $m_{\pi^+}^2 - m_{\pi^0}^2$ 

$$\delta m_\pi^2 = \left(\frac{e}{4\pi}\right)^2 (4\pi f_\pi)^2$$

But our m_H is finite! This explains why $\Lambda = 4\pi v$ replaced by $\Lambda' = \dfrac{g_4 v}{N}$

For a realistic model: $\left\{\begin{array}{l}\text{negative } m^2 \\ \text{fundamentals, not adjoi} \\ \text{avoid } U(1) \text{ div}\end{array}\right.$

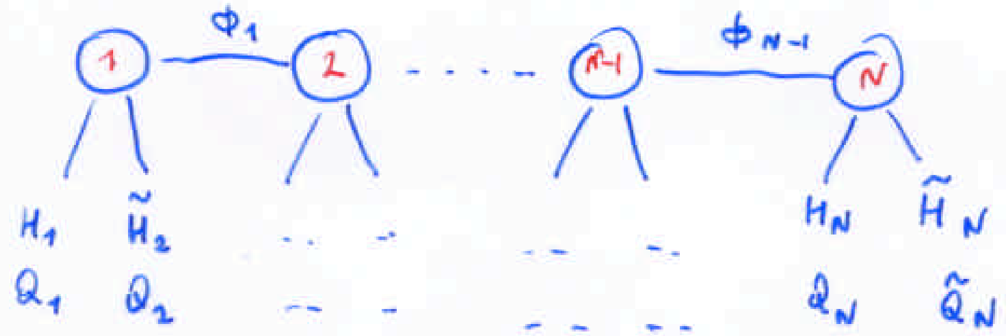
More interactions ⇒ Higgs mass no longer fully protected but realistic models with no quadratic div at one loop can be constru

(Falkowski, Grojean, SP)

$SU(2)$ toy model

An analog to Scherk - Schwarz mechanism

Hard supersymmetry breaking by removing some of the degrees of freedom within supermultiplets



$$H_N \equiv (h_N, \psi_{HN}) \qquad Q_N \equiv (q_N, \psi_{QN})$$

$$\tilde{H}_N \equiv (\cancel{\tilde{h}_N}, \tilde{\psi}_{HN}) \qquad \tilde{Q}_N \equiv (\tilde{q}_N, \cancel{\psi_{QN}})$$

Yukawa only at 1

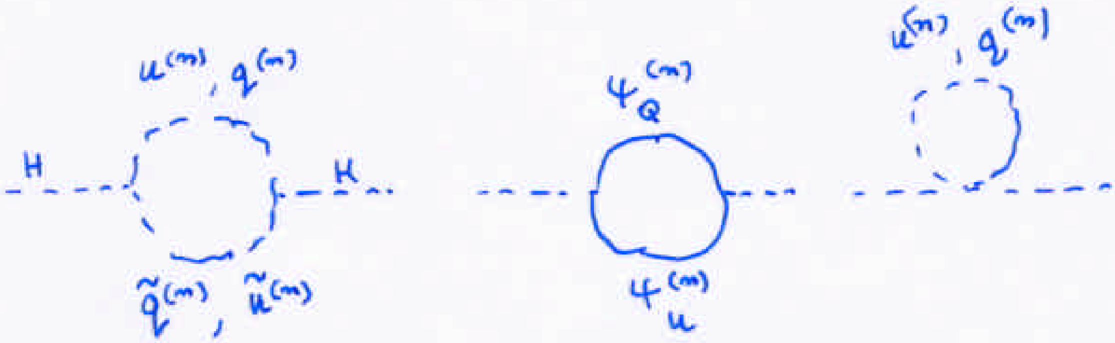
$$W = \lambda\, U_1 Q_1 H_1$$

Non - supersymmetric spectrum but finite Yukawa 1- loop contribution to the Higgs mass and the electroweak symmetry breaking is triggered by the top Yukawa coupling (no regularization problems)

$$W = \sum_i g_4 \tilde{H}_i \phi_i H_{i+1} - \sum m \tilde{H}_i H_i$$

$$+ \sum_i g_4 \tilde{Q}_i \phi_i Q_{i+1} - \sum m \tilde{Q}_i Q_i$$

$$m = g_4 \langle \phi \rangle$$



$$m_\phi^2 = - \frac{\lambda_D^2}{16\pi^2} \frac{g_D^2 v^2}{N} * O(1)$$

Interesting link: $U(n)^N$ with bi-fundamental matter constructed in open string compactification

Douglas & Moore, Vafa et al, Klebanov & Witten, Ibanez et al

So-called quiver theories are constructed by studying a stack of D3 branes on orbifolds of R^6 transverse to the D-branes

Equivalently, quiver theories are obtained by orbifolding $\mathcal{N}=4$ SUSY $U(Nn)$ in 4d by a discrete Z_N group embedded in the $SU(4)$ R-symmetry group of $\mathcal{N}=4$ $U(Nn)$.

Field content of $\mathcal{N}=4$: gauge field A
gaugino λ
three fermions ψ_i
— " — scalars ϕ_i
all in the adjoint of $U(Nn)$

$\mathcal{N} = 2, 1, 0$ $\quad U(n)^N$ theories can be constructed.

$\mathcal{N} = 2, 1$ — superconformal (finite)

$\mathcal{N} = 0$: equal number of bosons and fermions but they are in different gauge group representations

fermions $(n_p, \bar{n}_{p+a_i})$ $i = 1..4$
bosons $(n_p, \bar{n}_{p+a_i+a_4})$ $i = 1, 2, 3$
gauge $(n_p, \bar{n}_p)$

$$\sum_{i=1}^{4} a_i = 0$$

Scalar potential has flat directions, in particular $\quad \phi_i = v_i \mathbb{1}_{n \times n}$

which breaks $\quad U(n)^N \to U(n)_{\text{diagonal}}$

Stringy interpretation of diagonal breaking: moving branes away from the fixed points of $\mathbb{Z}_N$
Klebanov & Witten

$\mathcal{N} = 1$: realistic low energy models embedded in superconformal (finite) theories?

$\underline{\mathcal{N} = 0}$ (Brax, Falkowski, Lalak, SP)

Non-supersymmetric quiver theory
$U(n)^N$

but we find:

no one-loop quadratic divergences in the effective potential ($\mathrm{STr}\, M^2 = 0$ identically)
for any pattern of symmetry breaking
if there are no scalars in the adjoint representation of one of $U(n)$'s

(related to "weak" solution to the hierarchy problem)

Moreover, in the deconstruction phase, with

$$\langle \phi_i \rangle = v_i \mathbb{1}_{n\times n}$$

and $U(n)^N \to U(n)_D$ one can identify "custodial" supersymmetry: all terms in the lagrangian up to terms quadratic in the heavy modes match the structure of $\mathcal{N}=1$ supersymmetric theory (the zero mode sector has $\mathcal{N}=4$)

- n th level spectrum is boson-fermion degenerate
- the universal vevs are a flat direction at 1-loop. Indeed, $\mathrm{STr}\, M^{2q} = 0$ and 1-loop effective potential is zero. $v \ll \Lambda^2$ protected
- 1-loop corrections to the zero mode masses vanish

Custodial susy is explicitly violated by triple and quartic self-interactions of the heavy modes. So UV behaviour of higher loops - not protected

The low-energy, broken, phase of the model is more symmetric than the unbroken, high-energy phase

Theories with replicated gauge groups broken down to the diagonal subgroup ($SU(3)\times SU(2)\times U(1)$?) –

a useful alternative to extra dimensions

Interesting link to open string theories