SUSY

Supersymmetric field theory

Superspace $x^\mu, \theta^\alpha, \bar{\theta}_{\dot\alpha}$

Superfields : $\phi(x, \theta, \bar{\theta})$

Gauge Transf : $\delta\phi = i\Lambda(x, \theta, \bar{\theta})\,\phi$

N.C.T.T :

N.C. Coordinates $\hat{x}^\mu$

$[\hat{x}^\mu, \hat{x}^\nu] = i C^{\mu\nu}(\hat{x})$

N.C field : $\phi(\hat{x})$

N.C gauge transformation $\delta\phi = i\Lambda(\hat{x})\phi$

$C^{\mu\nu}(\hat{x}) = \theta^{\mu\nu}$: constant : canonical c

$C^{\mu\nu}(\hat{x}) = f^{\mu\nu}{}_\lambda \hat{x}^\lambda$: linear, : Lie case

$C^{\mu\nu}(\hat{x}) = \{\delta^\mu_\rho \delta^\nu_\sigma + R^{\mu\nu}{}_{\rho\sigma}\}\hat{x}^\rho \hat{x}^\sigma$: Quantum c

To every gauge theory on commutative coordinates there corresponds one on n.c coordinates

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$$[\hat{x}^\mu, \hat{x}^\nu] = ih\theta^{\mu\nu}$$

this leads to the following new terms in the Lagrangian: (to first order in h)

$$\begin{aligned}
\int \bar{\psi} \star (\gamma^\mu \mathcal{D}_\mu - m)\psi \mathrm{d}x &= \int \bar{\psi}^0 (\gamma^\mu \mathcal{D}_\mu - m)\psi^0 \mathrm{d}x \\
&\quad - \frac{1}{4} h\theta^{\kappa\lambda} \int \bar{\psi}^0 F^0_{\kappa\lambda} (\gamma^\mu \mathcal{D}_\mu - m)\psi \\
&\quad - \frac{1}{4} h\theta^{\kappa\lambda} \int \bar{\psi}^0 \gamma^\mu F^0_{\mu\kappa} \mathcal{D}_\lambda \psi^0 \mathrm{d}x \\
-\frac{1}{4} \mathrm{Tr} \int F_{\mu\nu} \star F^{\mu\nu} \mathrm{d}x &= -\frac{1}{4} \mathrm{Tr} \int F^0_{\mu\nu} F^{0\mu\nu} \mathrm{d}x \\
&\quad + \frac{1}{8} h\theta^{\kappa\lambda} \mathrm{Tr} \int F^0_{\kappa\lambda} F^0_{\mu\nu} F^{0\mu\nu} \mathrm{d}x \\
&\quad - \frac{1}{2} h\theta^{\kappa\lambda} \mathrm{Tr} \int F^0_{\mu\kappa} F^0_{\nu\lambda} F^{0\mu\nu} \mathrm{d}x
\end{aligned}$$

B. Jurco, L. Möller, S. Schraml, P. Schupp, J. Wess

NC Standard Model

- gauge group $SU(3) \times SU(2) \times U(1)$
- particle content the same as ordinary SM
- zeroth order reproduces the ordinary SM

Lorentz symmetry violation relative to $\theta^{\mu\nu}$

Calmet, Jurčo, Schupp, Wess, Wohlgenannt

Freedom in the choice of traces in kinetic terms – minimal choice (minimal deviation from commutative SM)

$$F^0_{\mu\nu} = F^i_{\mu\nu} T^i$$

$\mathrm{Tr}\, T\, T\, T$ is representation dependent

Possible phenomenology
including non-minimal versions

- SM forbidden tree level decays
- $Z \to 2\gamma$
- $\bar{Q}Q \to 2\gamma$

- SM spin forbidden decays from flavour changing neutral currents
- $K \to \pi\gamma$
- $B \to K\gamma$

CPT

$\gamma^* \to \nu\bar{\nu}$

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standard model

$$\Gamma_{Z\to\gamma\gamma} = \frac{\alpha}{12} M_Z^5 \sin^2 2\theta_W \, K^2 \left[\frac{7}{3}(\vec{\theta}_T)^2 + (\vec{\theta}_S)^2\right]$$

rest frame, spin averaged

tree level $\mathcal{L}_{Z\gamma\gamma} \sim$

$$\theta^{kl}[2(-\partial_i Z_k + \partial_k Z_i)\partial_j A_l(\partial^i A^j - \partial^j A^i)$$

$$+(\partial_i A_k \partial_j A_l + \partial_k A_i \partial_l A_j - 2\partial_k A_i \partial_j A_l)(-\partial^i Z^j + \partial^j Z^i)$$

$$+(-2\partial_k Z_i \partial_l A_j + 2\partial_j Z_l \partial_k A_i + 2\partial_i Z_j \partial_k A_l + \partial_k Z_l \partial_i A_j)$$

$$\times(\partial^i A^j - \partial^j A^i)]$$

W. Behr, N. G. Deshpande,
G. Duplančič, P. Schupp,
J. Trampetić, J. Wess

New ideas :

1) Noncommutative coordinates

$$[\hat{x}^\mu, \hat{x}^\nu] = i C^{\mu\nu}(\hat{x})$$

e.g. $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$: canonical

H. S. Snyder (1947)   Heisenberg, Pauli

2) $*$ Product representation

$f(x^1 \dots x^n)$: objects in algebra in Physics (fields)

$f * g$ such that $x^\mu * x^\nu - x^\nu * x^\mu = i C^{\mu\nu}(x)$

Canonical

$$f * g(x) = e^{\frac{i}{2} \frac{\partial}{\partial y^\nu} \theta^{\nu\mu} \frac{\partial}{\partial x^\mu}} f(y)\, g(x) \Big|_{y \to x}$$

link to deformation quantization

[861] Pauli an Bohr

Zürich, 28. Januar 1947
[Maschinenschrift]

Dear Bohr!

Many thanks for your letter and for the abstract of your talk at the Cambridge meeting. I read this abstract with the greatest interest and it seems to me that in its present form your views are much clearer expressed. So I hope, that it will appear soon just as it is now. Obviously only the future can show whether your view on the open questions is correct. Perhaps I had still more stressed the necessity of new ideas (in contrast to a procedure on the old lines with better mathematics) than you did already. On the other hand I am looking as critical as you on this idea of a so-called *universal length*. If this length – let us call it l_0 – is understood to be of a geometrical nature, such theories or models will always lead to strange consequences for large momenta of the order h/l_0 in a field of purely classical experiments where the quantum of action should not play any role. Recently we discussed here in Zürich a mathematically *ingenious* proposal of Snyder, which, however, seems to be a failure for reasons of physics of the type just mentioned.

3) Enveloping algebra valued gauge transformations

Lie algebra: $[L^i, L^j] = i f^{ij}_k L^k$

$\delta \psi = i \Lambda(x) * \psi$

$\Lambda = \alpha^{(1)}_i(x) L^i + \alpha^{(2)}_{ij}(x) :L^i L^j: + \dots$

$A^i_\mu(x)$: usual gauge potential

$\alpha^{(r)}_{i_1 \dots i_r}(x) = \Lambda^{(r)}[\alpha^{(1)}_i, A^i_\mu(x), \partial]$

$\Lambda \to \Lambda_\alpha$, $\Lambda_\alpha = \alpha_0 + \frac{1}{2} \theta^{\mu\nu} : \partial_\mu \alpha^0(x) A_\nu$

$\delta_\alpha \psi = i \Lambda_\alpha * \psi$

$(\delta_\alpha \delta_\beta - \delta_\beta \delta_\alpha) \psi = \delta_{\alpha \times \beta} \psi$

N. Seiberg, E. Witten

J. Madore, S. Schraml, P. Schupp, J. Wess

4. Seiberg Witten map

$$\Lambda_\alpha(x) = \Lambda_\alpha\{\alpha^i(x), A^i_\mu(x), \partial\}$$

$$\delta\psi = i\alpha^i(x)T^i\psi$$

$$\delta A^i_\mu(x) = \partial_\mu\alpha^i(x) - \alpha_b a_{\mu c} f^{ibc}$$

$$\hat\psi = \hat\psi\{\alpha, A, \partial\}$$

$$\delta\hat\psi = i\Lambda_\alpha * \hat\psi$$

$$\hat A^i_\mu\{\alpha, A, \partial\}$$

$$\delta\hat A^*_\mu = -i[x_\mu \overset{*}{,} \Lambda_\alpha] + i[\Lambda_\alpha \overset{*}{,} \hat A_\mu]$$

canonical:

$$\hat A_\mu = A_\mu + \frac{1}{4}\theta^{\rho\nu} :\{A_\nu, (\partial_\rho A_\mu + F_{\rho\mu})\}: + \dots$$

$$\hat\psi = \psi + \frac{1}{2}\theta^{\mu\nu}A_\nu\partial_\mu\psi + \frac{1}{4}\theta^{\mu\nu}\partial_\mu A_\nu\psi + \dots$$

5 Covariant coordinates

$$\delta \hat\psi = i \alpha * \hat\psi$$

$$\delta \hat x^\mu * \hat\psi = \hat x^\mu * i\alpha * \hat\psi$$

$$\neq i\alpha * \hat x^\mu * \hat\psi$$

$$\hat X^\mu = \hat x^\mu + \hat A^\mu$$: covariant coordinates

$$\delta \hat X^\mu * \hat\psi = i\alpha * \hat X^\mu * \psi$$

$$\delta A^\mu = -i[x^\mu \overset{*}{,} \alpha] + i[\alpha \overset{*}{,} A^\mu]$$

Fieldstrength:

$$\hat X^\mu \hat X^\nu - \hat X^\nu \hat X^\mu = i\theta^{\mu\nu} + \hat F^{\mu\nu}$$

$$\delta \hat F^{\mu\nu} = [\alpha \overset{*}{,} \hat F^{\mu\nu}]$$

Seiberg Witten map.

$$\Lambda_\alpha(x) = \Lambda_\alpha\{\alpha^i(x), A_\mu^i(x), \partial\}$$

$$\delta_\alpha \psi = i\Lambda_\alpha * \psi$$

Defining condition:

$$(\delta_\alpha\delta_\beta - \delta_\beta\delta_\alpha)\psi = \delta_{\alpha\times\beta}\psi$$

$$\Lambda_\alpha * \Lambda_\beta - \Lambda_\beta * \Lambda_\alpha + i(\delta_\alpha\Lambda_\beta - \delta_\beta\Lambda_\alpha) = i\Lambda_{\alpha\times\beta}$$

$$[\hat{x}^i \overset{*}{,} \hat{x}^j] = i h \hat{c}^{ij}(\hat{x})$$

Expansion in h

$$\Lambda_\alpha = \Lambda_\alpha^0 + h\Lambda_\alpha^1 + h^2\Lambda_\alpha^2 + \ldots$$

Lowest order in h:

$$\Lambda^0 = \alpha(x)$$

first order in h

$$\Lambda^0_\alpha * \Lambda^0_\beta - \Lambda^0_\beta * \Lambda^0_\alpha \Big|_{\text{first order in } h}$$

$$+ \Lambda^0_\alpha \Lambda^1_\beta + \Lambda^1_\alpha \Lambda^0_\beta - \Lambda^0_\beta \Lambda^1_\alpha - \Lambda^1_\beta \Lambda^0_\alpha$$

$$+ i (\delta_\alpha \Lambda^1_\beta - \delta_\beta \Lambda^1_\alpha) = i \Lambda^1_{\alpha * \beta}$$

Inhomogeneous linear equation for Λ^1

Canonical model:

$$\Lambda^1_\alpha = \frac{1}{4} \theta^{\xi \gamma} \{ \partial_\xi \alpha , A_\gamma \}$$

+ solution of the homogeneous eqn.

$$\Lambda^0_\alpha \Lambda^n_\beta + \Lambda^n_\alpha \Lambda^0_\beta - \Lambda^0_\beta \Lambda^n_\alpha - \Lambda^n_\beta \Lambda^0_\alpha$$

$$+ i (\delta_\alpha \Lambda^n_\beta - \delta_\beta \Lambda^n_\alpha) = F(\Lambda^0 \ldots \Lambda^{n-1})$$

The same homogeneous eqn

Stora ....

BRS :

$$s\,\omega = -\omega\cdot\omega$$

$$s\,a = -d\omega - a\omega - \omega a$$

$$s\,\psi = -\rho(\omega)\psi$$

$$s\,\Omega = -\Omega * \Omega$$

$$s\,A = -d\Omega - A*\Omega - \Omega*A$$

$$s\,\psi = -\rho(\Omega)*\psi$$

Ω, A : enveloping algebra valued

$$\Omega = \sum_n h^n \Omega^{(n)}$$

$$A = \sum_n h^n A^{(n)}$$

$$\psi = \sum_n h^n \psi^{(n)}$$

$$\Delta = s + [\omega, \quad]$$

$$\Delta^2 = 0$$

$$\Delta \Omega^{(n)} = M^{(n)}$$

Consistency : $\Delta M^{(n)} = 0$

$$\Omega^{(n)\prime} = \Omega^{(n)} + \Delta \tilde{f}^{(n)}$$

$$\Delta \Omega^{\prime(n)} = M^{(n)}$$

Zumino : . . . .

Formal integration :

$$K\Delta + \Delta K = 1$$

$$(K\Delta + \Delta K) M = \Delta K M = 1 M$$

$$\Omega = k M$$

Gauge Theories
on Differential Manifolds $(\mathbb{R}^n)$

$\Downarrow$

Algebras

Gauge Theories on Algebras

we loose information:
on the concept of Topology
Neighborhood
Points

might be right for:
Physics at very short distances

Maintain as much as possible
the principles of
gauge theories

L

Gauge theories from an algebraic point of view:

$x^1 \ldots x^N$: commuting coordinates

$[x^i, x^j] = 0$: Relations : R

$$\mathcal{A}_x = \mathbb{C}[[x^1 \ldots x^N]] / I_R$$

Algebra of polynomials and formal power series.

$f \in \mathcal{A}_x$ Basis:

$$f = \sum c_{i_1 \ldots i_N} (x^1)^{i_1} \ldots (x^N)^{i_N}$$

$$= \sum c_{i_1 \ldots i_n} : (x^1)^{i_1} \ldots (x^n)^{i_n} :$$

$f \cdot g = h$ $f \in \mathcal{A}_x$, $g \in \mathcal{A}_x$

$h \in \mathcal{A}_x$

Noncommutative coordinates

$\hat{x}^1 \dots \hat{x}^n$

$[\hat{x}^i, \hat{x}^j] = i\, C^{ij}(\hat{x})$ $\qquad R_c$

$$\hat{\mathcal{A}}_{\hat{x}} = \mathbb{C}[[\hat{x}^1 \dots \hat{x}^N]] \Big/ I_{R_c}$$

formal power series.

Polynomials of degree L form a vectorspace

V_L, it should have the same dimension as V_L

$$V_L \sim V_L$$

: Poincaré, Birkhoff, Witt property

$\hat{f} \in \hat{\mathcal{A}}_{\hat{x}}$

$$\hat{f} = \sum c_{i_1} \dots c_{i_n} \underbrace{:(\hat{x}^1)^{i_1} \dots (\hat{x}^N)^{i_N}:}_{\text{Basis}}$$

$\hat{f} \longleftrightarrow f$ Vectorspace isomorphism

$$f = \sum c_{i_1 \dots i_n} :(x^1)^{i_1} \dots (x^n)^{i_n}:$$

Algebra isomorphism

$$\hat{f} \cdot \hat{g} = \hat{h} \qquad \hat{f} \sim f$$
$$\hat{g} \sim g$$
$$\hat{h} \sim h$$

$$\Big\downarrow$$

$$f * g = h \quad : \text{Star product}$$

$$f \in \mathcal{A}^*_x$$

Examples:

Canonical case

$$[\hat{x}^i, \hat{x}^j] = i\theta^{ij}$$

Sym. Basis

$$f * g = f(x) e^{\frac{i}{2} \frac{\overleftarrow{\partial}}{\partial x^i} \theta^{ij} \frac{\overrightarrow{\partial}}{\partial x^j}} g(x)$$

Moyal-Weyl product

Lie Algebra:

$\hat{x}^i \to T^i \qquad x^i \to t^i$

$$[T^i, T^j] = i f^{ij}_{\ \ k} T^k \qquad \text{P.B.W}$$

$$e^{ik\cdot T} e^{ip\cdot T} = e^{i\left(k+p+\frac{i}{2} g(k,p)\right) T}$$

Baker Campbell Hausdorf

$$f * g(t) = e^{\frac{i}{2} t\, g\left(i\frac{\partial}{\partial t'}, i\frac{\partial}{\partial t''}\right)} f(t') g(t'')\Big|_{t''=t'=t}$$

: Enveloping algebra of the Lie algebra

Quantum space $\hat{x}, \hat{y}$, $\hat{x}\hat{y} = q\hat{y}\hat{x}$

$$f * g(x,y) = q^{-x'\frac{\partial}{\partial x'} y\frac{\partial}{\partial y}} f(x,y) g(x',y')\Big|_{\substack{x'\to x \\ y'\to y}}$$

Basis: symmetrized products.

[247] Heisenberg an Peierls

Helgoland[a], 13. Juni [1930]

Lieber Herr Peierls!

Im ganzen glaub' ich aber doch nicht daran, daß mit den bisherigen Vertauschungs-Relationen eine vernünftige Lösung der Gleichungen möglich ist. Ich möchte in ähnlicher Weise, wie wir in Kopenhagen besprochen haben[b], doch Ungenauigkeiten der Ortskoordinanten einführen; aber diesmal ohne neue universelle Länge, sondern etwa nach dem Schema $\{x_i, x_k\} = \delta_{ik}$ oder $\{f_{ik} | x_r x_s\}_- = \delta_{ik}\delta_{ks}$.

Mir ist es bisher nicht gelungen, solchen Vertauschungs-Relationen einen vernünftigen mathematischen Sinn zuzuordnen. Dagegen wären die zugehörigen Ungenauigkeitsrelationen äußerst vernünftig; auch wären die Vertauschungs-Relationen relativistisch invariant und es gäbe beliebig kurze Wellen. Fällt Ihnen oder Pauli nicht vielleicht etwas über den mathematischen Sinn solcher Vertauschungs-Relationen ein?

Ich freue mich sehr auf Ihr Kommen zu den Leipziger Festspielen und hoffe dringend, daß Sie auch Ihren Chef mitbringen[c]. Ihnen und Pauli also die herzlichsten Grüße

von Ihrem W. Heisenberg

Non commutative coordinates