

# SUSY

Supersymmetric field theory

Superspace  $x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}$

Superfields :  $\phi(x, \theta, \bar{\theta})$

Gauge Transform :  $\delta\phi = i\Lambda(x, \theta, \bar{\theta})\phi$

N.C.T.T :

N.C. Coordinates  $\hat{x}^\mu$

$$[\hat{x}^\mu, \hat{x}^\nu] = i C^{\mu\nu}(\hat{x})$$

N.C field :  $\phi(\hat{x})$

N.C gauge transformation  $\delta\phi = i\Lambda(\hat{x})\phi$

$C^{\mu\nu}(\hat{x}) = \theta^{\mu\nu}$  : constant : canonical

$C^{\mu\nu}(\hat{x}) = f^{\mu\nu}_\lambda \hat{x}^\lambda$  : linear, : Lie case

$C^{\mu\nu}(\hat{x}) = \left\{ \begin{matrix} S^{\mu\nu} \\ J^{\mu\nu} \\ R^{\mu\nu} \\ SO \end{matrix} \right\} \hat{x}^\sigma \hat{x}^\tau$  : Quantum

To every gauge theory on commutative coordinates there corresponds one on n.c. coordinates

$$[\hat{x}^\mu, \hat{x}^\nu] = i\hbar\theta^{\mu\nu}$$

this leads to the following new terms  
in the Lagrangian: (to first order in  $\hbar$ )

$$\begin{aligned} \int \bar{\psi} \star (\gamma^\mu \mathcal{D}_\mu - m) \psi dx &= \int \bar{\psi}^0 (\gamma^\mu \mathcal{D}_\mu - m) \psi^0 dx \\ &\quad - \frac{1}{4} \hbar \theta^{\kappa\lambda} \int \bar{\psi}^0 F_{\kappa\lambda}^0 (\gamma^\mu \mathcal{D}_\mu - m) \psi^0 dx \\ &\quad - \frac{1}{4} \hbar \theta^{\kappa\lambda} \int \bar{\psi}^0 \gamma^\mu F_{\mu\kappa}^0 \mathcal{D}_\lambda \psi^0 dx \\ - \frac{1}{4} \text{Tr} \int F_{\mu\nu} \star F^{\mu\nu} dx &= - \frac{1}{4} \text{Tr} \int F_{\mu\nu}^0 F^{0\mu\nu} dx \\ &\quad + \frac{1}{8} \hbar \theta^{\kappa\lambda} \text{Tr} \int F_{\kappa\lambda}^0 F_{\mu\nu}^0 F^{0\mu\nu} dx \\ &\quad - \frac{1}{2} \hbar \theta^{\kappa\lambda} \text{Tr} \int F_{\mu\kappa}^0 F_{\nu\lambda}^0 F^{0\mu\nu} dx \end{aligned}$$

B. Jurco, L. Möller, S. Schraml.

P. Schupp, J. Wess

## NC Standard Model

- gauge group  $SU(3) \times SU(2) \times U(1)$
- particle content the same as ordinary SM
- zeroth order reproduces the ordinary SM

Lorentz symmetry violation relative to  $\theta^{\mu\nu}$

Colmet, Jurčo, Schupp, Wess, Wohlgenannt

Freedom in the choice of traces in kinetic terms – minimal choice (minimal deviation from commutative SM)

$$F_{\mu\nu}^0 = F_{\mu\nu}^i T^i$$

$$\text{Tr } T T T$$

is representation dependent

## Possible phenomenology including non-minimal versions

- SM forbidden tree level decays
  - $Z \rightarrow 2\gamma$
  - $\bar{Q}Q \rightarrow 2\gamma$
- SM spin forbidden decays from flavour changing neutral currents
  - $K \rightarrow \pi\gamma$
  - $B \rightarrow K\gamma$

CPT

$$\gamma \rightarrow \nu \bar{\nu}$$

standard model  
↓

$$\Gamma_{Z \rightarrow \gamma\gamma} = \frac{\alpha}{12} M_Z^5 \sin^2 2\theta_w K^2 \left[ \frac{7}{3} (\vec{\Theta}_T)^2 + (\vec{\Theta}_S)^2 \right]$$

rest frame, spin averaged

tree level  $\mathcal{L}_{Z\gamma\gamma} \sim$

$$\begin{aligned} & \theta^{kl} [2(-\partial_i Z_k + \partial_k Z_i) \partial_j A_l (\partial^i A^j - \partial^j A^i) \\ & + (\partial_i A_k \partial_j A_l + \partial_k A_i \partial_l A_j - 2\partial_k A_i \partial_j A_l) (-\partial^i Z^j + \partial^j Z^i) \\ & + (-2\partial_k Z_i \partial_l A_j + 2\partial_j Z_l \partial_k A_i + 2\partial_i Z_j \partial_k A_l + \partial_k Z_l \partial_i A_j) \\ & \quad \times (\partial^i A^j - \partial^j A^i)] \end{aligned}$$

W. Behr, N. G. Deshpande,  
G. Duplancić, P. Schupp,  
J. Trampetic, J. Weiss

## New ideas :

1) Noncommutative coordinates

$$[\hat{x}^\mu, \hat{x}^\nu] = iC^{\mu\nu}(\hat{x})$$

e.g.  $[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu}$  : canonical

H.S. Snyder (1947)

Heisenberg, Pauli

2) \* Product representation

$f(x^1 \dots x^n)$  : objects in algebra  
in physics (fields)

$f * g$  such that  $x^\mu * x^\nu - x^\nu * x^\mu = iC^{\mu\nu}(x)$

canonical

$$f * g(x) = \left. e^{\frac{i}{2} \frac{\partial}{\partial y^\mu} \Theta^{\mu\nu} \frac{\partial}{\partial x^\nu}} f(y) g(x) \right|_{y \rightarrow x}$$

link to deformation quantization

[861] PAULI AN BOHR

Zürich, 28. Januar 1947  
[Maschinenschrift]

Dear Bohr!

Many thanks for your letter and for the abstract of your talk at the Cambridge meeting. I read this abstract with the greatest interest and it seems to me that in its present form your views are much clearer expressed. So I hope, that it will appear soon just as it is now. Obviously only the future can show whether your view on the open questions is correct. Perhaps I had still more stressed the necessity of new ideas (in contrast to a procedure on the old lines with better mathematics) than you did already. On the other hand I am looking as critical as you on this idea of a so-called *universal length*. If this length - let us call it  $l_0$  - is understood to be of a geometrical nature, such theories or models will always lead to strange consequences for large momenta of the order  $h/l_0$  in a field of purely classical experiments where the quantum of action should not play any role. Recently we discussed here in Zürich a mathematically *ingenious* proposal of Snyder, which, however, seems to be a failure for reasons of physics of the type just mentioned.

3) Enveloping algebra values  
gauge transformations

$$\text{Lie algebra: } [L^i, L^d] = i f^i{}^d{}_k L^k$$

$$\mathcal{J}\psi = i \Lambda(x) * \psi$$

$$\Lambda = \alpha_{i_1}^{(1)}(x) L^{i_1} + \alpha_{i_1 i_2}^{(2)}(x) L^{i_1} L^{i_2} + \dots$$

$A_{\mu}^i(x)$  : usual gauge potential

$$\alpha_{i_1 \dots i_r}^{(r)}(x) = \Lambda^{(r)}[\alpha_{i_1}^{(1)}, A_{\mu}^i(x), \partial]$$

$$\Lambda \rightarrow \Lambda_{\alpha}, \quad \Lambda_{\alpha} = \alpha_0 + \frac{1}{2} \theta^{\mu\nu} \partial_{\mu} \alpha^{\rho}(x) A_{\nu}^{\rho}$$

$$\mathcal{J}_2 \psi = i \Lambda_{\alpha} * \psi$$

$$(\mathcal{J}_{\alpha} \mathcal{J}_{\beta} - \mathcal{J}_{\beta} \mathcal{J}_{\alpha}) \psi = \mathcal{J}_{\alpha \times \beta} \psi$$

N. Seiberg, E. Witten

J. Madore, S. Schraml, P. Schupp, J. Wenz

## 4. Seiberg Witten map

$$\Lambda_\alpha(x) = \Lambda_\alpha \{ \alpha^i(x), A_\mu^i(x), \partial \}$$

$$\delta \psi = i \alpha^i(x) T^i \psi$$

$$\delta A_\mu^i(x) = \partial_\mu \alpha^i(x) - \alpha^j \alpha_{\mu c}^i \}^{jbc}$$

$$\hat{\psi} = \hat{\psi} \{ \alpha, A, \partial \}$$

$$\delta \hat{\psi} = i \Lambda_\alpha * \hat{\psi}$$

$$\hat{A}_\mu^i \{ \alpha, A, \partial \}$$

$$\delta \hat{A}_\mu^i = -i [x_\mu, * \Lambda_\alpha] + i [\Lambda_\alpha, * \hat{A}_\mu^i]$$

canonical:

$$\hat{A}_\mu^i = A_\mu^i + \frac{1}{4} \theta^{\nu\rho} \{ A_\nu^i (\partial_\rho A_\mu^i + F_{\rho\mu}^i) \} + \dots$$

$$\hat{\psi} = \psi + \frac{1}{2} \theta^{\mu\nu} A_\nu \partial_\mu \psi + \frac{1}{4} \theta^{\mu\nu} \partial_\mu A_\nu \psi + \dots$$

## 5 Covariant coordinates

$$\delta \hat{\psi} = i \alpha * \hat{\psi}$$

$$\begin{aligned} \delta \hat{x}^\mu * \hat{\psi} &= \hat{x}^\mu * i \alpha * \hat{\psi} \\ &\neq i \alpha * \hat{x}^\mu * \hat{\psi} \end{aligned}$$

$$\hat{x}^\mu = \hat{x}^\mu + \hat{A}^\mu \quad : \text{covariant coordinates}$$

$$\delta \hat{x}^\mu * \hat{\psi} = i \alpha * \hat{x}^\mu * \hat{\psi}$$

$$\delta \hat{A}^\mu = -i [\hat{x}^\mu, \alpha] + i [\alpha, \hat{A}^\mu]$$

Field strength:

$$\hat{x}^\mu \hat{x}^\nu - \hat{x}^\nu \hat{x}^\mu = i \theta^{\mu\nu} + \hat{F}^{\mu\nu}$$

$$\delta \hat{F}^{\mu\nu} = [\alpha, \hat{F}^{\mu\nu}]$$

## Seiberg Witten map.

$$\Lambda_{\alpha}(x) = \Lambda_{\alpha} \{ \alpha^i(x), A_{\mu}^i(x), 0 \}$$

$$\mathcal{J}_{\alpha} \psi = i \Lambda_{\alpha} * \psi$$

Defining condition:

$$(\mathcal{J}_{\alpha} \mathcal{J}_{\beta} - \mathcal{J}_{\beta} \mathcal{J}_{\alpha}) \psi = \mathcal{J}_{\alpha \times \beta} \psi$$

$$\begin{aligned} & \Lambda_{\alpha} * \Lambda_{\beta} - \Lambda_{\beta} * \Lambda_{\alpha} \\ & + i(\mathcal{J}_{\alpha} \Lambda_{\beta} - \mathcal{J}_{\beta} \Lambda_{\alpha}) = i \Lambda_{\alpha \times \beta} \end{aligned}$$

$$[\hat{X}^i, \hat{X}^j] = i \hbar \hat{C}^{ij}(x)$$

Expansion in  $\hbar$

$$\Lambda_{\alpha} = \Lambda_{\alpha}^0 + \hbar \Lambda_{\alpha}^1 + \hbar^2 \Lambda_{\alpha}^2 + \dots$$

Lowest order in  $\hbar$ :

$$\Lambda_{\alpha}^0 = \alpha(x)$$

first order in  $\hbar$

$$\Lambda_{\alpha}^0 * \Lambda_{\beta}^0 - \Lambda_{\beta}^0 * \Lambda_{\alpha}^0 \Big| \text{ first order in } \hbar$$

$$+ \Lambda_{\alpha}^0 \Lambda_{\beta}^1 + \Lambda_{\alpha}^1 \Lambda_{\beta}^0 - \Lambda_{\beta}^0 \Lambda_{\alpha}^1 - \Lambda_{\beta}^1 \Lambda_{\alpha}^0$$

$$+ i(\mathcal{J}_{\alpha} \Lambda_{\beta}^1 - \mathcal{J}_{\beta} \Lambda_{\alpha}^1) = i \Lambda_{\alpha * \beta}^1$$

Inhomogeneous linear equation for  $\Lambda^1$

Canonical model:

$$\Lambda_{\alpha}^1 = \frac{1}{4} \Theta^{\beta\gamma} \{ \mathcal{D}_{\beta} \alpha, A_{\gamma} \}$$

+ solutions of the homogeneous equ.

$$\Lambda_{\alpha}^0 \Lambda_{\beta}^n + \Lambda_{\alpha}^n \Lambda_{\beta}^0 - \Lambda_{\beta}^0 \Lambda_{\alpha}^n - \Lambda_{\beta}^n \Lambda_{\alpha}^0$$

$$+ i(\mathcal{J}_{\alpha} \Lambda_{\beta}^n - \mathcal{J}_{\beta} \Lambda_{\alpha}^n) = F(\Lambda^0 \dots \Lambda^{n-1})$$

The same homogeneous equ

Stora . . . .

BRS :

$$S \omega = -\omega \cdot \omega$$

$$S \alpha = -d\omega - \alpha \omega - \omega \alpha$$

$$S \psi = -\rho(\omega) \psi$$

$$S \Omega = -\Omega * \Omega$$

$$S A = -d\Omega - A * \Omega - \Omega * A$$

$$S \Psi = -\rho(\Omega) * \Psi$$

$\Omega, A$  : enveloping algebra valued

$$\Omega = \sum_n h^n \Omega^{(n)}$$

$$A = \sum_n h^n A^{(n)}$$

$$\Psi = \sum_n h^n \Psi^{(n)}$$

$$\Delta = S + [\omega, \quad ]$$

$$\Delta^2 = 0$$

$$\Delta \Omega^{(n)} = M^{(n)}$$

$$\text{Consistency: } \Delta M^{(n)} = 0$$

$$\Omega^{(n)'} = \Omega^{(n)} + \Delta \widetilde{F}^{(n)}$$

$$\Delta \Omega^{(n)'} = M^{(n)}$$

Zumino: . . . .

Formal integration:

$$K \Delta + \Delta K = 1$$

$$(K \Delta + \Delta K) M = \Delta K M = 1 M$$

$$\Omega = K M$$

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Gauge Theories

on Differential Manifolds ( $\mathbb{R}^n$ )

↓

Algebras

Gauge Theories on Algebras

we lose information:  
on the concept of Topology  
Neighborhood  
Points

might be right for:

Physics at very short distances

Maintain as much as possible  
the principles of  
gauge theories

Gauge theories from an  
algebraic point of view:

$x^1 \dots x^n$  : commuting coordinates

$[x^i, x^j] = 0$  : Relations :  $R$

$$\mathcal{A}_x = \mathbb{C}[x^1 \dots x^n] / I_R$$

Algebra of polynomials and  
formal power series.

$f \in \mathcal{A}_x$  Basis :

$$\begin{aligned} f &= \sum c_{i_1 \dots i_n} (x^1)^{i_1} \dots (x^n)^{i_n} \\ &= \sum c_{i_1 \dots i_n} : (x^1)^{i_1} \dots (x^n)^{i_n} : \end{aligned}$$

$$f \cdot g = h$$

$$f \in \mathcal{A}_x, g \in \mathcal{A}_x$$

$$h \in \mathcal{A}_x$$

Noncommutative coordinates

$$\hat{x}^1 \dots \hat{x}^n$$

$$[\hat{x}^i, \hat{x}^j] = i C^{ij}(\hat{x}) \quad \mathbb{R}_c$$

$$\hat{\mathcal{A}}_{\hat{x}} = \mathbb{C}[[\hat{x}^1 \dots \hat{x}^n]] / \mathbb{I}_{\mathbb{R}_c}$$

formal power series.

Polynomials of degree  $L$  form a vector space

$V_L$ , it should have the same dimension as  $V_L$

$$V_L \sim V_L$$

: Poincaré, Birkhoff, Witt property

$$\hat{f} \in \hat{\mathcal{A}}_{\hat{x}}$$

$$\hat{f} = \sum c_{i_1 \dots i_n} \underbrace{(\hat{x}^1)^{i_1} \dots (\hat{x}^n)^{i_n}}_{\text{Basis}}$$

$\hat{f} \leftrightarrow f$  Vector space isomorphism

$$f = \sum c_{i_1 \dots i_n} (x^1)^{i_1} \dots (x^n)^{i_n}$$

Algebra isomorphism

$$\hat{f} \cdot \hat{g} = \hat{h}$$

$$\hat{f} \sim f$$

$$\hat{g} \sim g$$

$$\hat{h} \sim h$$



$f * g = h$  : Star product

$$f \in \mathcal{A}_x^*$$

Examples:

Canonical case

$$[\hat{x}^i, \hat{x}^j] = i\theta^{ij}$$

Sym. Basis

$$f * g = f(x) e^{\frac{i}{2} \overleftarrow{\partial}_i \theta^{ij} \overrightarrow{\partial}_j} g(x)$$

Moyal - Weyl product

# Lie Algebra :

$$\hat{x}^i \rightarrow T^i \quad x^i \rightarrow t^i$$

$$[T^i, T^j] = i f_{ij}^k T^k \quad \text{P.B.W}$$

$$e^{i k \cdot T} e^{i p \cdot T} = e^{i (k+p + \frac{i}{2} g(k,p)) T}$$

Baker-Campbell-Hausdorff

$$f * g(t) = e^{\frac{i}{2} t} g\left(i \frac{\partial}{\partial t'}, i \frac{\partial}{\partial t''}\right) f(t') g(t'') \Big|_{t''=t'=t}$$

Enveloping algebra of the Lie algebra

Quantum space  $\hat{x}, \hat{y}$  ,  $\hat{x}\hat{y} = \hat{y}\hat{x} + \hbar$

$$f * g(x, y) = \frac{\hbar^{-x}}{2} \frac{\partial}{\partial x'} \frac{\partial}{\partial y} f(x, y) g(x', y') \Big|_{\substack{x' \rightarrow x \\ y' \rightarrow y}}$$

Basis: symmetrized products.

[247] HEISENBERG AN PEIERLS

Helgoland<sup>a</sup>, 13. Juni [1930]

Lieber Herr Peierls!

Im ganzen glaub' ich aber doch nicht daran, daß mit den bisherigen Vertauschungs-Relationen eine vernünftige Lösung der Gleichungen möglich ist. Ich möchte in ähnlicher Weise, wie wir in Kopenhagen besprochen haben<sup>b</sup>, doch Ungenauigkeiten der Ortskoordinanten einführen; aber diesmal ohne neue universelle Länge, sondern etwa nach dem Schema  $\{x_i, x_k\} = \delta_{ik}$  oder  $\{f_{ik}, x_r, x_s\} = \delta_{ik} \delta_{rs}$ .

Mir ist es bisher nicht gelungen, solchen Vertauschungs-Relationen einen vernünftigen mathematischen Sinn zuzuordnen. Dagegen wären die zugehörigen Ungenauigkeitsrelationen äußerst vernünftig; auch wären die Vertauschungs-Relationen relativistisch invariant und es gäbe beliebig kurze Wellen. Fällt Ihnen oder Pauli nicht vielleicht etwas über den mathematischen Sinn solcher Vertauschungs-Relationen ein?

Ich freue mich sehr auf Ihr Kommen zu den Leipziger Festspielen und hoffe dringend, daß Sie auch Ihren Chef mitbringen<sup>c</sup>. Ihnen und Pauli also die herzlichsten Grüße  
von Ihrem W. Heisenberg

*Non commutative coordinates*