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QCD Mini-jet contribution to the total cross section

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Abstract
We present the predictions of a model for proton-proton total cross-section at LHC. It takes into account both hard partonic processes and soft gluon emission effects to describe the proper high energy behavior and to respect the Froissart bound.

1 Introduction

A reliable prediction of the total proton-proton cross section is fundamental to know which will be the underlying activity at the LHC and for new discoveries in physics from the LHC data. In this article, we shall describe a model [1] [2] for the hadronic total cross section based on QCD minijet formalism. The model includes a resummation of soft gluon radiation which is necessary to tame the fast high-energy rise typical of a purely perturbative minijet model. It is called the BN model from the Bloch and Nordsiek discussion of the infrared catastrophe in QED. In the first section, results are presented concerning the behavior of the QCD minijet cross section. It will then be explained how this term is included into an eikonal formalism where infrared soft gluon emission effects are added. The last section is devoted to the link between the total cross-section asymptotic high energy behavior predicted by our model and the model parameters. This relation also shows that our prediction is in agreement with the limit imposed by the Froissart bound.

2 Mini-jet cross section

Hard processes involving high-energy partonic collisions drive the rise of the total cross section [3]. These jet-producing collisions are typical perturbative processes and we can describe them through the usual QCD expression:

$$\sigma_{\rm jet}^{AB}(s, p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s/2}} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \times \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p_t^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}, \tag{1}$$

with $A, B = p, \bar{p}$. This expression depends on the parameter p_{tmin} which represents the minimum transverse momentum of the scattered partons for which one allows a perturbative QCD treatment. Its value is usually around $\approx 1-2$ GeV and it distinguishes hard processes (that are processes for which a perturbative approach is used) from the soft ones that dominate at low

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energy, typically for $\sqrt{s} \leq 10 \div 20\ GeV$, i.e, well before the cross-section starts rising. The Minijet expression also depends on the DGLAP evoluted Partonic Densities Functions $f_{i|A}$ for which there exist in the literature different LO parameterizations(GRV, MRST, CTEQ [4]). We obtain an asymptotic growth of σ_{jet} with energy as a power of s. As shown in figure 1, the value of the exponent depends on the PDF used and one has

$$\sigma_{jet}^{GRV} \approx s^{0.4} \quad \sigma_{jet}^{MRST} \approx s^{0.3} \quad \sigma_{jet}^{CTEQ} \approx s^{0.3}.$$

This result can be derived by considering the relevant contribution to the integral in (1) in the $\sqrt{s} >> p_{tmin}$ limit. In this limit, the major contribution comes from the small fractions of momentum carried by the colliding gluons with $x_{1,2} << 1$. In this limit we know that the relevant PDF's behave approximately like powers of the momentum fraction x^{-J} with $J \sim 1.3$ [5]. From the previous consideration and noting that $\frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t} \propto \frac{1}{p_t^3}$ we obtain from (1) the following asymptotic high-energy expression for σ_{jet}:

$$\sigma_{jet} \propto \frac{1}{p_{t\,\min}^2} \left[\frac{s}{4p_{t\,\min}^2} \right]^{J-1} . \qquad (2)$$

The dominant term is just a power of s and the estimate obtained for the exponent $\epsilon = J-1 \sim 0.3$ is in agreement with our previous results. We now need to understand how to incorporate into a model for the total cross section this very fast rise at very high energy, which is present in the perturbative regime. Firstly it is important to note that σ_{jet} is an inclusive cross section and therefore contains in itself a multiplicity factor, linked to the average number $< n >$ of partonic collisions that take place during the hadronic scattering. We can approximate the energy driving term at high energy [6] $< n >$ as

$$< n > \approx \sigma_{jet} \cdot A, \qquad (3)$$

where A is a function representing the overlap between the two hadrons.

Now we can derive an expression for the total cross section as a function of $< n >$. Assuming that the number of partonic collisions follows a Poisson distribution, since each interaction is independent from the other, the probability of having k partonic collisions is:

$$P(k, < n >) = \frac{< n >^k \, e^{-<n>}}{k!}. \qquad (4)$$

The average number of partonic collisions should depend on the energy and on the impact parameter b relative to the hadronic process $< n > \equiv < n(b,s) >$. From the previous expression it is possible to obtain the inelastic hadronic cross section:

$$\sigma_{inelastic} = \int d^2b \sum_{k=1} P(k, < n(b,s) >) = \int d^2b \left[1 - e^{-<n(b,s)>} \right], \qquad (5)$$

which is the usual eikonal expression if we consider the link between $< n(b,s) >$ and the eikonal $\chi(b,s)$:

$$< n(b,s) > = 2 \mathrm{Im} \chi(b,s). \qquad (6)$$

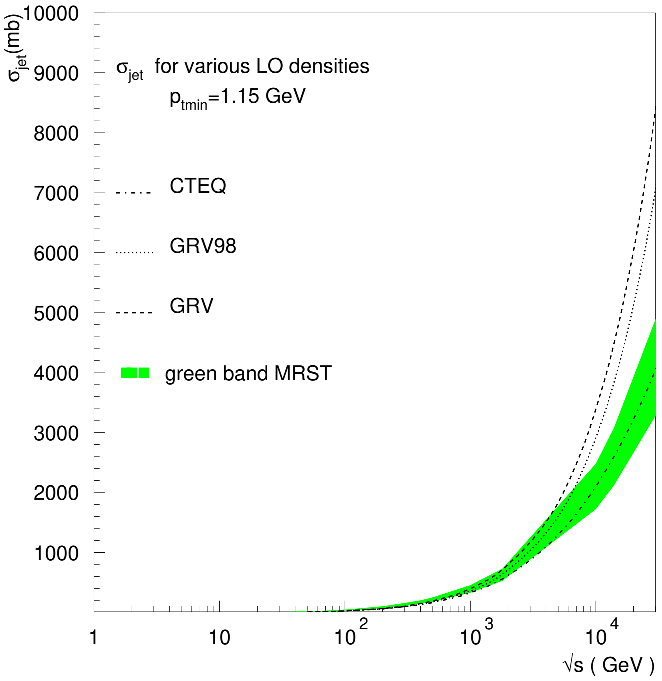

Fig. 1: minijet cross section for different input parton densities.

3 Eikonal model

The eikonal representation allows to implement multiple parton scattering and to restore a finite size of the interaction. Neglecting the real part of the eikonal function, an acceptable approximation in the high energy limit, the expression for the total cross section is

$$\sigma_{tot} = 2 \int d^2b \left[1 - e^{-n(b,s)/2}\right]. \tag{7}$$

The average number of partonic collisions receives contributions both from hard and soft physics processes and we write it in the form

$$n(b,s) = n_{soft}(b,s) + n_{hard}(b,s), \tag{8}$$

where the soft term parameterizes the contribution of all the processes for which the partons scatter with $p_t < p_{tmin}$. It is the only relevant term at low-energy and it establishes the overall normalization, while the hard term is responsible for the high-energy rise. From (3), we approximate this term with

$$n_{hard}(b,s) = A(b,s)\sigma_{jet}(s), \tag{9}$$

where the minijet cross section drives the rise due to the increase of the number of partonic collisions with the energy and $A(b,s)$ is the overlap function which depends on the (energy dependent) spatial distribution of partons inside the colliding hadrons. In some older models [6] a simpler factorized expression for $n(b,s)$ was used, with the overlap function depending only on b. However, when up-to-date realistic parton densities are used, such impact parameter distributions, inspired by constant hadronic form factors, led to an excessive rise of σ_{tot} with the energy. In our BN model we include an s-dependence in the overlap function that has to tame the strong growth due to the fast asymptotic rise of σ_{jet} [2].

We identify soft gluon emissions from the colliding partons as the physical effect responsible for the attenuation of the rise of the total cross section. These emissions influence matter distribution inside of the hadrons, hence changing the overlap function. They break collinearity between the colliding partons, diminishing the efficiency of the scattering process. The number of soft emissions increases with the energy and this makes their contribution important, also at very high energy. The calculation of this effect uses a semiclassical approach based on a Block-Nordsieck inspired formalism [7], the basic assumption of this technique is that all emissions are independent from each other, so the number of gluons emitted follows a Poisson distribution. Thereof one obtains a distribution of the colliding partons as function of the transverse momentum of the soft gluons emitted in the collision, i.e.

$$d^2P(\mathbf{K}_\perp) = d^2\mathbf{K}_\perp \frac{1}{(2\pi)^2}\int d^2\mathbf{b}\, e^{i\mathbf{K}_\perp\cdot\mathbf{b}-h(b)}\,, \tag{10}$$

the factor $h(b)$ is given by

$$h(b) = \int d^3n_g(k)[1-e^{-ik_\perp\cdot b}] = \int \frac{d^3k}{2k_0}\sum_{m,n=colors}|j^{\mu,m}(k)j_{\mu,n}(k)|[1-e^{-ik_\perp\cdot b}],$$

where $d^3n_g(k)$ is the distribution for single gluon emission in a scattering process and it is linked to the QCD current j^μ responsible for emission.

We have proposed to obtain the overlap function as the Fourier transform of the previous expression of the soft gluon transverse momentum resummed distribution, namely to put

$$A_{BN}(b,s) = N\int d^2\mathbf{K}_\perp\, e^{-i\mathbf{K}_\perp\cdot\mathbf{b}}\frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\, e^{-h(b,q_{max})}}, \tag{11}$$

with

$$h(b,q_{max}) = \frac{16}{3}\int_0^{q_{max}} \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\log\frac{2q_{\max}}{k_t}[1-J_0(k_tb)], \tag{12}$$

this integral is performed up to a maximum value which is linked to the maximum transverse momentum allowed by the kinematics for a single gluon emitted, q_{max} [8]. In principle, this parameter and the overlap function should be calculated for each partonic sub-process, but in the partial factorization of Eq.(9) we use an average value of q_{max} obtained considering all the sub-processes that can happen for a given energy of the main hadronic process [2]:

$$q_{\max}(s) = \sqrt{\frac{s}{2}}\,\frac{\sum_{i,j}\int\frac{dx_1}{x_1}\int\frac{dx_2}{x_2}\int_{z_{min}}^1 dz f_i(x_1)f_j(x_2)\sqrt{x_1x_2}(1-z)}{\sum_{i,j}\int\frac{dx_1}{x_1}\int\frac{dx_2}{x_2}\int_{z_{min}}^1 dz f_i(x_1)f_j(x_2)}, \tag{13}$$

with $z_{min} = 4p_{tmin}^2/(sx_1x_2)$. Notice that consistency of the calculation requires that the PDF's used in Eq. (13) be the same as those used in σ_{jet}. In Fig.2 are presented our results for q_{max} as function of $\sqrt{s}$ using $p_{tmin} = 1.15$ GeV.

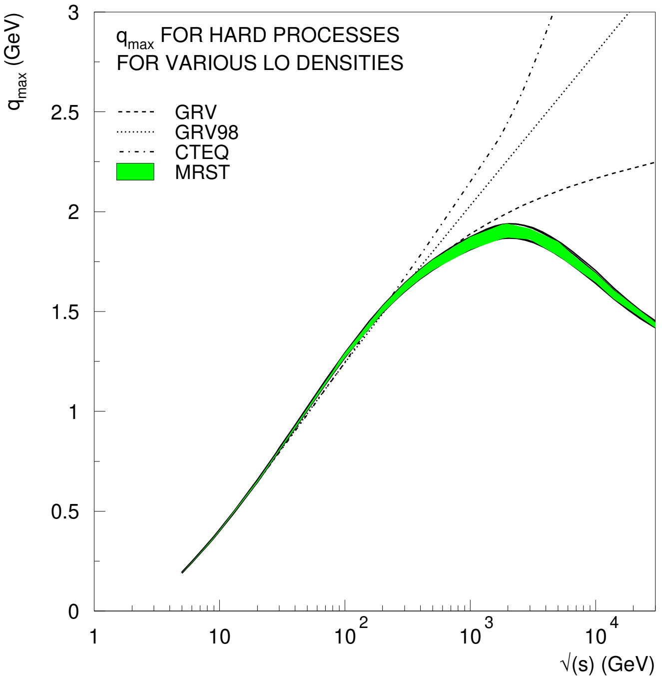

Fig. 2: Results for q_{max} using different input parton densities with $p_{tmin} = 1.15$ GeV.

The integral in (12) has another relevant feature, it extends down to zero momentum values, and to calculate it we have to take an expression of α_s different from the perturbative QCD expression which is singular and not integrable in (12). We use a phenomenological expression [9], which coincides with the usual QCD limit for large k_t, and is singular but integrable for $k_t \to 0$:

$$\alpha_s(k_t^2) = \frac{12\pi}{33 - 2N_f} \frac{p}{\ln[1 + p(\frac{k_t}{\Lambda})^{2p}]}. \quad (14)$$

This expression for α_s is inspired by the Richardson expression for a linear confining potential [10], and we find for the parameter p that

- $p < 1$ to have a convergent integral (unlike the case of the Richardson potential where $p = 1$)
- $p > 1/2$ for the correct analyticity in the momentum transfer variable.

Fig.3 [1] shows our predictions, obtained for the total cross-section using a set of phenomenological values for p_{tmin} and p, and varying the parton densities. We also make a comparison with data and other current models.

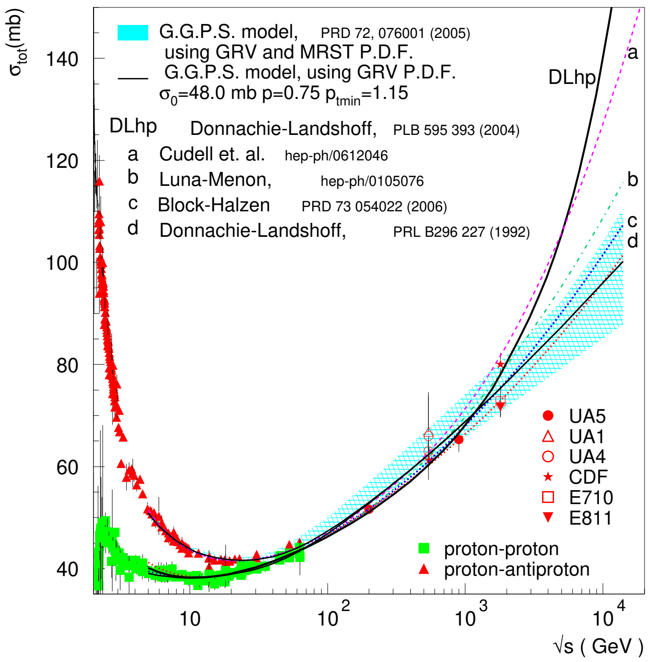

Fig. 3: Results from our total cross-section model (for different parton densities) compared with data [11] and with other models [12].

4 Restoration of Froissart Bound

The Froissart Martin Bound [13] states that σ_{tot} cannot rise faster than a function which is proportional to $log^2(s)$. In order to see that in our model this bound is respected, we approximate our total cross section at very large energies as

$$\sigma_{tot} \approx 2\pi \int db^2 \left[1 - e^{-n_{hard}(b,s)/2}\right], \tag{15}$$

with $n_{hard}(b,s) \approx \sigma_{jet}(s) A_{hard}(b,s)$. We then take for σ_{jet} the asymptotic high energy expression:

$$\sigma_{jet} = \sigma_1 \left(\frac{s}{GeV^2}\right)^{\varepsilon},$$

with σ_1 =constant and $\epsilon \sim 0.3 - 0.4$. Being $A_{hard}(b,s) \propto e^{-h(b,s)}$, we can consider in (12) the infrared limit $k_t \to 0$ where the integral receives the dominant contribution. In this limit we have

$$\alpha_s(k_t^2) \approx \left(\frac{\Lambda}{k_t}\right)^{2p},$$

apart from logarithmic terms. Then, with $h(b,s) \propto (b\bar{\Lambda})^{2p}$ [2] (again apart from logarithmic terms), we have

$$A_{hard}(b) \propto e^{-(b\bar{\Lambda})^{2p}},$$

and from this expression

$$n_{hard} = 2C(s)e^{-(b\bar{\Lambda})^{2p}},$$

with $C(s) = \frac{A_0\sigma_1}{2}\left(\frac{s}{GeV^2}\right)^{\varepsilon}$. The very high energy limit of Eq. (15) then gives

$$\sigma_{tot} \approx 2\pi \int_0^{\infty} db^2[1 - e^{-C(s)e^{-(b\bar{\Lambda})^{2p}}}] \to \left[\varepsilon \ln\left(\frac{s}{GeV^2}\right)\right]^{1/p}. \tag{16}$$

The asymptotic growth of σ_{tot} in our model depends on the parameter ϵ which fixes the asymptotic rise of the minijet cross section, and on p which modulates the infrared behavior of α_s. Notice that $1/2 < p < 1$ and thus this approximated result links the restoration of the Froissart bound in our model with the infrared behavior of α_s. We can now understand why a knowledge of the confining phase of the strong interaction is necessary if we want to restore the finite size of the hadronic interaction.

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