Grand Unification with Anomalous U(1) Symmetry.

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SO(10)	hep-ph/0104200 (PTP)	N.M.
μ	hep-ph/0107313 (PLB)	N.M.
E_6 (matter)	hep-ph/0109018 (PTP)	M. Bando & N.M.
Coupling Unification	hep-ph/0111205 (PTP)	N.M.
E_6 (Higgs)	hep-ph/0202050 (PTP)	N.M. & T. Yamashita
$SU(3)^3$	hep-ph/0204030	N.M. & Q. Shafi
2loop RGE.	hep-ph/0205185	N.M. & T. Yamashita.

I Overview of Our Scenario

II. Anomalous U(1) Gauge Symmetry

III. Doublet-Triplet Splitting

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V. Quark & Lepton Mass Matrices

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I Overview of our scenario

★ "Generic" interactions.

Symmetry $\begin{pmatrix} SO(10) \\ E_6 \end{pmatrix} \times U(1)_A$ — Anomalous U(1) Gauge Symmetry

$\Downarrow$

Definition of the theory (except O(1) coefficients).

Input (# Anomalous U(1) charges (integer)).

	matter	Higgs
SO(10)	$4\,[3\times\mathbf{16}+1\times\mathbf{10}]$	$8\,[2\times\mathbf{45}+2\times(\mathbf{16}+\overline{\mathbf{16}})+2\times\mathbf{10}]$
E_6	$3\,[3\times\mathbf{27}]$	$8\,[2\times\mathbf{78}+3\times(\mathbf{27}+\overline{\mathbf{27}})]$

Output

- GUT scales, other symmetry breaking scales
- Doublet-triplet splitting.
- Proton decay via dim. 5 operators is suppressed.
- MSSM is obtained at the low energy scale.
- Mass spectrum of superheavy fields. $m_{SH} < \Lambda_G$
- Gauge coupling unification in MSSM is naturally explained.
- Realistic quark & lepton masses & mixings (no GUT relation)

  Neutrino Bi-large mixing (LMA)

+ SUSY

- μ problem
- Suppression of FCNC (in E_6)

Predictions.

* Neutrino $\longrightarrow$ bi-large neutrino mixing
  - $\lambda\, m_{\nu_\tau} \sim m_{\nu_\mu}$ (LMA)
  - $\lambda\, m_{\nu_\mu} \sim m_{\nu_e}$ $\qquad \underline{\lambda \sim \sin\theta_c \sim 0.22}$
  - $U_{e3} \sim \lambda \sim O(0.1)$. $\qquad$ · $\delta_{cp} \sim O(1)$
* small $\tan\beta \equiv \dfrac{v_u}{v_d}$
* Unification scale $\Lambda_u < \Lambda_G = 2\times 10^{16}$ GeV

  $\Rightarrow$ Proton decay via dimension 6 operators

$$\tau(p\to e\pi) \sim \begin{cases} 5\times 10^{33} \text{ years} & (a=-1) \\ 8\times 10^{34} \text{ years} & (a=-\frac{1}{2}) \end{cases}$$

$$\Updownarrow$$

$$\tau_{exp}(p\to e\pi) \gtrsim 5\times 10^{33} \text{ years.}$$

* Cutoff scale $\Lambda \sim \Lambda_G = 2\times 10^{16}$ GeV $< \Lambda_{Pl}$

  $\Rightarrow$ Extra-dimensions? $\qquad$ Horava - Witten

* SUSY

II Anomalous U(1) Gauge Symmetry

- The gauge anomaly is cancelled by Green-Schwarz mechanism.
- D flatness.

charge $\theta = -1$

$$D_A = \xi_{FI}^2 - |\Theta|^2 = 0 \longrightarrow \langle \Theta \rangle = \xi_{FI.} \equiv \lambda \Lambda \qquad \lambda < 1.$$

Anomalous U(1) is broken at just below the cutoff.

$\boxed{\lambda \sim 0.2}$

- Yukawa hierarchy in SSM. Ibanez - Ross.

$$W = \left(\frac{\Theta}{\Lambda}\right)^{x+y+z} XYZ \longrightarrow \lambda^{x+y+z} XYZ$$

(Small letter .... charges.)

- SUSY zero

$x+y+z<0 \Rightarrow$ No gauge invariant interaction with positive powered Θ.

- Vacuum expectation value (VEV)

Z : GUT gauge singlet operator

$$\langle Z \rangle = \begin{cases} 0 & z>0 \\ \lambda^{-z} & z<0 \end{cases}$$

"The GUT scale is determined by the Higgs negative charge."

III Doublet-Triplet Splitting.

One interesting point

- 2 adjoint fields. 45 $\quad A'(a'=3) \quad A(a=-1)$

$\Rightarrow \underline{\langle A' \rangle = 0}$

$$W_{A'} = \lambda^{a+a'} A'A + \lambda^{3a+a'} A'A^3$$

$\boxed{\Delta = 1 \text{ unit}}$

$$\langle A \rangle = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \otimes \begin{pmatrix} x_1 & & & & \\ & x_2 & & & \\ & & x_3 & & \\ & & & x_4 & \\ & & & & x_5 \end{pmatrix}$$

$$\frac{\partial W_{A'}}{\partial A'} = 0 \Rightarrow \lambda^{a+a} x_i (1 + x_i^2 \lambda^{2a}) = 0 . \text{ for all } i$$

$$\Rightarrow x_i = 0 \text{ or } \lambda^{-a}$$ only two solution.

The vacuum is classified by # of zero solution.

$N_0 = 2$

$$\langle A \rangle = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \otimes \begin{pmatrix} v & & & & \\ & v & & & \\ & & v & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}, \quad \underline{v \sim \lambda^{-a}}$$

Ht, Hd

Dimopoulos - Wilczek type VEV

- $A'A^{2L+1}$ $(L \geq 2)$ $\rightarrow$ $\frac{\partial W}{\partial A'} = 0$ have more solutions

$\Rightarrow$ less natural to obtain DW VEV

$\boxed{\text{SUSY zero forbids these terms.}}$

$v \sim \lambda^{-a}$

$\boxed{\text{GUT scale is determined by the charge of } A.}$

- Doublet Higgs is included in **10** $H\ (h=-3)$ $+$

$\langle H' \rangle = 0$ **10** $H'\ (h=4)$ $-$

$$W = \lambda^{h+h'+a} H' A H + \lambda^{2h'} H'^2 \qquad A\ (a=-1) \quad -$$

$$\Rightarrow \quad M_H = \begin{array}{c|cc} & H & H' \\ \hline H & 0 & \lambda^{h+h'+a}\langle A \rangle \\ H' & \lambda^{h+h'+a}\langle A \rangle & \lambda^{2h'} \end{array}$$

SUSY zero.

$$\langle A \rangle = \begin{cases} 0 & \text{for doublet} \\ \lambda^{a} & \text{for triplet} \end{cases}$$

- $\boxed{\text{Only 1 pair of doublet Higgs is massless.}}$

- Proton decay (dim. 5) is naturally suppressed.

$$m_{H^c}^{\mathrm{eff}} \sim \lambda^{2h} > \Lambda \qquad (\because\ h<0)$$

$$\left(2h' > h+h' \rightarrow \lambda^{2h'} < \lambda^{h+h'} \sim \lambda^{h+h'+a}\langle A \rangle \right)$$

IV Gauge Coupling Unification

Anomalous U(1) charges ⇒ mass spectrum (superheavy)

$SO(10) \supset SU(5)$

$45 = 24 + 10 + \overline{10} + 1.$

24,

$$M_I = \begin{array}{c|cc} & I_A & I_{A'} \\ \hline \bar{I}_A & 0 & \lambda^{a+a'}\alpha_I \\ \bar{I}_{A'} & \lambda^{a+a'}\alpha_I & \lambda^{2a'} \end{array} \qquad I = G(8,1)_0,\ W(1,3)_0,\ X(3,2)_{\frac{5}{6}}$$

$\underline{\alpha_X = 0}$

c.f. $\underline{\langle A\rangle \sim \lambda^{-a} \sim \lambda}$

$I = G, W \qquad (\lambda^{a+a'}, \lambda^{a+a'}) = (\lambda^2, \lambda^2)$

$I = X \qquad (0, \lambda^{2a'}) = (0, \lambda^6)$

↳ absorbed by Higgs mechanism.

⇒ The spectrum does not respect SU(5).

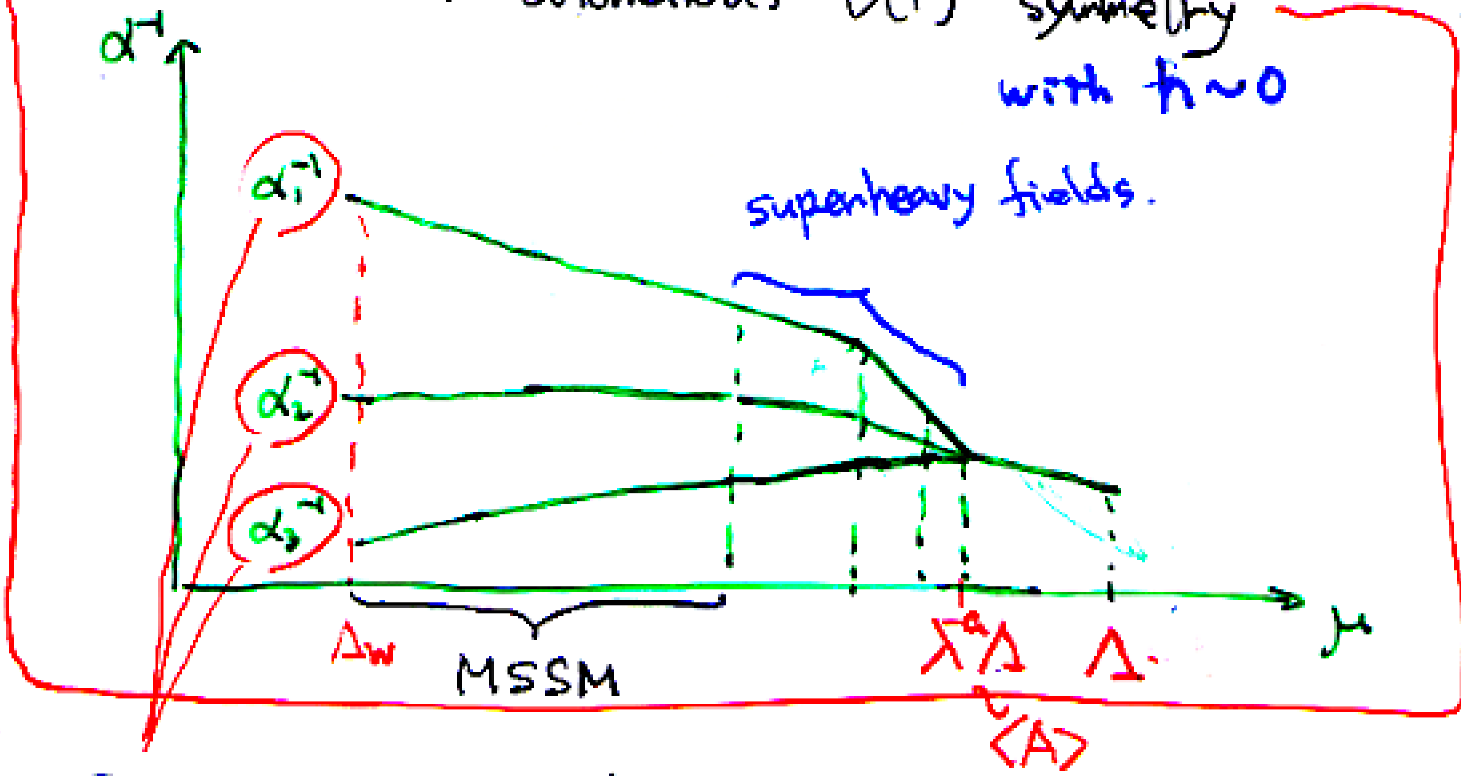
⇒ Is the success of gauge coupling unification in MSSM spoiled in our scenario?

No!!

mass spectrum
GUT breaking scales } are determined by anomalous U(1) charges.

Straightforward calculation of running gauge couplings.

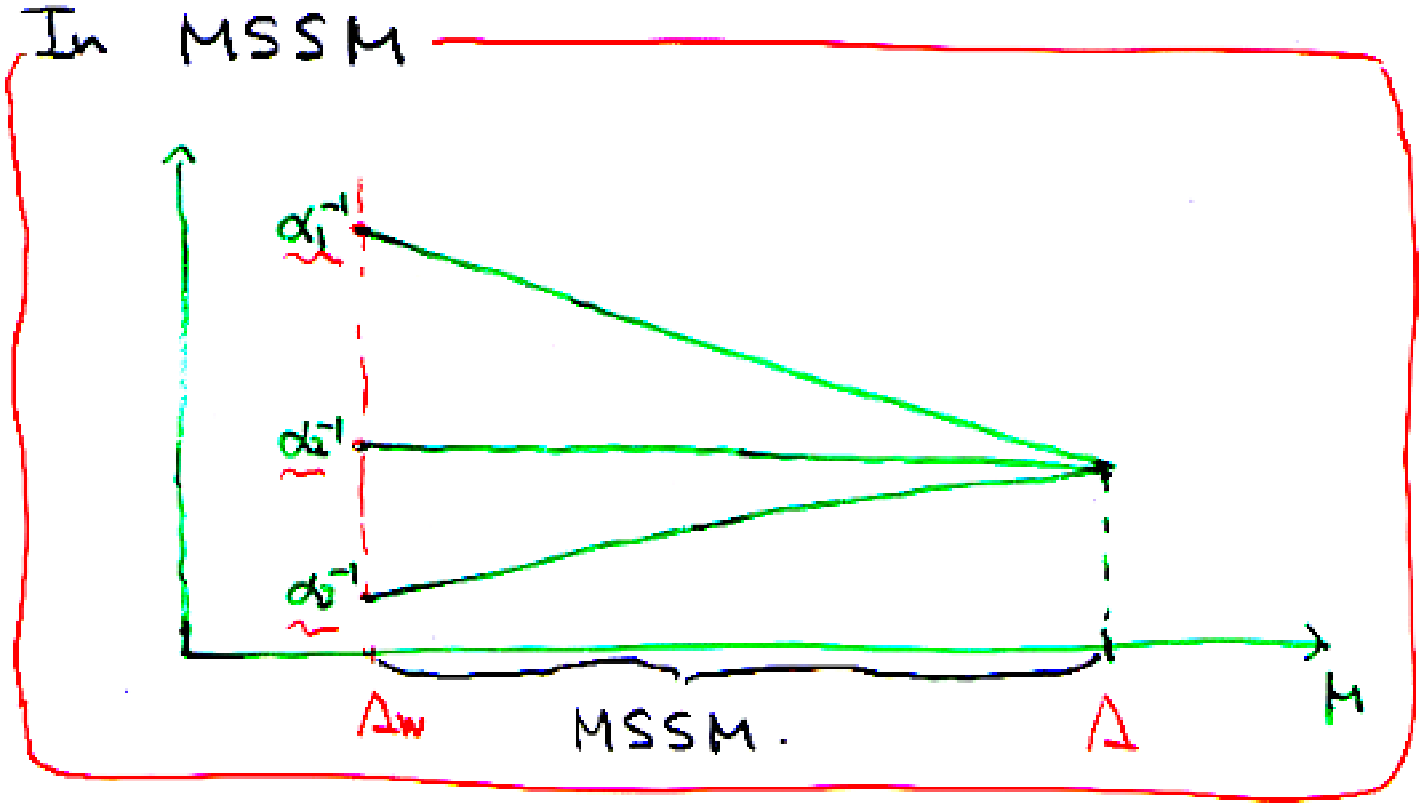
In GUT with anomalous U(1) symmetry — for $\forall\Lambda$.



real

By using these values

In MSSM



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The real unification scale $\Lambda_u \sim \lambda^a \Lambda_G < \Lambda_G \sim 2\times 10^{16}$ GeV

$\Rightarrow$ Proton decay (dim. 6) $\tau(p \to e\pi) \sim \begin{cases} 3\times 10^{33} \text{ yrs} & a=-1 \\ 5\times 10^{34} \text{ yrs} & a=-\frac{1}{2} \end{cases}$

V Quark & Lepton Mass Matrices

$$M_u = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle , \quad M_d = \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix} \langle H_d \rangle$$

$$M_\nu = \lambda^{-(3\sim4)} \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \frac{\eta^2 \langle H_u \rangle^2}{\Lambda} , \quad M_e^T = \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix} \langle H_d \rangle$$

$$U_{CKM} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim \sin\theta_c \sim 0.22 .$$

$$U_{MNS} = \begin{pmatrix} 1 & \sqrt{\lambda} & \lambda \\ \sqrt{\lambda} & 1 & \sqrt{\lambda} \\ \lambda & \sqrt{\lambda} & 1 \end{pmatrix} \begin{pmatrix} 1 & \sqrt{\lambda} & \lambda \\ \sqrt{\lambda} & 1 & \sqrt{\lambda} \\ \lambda & \sqrt{\lambda} & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & \sqrt{\lambda} & \lambda \\ \sqrt{\lambda} & 1 & \sqrt{\lambda} \\ \lambda & \sqrt{\lambda} & 1 \end{pmatrix} \qquad \sqrt{\lambda} + \sqrt{\lambda} \sim \sqrt{\lambda}$$

* bi-large neutrino mixing

  $\sqrt{\lambda} \sim 0.5$, $\quad \sqrt{\lambda} + \sqrt{\lambda} \sim \sqrt{\lambda}$

* LMA solution for solar neutrino problem.

  $\lambda m_{\nu_\tau} \sim m_{\nu_\mu}$.

* leptgenesis ? $\quad \lambda m_{\nu_\mu} \sim m_{\nu_e} \qquad [(M_{\nu_{R1}}, M_{\nu_{R2}}, M_{\nu_{R3}}) = (\lambda^{12}, \lambda^{10}, \lambda^6)]$

* $U_{e3} \sim \lambda \sim O(0.1)$ $\qquad$ * $\delta_{CP} \sim O(1)$

* small $\tan\beta$

* unrealistic GUT relation $Y_u = Y_d = Y_e$ is avoided.

How to avoid the unrealistic GUT relation?

$$\Psi_i : \mathbf{16} = \mathbf{10} + \bar{\mathbf{5}} + \mathbf{1} \qquad H : \mathbf{10} = \mathbf{5} + \bar{\mathbf{5}}$$

$$i = 1, 2, 3$$

$$W_Y = \lambda^{\varphi_i + \varphi_j + h}\, \Psi_i \Psi_j H.$$

$$\longrightarrow \quad \underbrace{Y_u = Y_d}_{SO(10)} = Y_e \qquad Y_d = Y_e \;:\; SU(5)$$

1. An additional $\mathbf{10}$ matter T

$$\mathbf{16} = 10 + \bar{5} + 1$$

Nomura-Yanagida.

$$\mathbf{10} = 5 + \bar{5}$$

We can pick up SO(10) breaking VEV in the mass matrix of $(5_T, (\bar{5}_T, \bar{5}_1, \bar{5}_2, \bar{5}_3))$.

2. Non-renormalizable operators.

$$\Delta W_Y = \lambda^{\varphi_i + \varphi_j + h + 2a}\, \Psi_i \langle A \rangle \Psi_j H. \qquad \frac{\langle A \rangle}{\Lambda} \sim \lambda^{-a}$$

$$\langle A \rangle^2 \sim \lambda^{-2a}$$

$$\boxed{\text{Usually the effect from non-renormalizable terms is suppressed. But in our scenario, this suppression is cancelled by enhancement factor } \lambda^{2a}}$$

VI Prediction & Summary

* Completely consistent GUT with anomalous $U(1)$.

  "Generic" $\longrightarrow$ input : 3~4 (matter) + 8 (Higgs)

  It is surprising that such a simple set up can solve most of the problems of GUT. (DT splitting, unrealistic GUT relation…)

* Predictions.
  * Neutrino. $\rightarrow$ bi-large neutrino mixing.
    * $\lambda m_{\nu_\tau} \sim m_{\nu_\mu}$ (LMA)
    * $\lambda m_{\nu_\mu} \sim m_{\nu_e}$
    * $U_{e3} \sim \lambda$
  * small $\tan\beta$
  * Proton decay (dim 6 though SUSY).

    rough estimation

$$\tau(p \to e\pi) \sim \begin{cases} 3 \times 10^{33} \text{ years} & (a=-1) \\ 5 \times 10^{34} \text{ years} & (a=-\frac{1}{2}) \end{cases}$$

* How is the GUT induced from superstring theory?
  * 2 adjoint Higgs.
  * "Generic" superpotential.
    * × negative powered interactions. Θ^{-n}
    * × non-perturbative interaction of dilaton. e^{-D}

  $\Rightarrow$ These terms spoil SUSY zero mechanism.