Instabilities in 5-dimensional supersymmetric orbifold models

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collaboration with

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based on:

PLB B536(2002)270 [hep-th/0203055]

hep-th/0205012

Supersymmetric theories on an orbifold

- the supersymmetry in 5D with boundaries
- the Fayet-Iliopoulos tadpoles

Localization of charge bulk fields

- the zero mode shapes
- the fate of massive excitations

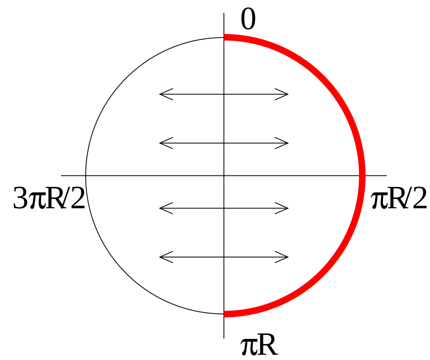
Anomalies in orbifold models

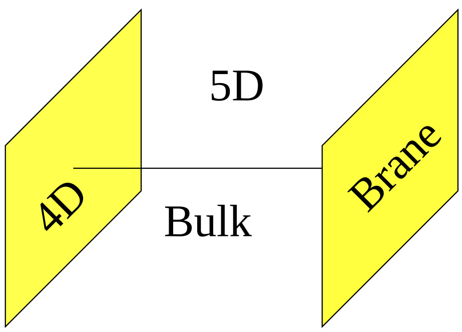
- five dimensional gauge invariance
- localization of chiral modes

Conclusions

The orbifold $S^1/\mathbb{Z}_2$ is defined as

- the circle $S^1 : y + 2\pi R \sim y$
- divided by $\mathbb{Z}_2 : y \sim -y$





Supersymmetry multiplets:

bulk	vector $\mathbb{V}$ and hyper $\mathbb{H}$
branes	chiral $\mathbb{C}_I = (\phi_I, \psi_{I\,L}, F_I)$

$\mathbb{V}$:

state	A_μ	A_5	Φ	λ_+	λ_-	D_3	$D_{1,2}$
parity	$+$	$-$	$-$	$+$	$-$	$+$	$-$

$\mathbb{H}$:

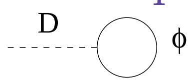
state	ϕ_+	ϕ_-	ψ_{+L}	ψ_{-L}	f_+	f_-
parity	$+$	$-$	$+$	$-$	$+$	$-$

One half of the supersymmetries are broken on the orbifold

$$\delta A_\mu = i\bar{\eta}_+\gamma_\mu\lambda_+, \qquad \delta\lambda_+ = \frac{1}{4}F^{\mu\nu}\gamma_{\mu\nu}\eta_+ - \frac{1}{2}i\tilde{D}_3\eta_+,$$

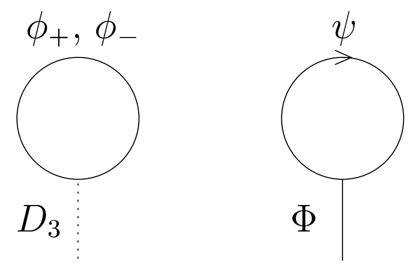
But with: $\tilde{D}_3 = D_3 - \partial_y\Phi$! Mirabelli,Peskin'98

For an abelian vector multiplet in 4D the Fayet,Iliopoulos'74 term: $\mathcal{L}_{FI} = \xi D$ is supersymmetric and gauge invariant.

- At one-loop this FI-term is quadratically divergent if $\sum$ 

$\propto \sum Y \neq 0$, Fischler,Nilles,Polchinski,Raby,Susskind'81
- But then a mixed gauge-gravitational anomaly arises!

In 5 dimensions there are tadpoles for D_3 Ghilencea,GN,Nilles'01, due to scalar loops Scrucca,Serone,Silvestrini,Zwirner'01, and for Φ due to fermion loops GN,Nilles,Olechowski'02:



A calculation in modes gives

$$\xi_{D_3} \sim g_5 q \sum_{n,n',n''} 2\delta_{n,n'+n''} \int \frac{d^4 p_4}{(2\pi)^4} \frac{\delta_{n',n''}}{p_4^2 - \frac{n'^2}{R^2}} D_n$$

$$\xi_{\Phi} \sim (-2) g_5 q \sum_{n,n',n''} 2\delta_{n,n'+n''} \int \frac{d^4 p_4}{(2\pi)^4} \frac{\delta_{n',n''}}{p_4^2 - \frac{n'^2}{R^2}} \left(-\frac{n''}{R}\right) \Phi_n$$

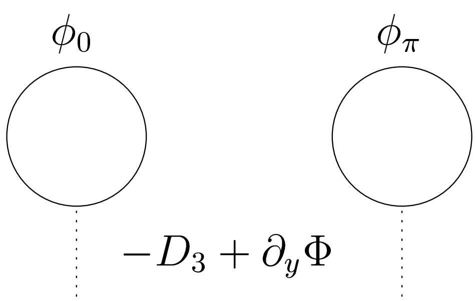
The last part of ξ_Φ can also be written as: $\left(\frac{1}{2}\partial_y \Phi(y)\right)_n$

The divergent part of the momentum integral is given by

$$\int \frac{d^4 p_4}{(2\pi)^4} \frac{1}{p_4^2 - n^2/R^2}\bigg|_{div} = \frac{1}{16\pi^2}\left(\Lambda^2 - \frac{n^2}{R^2} \ln \Lambda^2\right).$$

So that in terms of a coordinate space representation of the divergent tadpoles read

$$\xi_{bulk} = \int \mathrm{d}y g_5 \frac{\mathrm{tr}(q)}{2} (-D_3 + \partial_y \Phi) \left(\frac{\Lambda^2}{16\pi^2} + \frac{\ln \Lambda^2}{16\pi^2} \frac{1}{4} \partial_y^2 \right) \sum_I \delta(y - IR)$$



$$\xi_{branes} = g_5 \frac{\Lambda^2}{16\pi^2} \int \mathrm{d}y$$

$$(-D_3 + \partial_y \Phi) \sum_{I=0,\pi} \mathrm{tr}(q_I) \delta(y - IR)$$

The total FI parameter is therefore given by

$$\xi(y) = \sum \left(-\xi_I + \xi_I'' \partial_y^2 \right) \delta(y - IR) \quad \text{with}$$

$$\xi_I = g_5 \frac{\Lambda^2}{16\pi^2} \left(\frac{1}{2} \mathrm{tr}(q) + \mathrm{tr}(q_I) \right), \qquad \xi_I'' = \frac{1}{4} g_5 \frac{\ln \Lambda^2}{16\pi^2} \frac{1}{2} \mathrm{tr}(q)$$

A supersymmetric background is giving by

$$\partial_y \Phi = g_5(\phi_+^\dagger q \phi_+ - \phi_-^\dagger q \phi_-) + \sum_{I=0,\pi} \delta(y - IR) g_5 \phi_I^\dagger q_I \phi_I + \xi(y),$$

$$D_3 = \partial_y \Phi, \quad \xi(y) = \sum \left(-\xi_I + \xi_I'' \partial_y^2\right) \delta(y - IR)$$

- not $\langle D_3 \rangle$ but $\langle D_3 - \partial_y \Phi \rangle$ is decides on susy,
- integrability of equation for $\langle \Phi \rangle$ with $U(1)$ unbroken $\langle \phi_a \rangle = 0$:

$$0 = \int_0^{\pi R} \mathrm{d}y\, \partial_y \langle \Phi \rangle = \xi_0 + \xi_\pi \;\Rightarrow\; \mathrm{tr}(q) + \mathrm{tr}(q_0) + \mathrm{tr}(q_\pi) = 0$$

Localization due to (tree-level δ-like) FI-terms has been studied before Kaplan,Tait'01, Arkani-Hamed,Gregoire,Wacker'01.
We take the tadpole induced FI-terms as starting point:

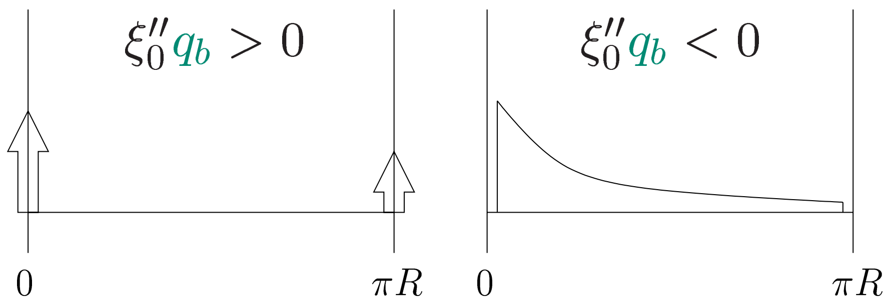
$$\partial_y \phi_{0+} - g_5 q_b \langle \Phi \rangle \phi_{0+} = 0$$

for the zero mode ϕ_{0+} with $\langle \Phi \rangle$ the solution of the equation above with no spontaneous gauge symmetry $\langle \phi_a \rangle = 0$.

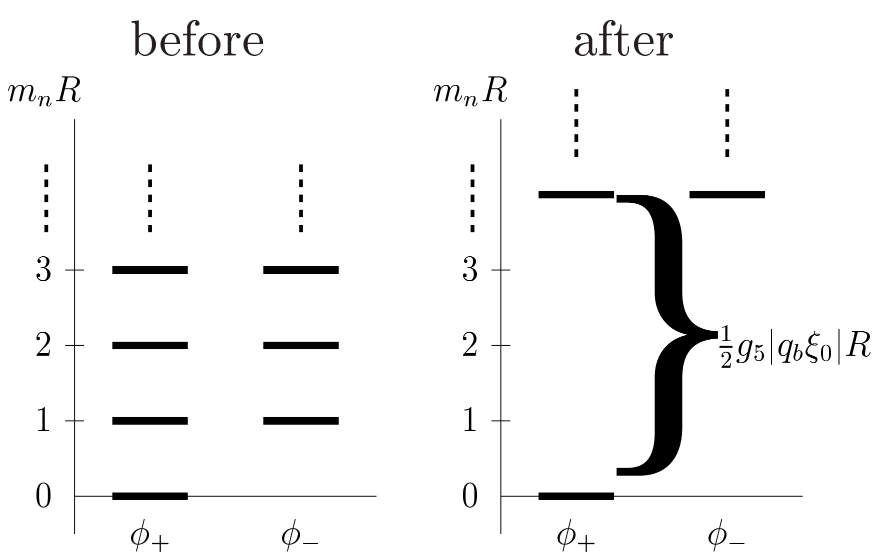
The solution of the zero mode with charge q_b equation reads

$$\phi_{0+} \propto \exp\left\{ g_5 q_b \int_0^y \mathrm{d}y \langle \Phi \rangle \right\}; \quad \int_0^y \mathrm{d}y \langle \Phi \rangle = \frac{\xi_0}{2} y + \xi_0'' \sum_I \delta(y - IR)$$

using that $U(1)$ is unbroken.
Pictorially the solutions take the form:



The spectrum of a bulk scalars (ϕ_+, ϕ_-) with charge q_b:



In a 5D theory there are no local anomalies

$$\delta_\Lambda \Gamma(A) = 2\pi i \int_{S^5} \mathrm{d}\Omega_4^1(A;\Lambda) = 0,$$

using the Wess-Zumino consistency conditions, unless the space has boundaries

$$\delta_\Lambda \Gamma(A) = \pi i \int_{\mathcal{M}_5} \left(\delta(y) + \delta(y-\pi R)\right) \Omega_4^1(A;\Lambda)\mathrm{d}y,$$

since we have the same orbifold projection on both branes Horava,Witten'96. A similar conclusion is reached using a perturbative calculation of the anomaly. Arkani-Hamed,Cohen,Georgi'01 Scrucca,Serone,Silvestrini,Zwirner'01, Pilo,Riotto'02, Barbieri, et al.'02.

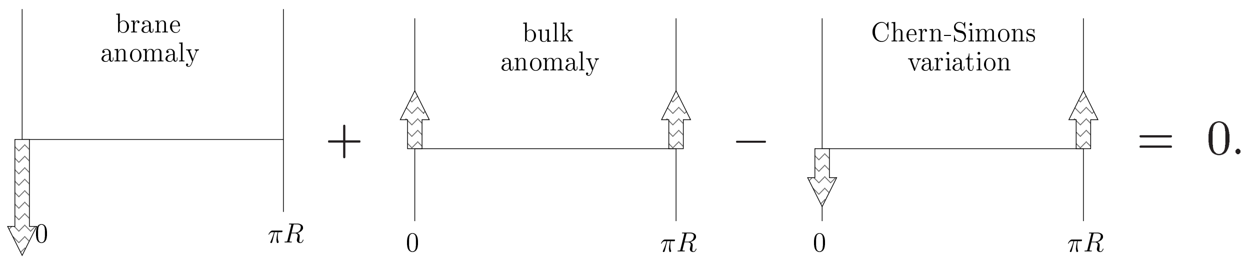
Local 5D anomaly cancelation can be achieved by adding an appropriate Chern-Simons term

$$\Gamma_{CS} = \pi i \int \Omega_5(A),$$

which transforms as

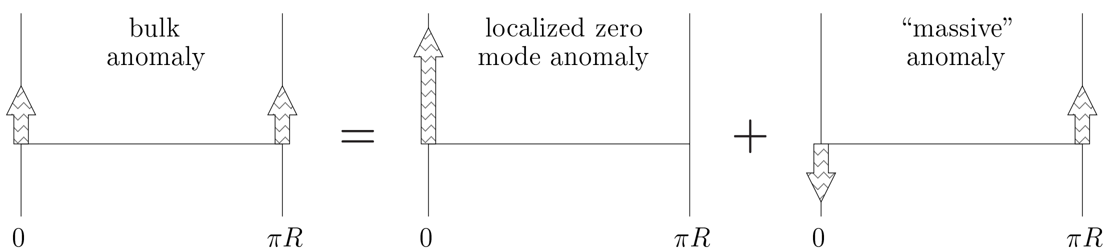
$$\delta_\Lambda \Gamma_{CS}(A) = \pi i \int \left(-\delta(y) + \delta(y-\pi R)\right) \Omega_4^1(A;\Lambda)\mathrm{d}y.$$

Consider a simple orbifold model with one hyper multiplet in the bulk, and one chiral multiplet with the opposite charge on the 0-brane.



Localization of the zero mode cannot change the shape of the bulk anomaly: the anomaly is independent on the complete set of modes used. Arkani-Hamed,Cohen,Georgi'01

The bulk anomaly splits up into the zero mode and the massive modes:



Thus zero-mode anomalies and "massive" anomalies (with CS variations) cancel separately.

How is the orbifold model $\mathcal{M}^4 \times S^1/\mathbb{Z}_2$ constructed?

- mod out the discrete $\mathbb{Z}_2$ symmetry: $y \to -y$ on S^1

$$\psi(-y) = i\gamma^5 \psi(y), \qquad \begin{array}{c} A_\mu(-y) = +A_\mu(y), \\ A_5(-y) = -A_5(y) \end{array}$$

Parity anomaly: what if the fermion determinant

$$e^{-\Gamma(A)} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{-S}, \quad S = -\int \mathrm{d}x\, \bar{\psi} \not{D}(A)\psi.$$

is not parity invariant?

Add the parity anomaly counter term Alvarez-Gaume,Della Pietra,Moore'85

$$\Gamma_{PAC}(A) = -\pi i \int_{S^4 \times S^1} \Omega_5(A).$$

But this breaks gauge invariance GN,Nilles,Olechowski'02 if:

The gauge group contains either

- an $SU(n)$ factor, with $n \geq 3$,
- or a $U(1)$ factor (and a simple Lie group factor).

the fermionic content consists of

- an odd number fundamental representations,
- charges that add up to an odd number.

5 dimensional Fayet-Iliopoulos terms were calculated

- tadpole D_3 (scalars) and tadpole $\partial_y \Phi$ (fermions)
- localized on branes as δ and δ'' contributions

The profile of Φ leads to localization of bulk zero modes

- the δ''-FI tadpole often gives delta-like localization
- 3 types of localization (for $\Lambda \to \infty$): at a brane, near a brane, and on both branes.

Anomalies of orbifold models were investigated

- the S^1 parity anomaly can make orbifold ill-defined,
- localization changes the appearance of anomalies.