

String Cosmology

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1. Dilaton-Moduli Cosmology (Pre Big Bang).
2. Dilaton-Moduli-Axion Cosmology.
3. Graceful Exit in String Cosmology.
4. Density perturbations.
5. Dilaton-Moduli including moving five brane.
6. Inflation in Brane worlds
7. Cyclic Universe
8. Rolling Tachyons

SUSY-02, June 20, 2002, DESY, Hamburg

Dilaton-Moduli Cosmology (PBB)

4+d dim metric on isotropic torus:

$$ds^2 = dt^2 + g_{ij} dx^i dx^j + e^{\sqrt{2/d} \phi} g_{ab} dX^a dX^b$$

Obtain 4 dim NS-NS string eff action:

$$S = \int d^4x \sqrt{|g|} e^{\phi} \left[R + \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} e^{2\phi} (\partial \mu)^2 \right]$$

ϕ effective dilaton in 4D

μ vol of int space

μ axion $[H^{\mu\nu\rho} = \partial^{\mu\nu\rho} e^{\phi} g_{\sigma\rho}]$

Note coupling

Pre Big Bang Solutions $\Lambda = 0$

Write: $ds_4^2 = a^2(\eta) \left[d\eta^2 + d\vec{x}^2 \right]$

Obtain:

$$a = a_0 |\eta|^{(1+\sqrt{3} \cos \alpha)}$$

$$e^{\phi} = e^{\phi_0} |\eta|^{(\sqrt{3} \cos \alpha)}$$

$$e^{\psi} = e^{\psi_0} |\eta|^{(\sqrt{3} \sin \alpha)}$$

(Mueller; Veneziano; Veneziano and Gasperini)

For $\cos \alpha < 1/\sqrt{3}$

1. Pole law inflation for $\alpha < 0$

2. Weak coupling as $\eta \rightarrow 0$, $(e^{\phi} \rightarrow 0)$

String coupling

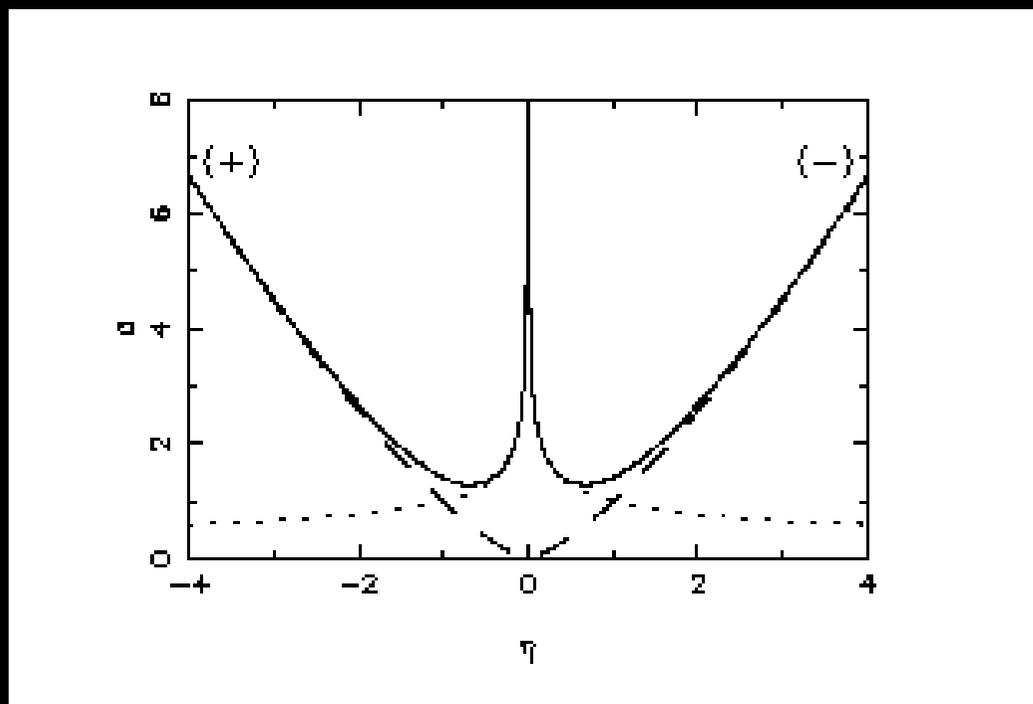
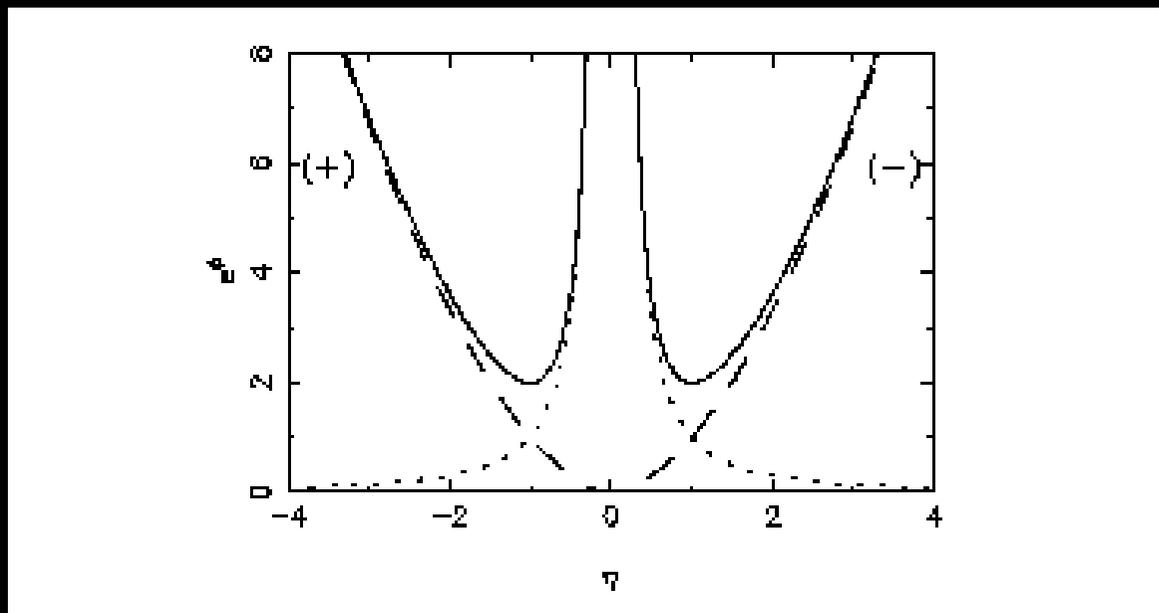
$$e^{\phi}$$

..... $\langle \sigma \rangle = \sigma$

----- $\langle \sigma \rangle = 0$

———— inc axion

String scale
factor



Solns: semi infinite proper lifetime

-ve branch: Start from singularity at $t=0$ for $t > 0$

+ve branch: Approach a singularity at $t=0$ for $t < 0$

Einstein frame: $\tilde{a} = \tilde{a}_0 |\phi|^{(1/2)}$

Both frames lead to inflation: Comoving Hubble length decreases with time

$$|d(\ln \tilde{a}) / d\phi|^{1/2} = 2|\phi|$$

Connecting the two branches – PBB to FRW –
Graceful Exit Problem in string cosmology

Dilaton-Moduli-Axion Cosmologies

EC, Lahiri, Wands

$$a^2 = \frac{a^2_0}{2} \left[x^{1+r} + x^{1+r} \right] ; \quad x \equiv \left| \frac{\phi}{\phi_0} \right|$$

$$e^\sigma = \frac{e^{\sigma_0}}{2} \left[x^r + x^{+r} \right]$$

$$e^\sigma = e^{\sigma_0} x^s$$

$$\phi = \phi_0 \pm e^{\sigma/\alpha} \frac{\phi_0 x^r - \phi_0 x^{+r}}{\phi_0 x^r + \phi_0 x^{+r}} ; \quad r^2 + s^2 = 3$$

Why important? $\left\{ \begin{matrix} \phi_0 \\ \phi_0 \end{matrix} \right\}, (e^\sigma \phi_0)$ No weak coupling limit

Axion interpolates between two strong coupling dilaton-moduli vac solutions. Bad for PBB.

Similar when five brane moves in Dil-Mod background.

Fine tuning issues.

Turner, Weinberg;
Kaloper, Linde,
Bousso; EC, Lahiri,
Wands; Veneziano

In PBB, $\dot{H}, \alpha > 0, \beta < 0$

Initial values must be very small implying a cold, empty, weakly coupled flat vacuum state.

But:

- Axion interpolates between two strong coupling dilaton-moduli vac solutions.
- Curvature tends to prevent enough inflation.

• String coupling must be initially v. small $g_s = e^{\alpha/2} \approx 10^{-26}$

Where does initial state come from?

Ensemble of grav and dilatonic grav waves? Milne

Universe ?

Buonanno, 7
Veneziano, Damour

Graceful Exit.

Joining the +ve and -ve branches through the singularity

$\alpha \rightarrow 0, e^{\alpha}$ grows, entering strong coupling regime.

Expect higher order corrections to become important.

g_s

Controls quantum loop corrections.

α'

Inverse string tension, controls finite string size corrections – higher derivative terms

Generally require combination of both sorts.

Gasperini, Maggiore, Veneziano; Kaloper, Madden, Olive;

Brustein, Madden; Cartier, EC, Madden

General conditions for exit:

1. i.c. of (+) branch and $H_S, \dot{\phi} > 0$, requires $H_E < 0$

$$\left(H_E = e^{\phi/2} \left(H_S - \frac{1}{2} \dot{\phi}^2 \right) \right)$$

2. Branch change from (+) to (-) has to occur while $H_E < 0$

3. Successful escape and exit completion requires NEC violation, with a bounce in Einstein frame after the branch change has occurred, ending up with $H_E > 0$

4. Dilaton stabilisation in radiation era requires potential or decay mechanism.

Example of exit: (Cartier, EC, Madden)

$$L^c = \frac{\alpha^2}{4} e^{\alpha^2} \left[a R_{GB}^2 + b \alpha^2 (\partial_\mu \alpha)^2 + c \left\{ R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right\} \partial_\mu \alpha \partial_\nu \alpha + d (\partial_\mu \alpha)^4 \right]$$

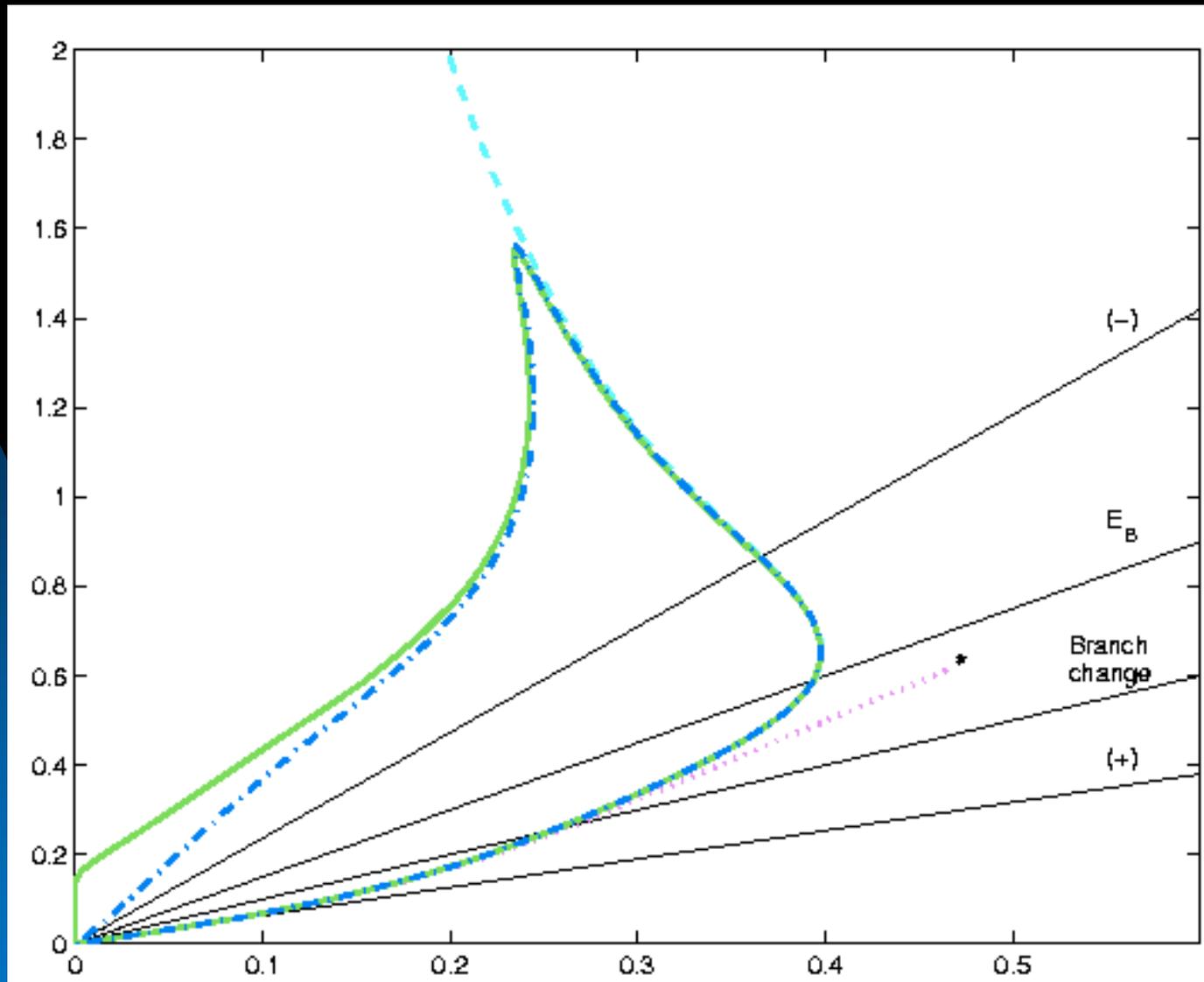
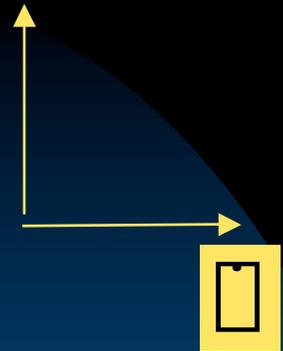
Most general correction up to 4th order in derivatives. (Meissner, Kaloper and Meissner)

Constraint: $2(b+c)+d = -16a$ --- reproduce string scattering amplitude.

Unfortunately, true loop corrections not known, so best approach to use plausible terms.

$$L = L^0 + L^c + A e^{\alpha^2} L^c + B e^{2\alpha^2} L^c$$

H_S



Generally difficult to go through the transition in the weak coupling regime. Typically $\square_{\text{final}} = .1 \square .3$

03.05.2003

Density perturbations in PDB

Comoving Hubble length decreases in PDB phase, vacuum fluctuations generated in flat space vacuum leave, become stretched and re-enter today.

Einstein frame:

$$ds^2 = \bar{a}^2(\eta) \left\{ \bar{\alpha} (1 + 2\bar{A}) d\eta^2 + 2\bar{B}_{,i} d\eta dx^i + (\bar{\alpha}_{ij} + h_{ij}) dx^i dx^j \right\}$$

Evolution of metric pertns about 4D dil-moduli vac solutions.

Single Fourier mode, co-moving k : $\bar{A}'' + 2\bar{h}\bar{A}' + k^2\bar{A} = 0$

+ constraint: $\bar{A} = \bar{\alpha}(\bar{B}' + 2\bar{h}\bar{B})$

with
$$\bar{A} = \frac{\bar{\alpha}\bar{\alpha}'}{4\bar{h}} + \frac{\bar{\alpha}\bar{\alpha}''}{4\bar{h}}$$

Brustein, Gasperini, Giovannini,
Mukhanov, Veneziano

Also: $\Omega = \frac{\bar{A}}{3}$

Curvature pertn on uniform energy density hypersurfaces as $k_{\text{pl}} = 0$

Curvature pertn on large scales:

$$P_{\Omega} = \frac{8}{\Omega^2} 1_{\text{pl}}^2 \bar{H}^2 (\Omega k_{\text{pl}})^3 [\ln(\Omega k_{\text{pl}})]^2$$

Spectral index:

$$n = 1 + \frac{d \ln P_{\Omega}}{d \ln k} = 4$$

cmp H-Z: n=1

Recent claim n=0 (n=1 with exponential potential inc).

Need to properly take account of matching conditions across singularity from collapse to expansion.

Durrer and Vernizzi

Similar, grav wave pertns: $n_T = 3$

Cmp sc-inv: $n_T=0$

(Gasperini, Veneziano)

Axion offers hope:

$$\square\square\square + 2\bar{h}\square\square\square + k^2\square\square = \square^2\square\square\square\square\square$$

$$n_\square = 3 \square 2\sqrt{3} |\cos \square_\square|$$

Can be sc-inv, but isocurvature – need to convert to adiabatic (Durrer, Gasperini, Sakellariadou, Veneziano)

Brane cosmology

Rapidly developing field, check hep-th!

Different approaches involve looking for inflation from colliding branes; no inflation from colliding branes; inflation from scalar fields living on brane, with gravity affected in bulk; pre big bang type models, cyclic models providing cosmology.

Exciting field.

Dilaton-Moduli Cosmology including moving five brane.

4D eff heterotic M-theory with moving five brane.

Induces transition between 2 asym rolling radii solns.

Always drives solns to strong coupling.

Can end in brane collision with small instanton transition.

Brane evolvn depends on dilaton and moduli.

EC, Gray, Lukas

$$S_E = \int d^4x \sqrt{|g|} \left[\frac{1}{2\alpha_p^2} R + \frac{1}{4} (\partial_\mu \phi)^2 + \frac{3}{4} (\partial_\mu \psi)^2 + \frac{q_5}{2} e^{\alpha\phi} (\partial_\mu z)^2 \right]$$

ϕ effective dilaton in 4D

ψ size of orbifold

z posn of five brane; $z \in [0,1]$

q_5 five brane charge

Note coupling

Metric: $ds^2 = e^{2\phi} d\mu^2 + e^{2\psi} dx^2$

$$\mu = \mu(\phi), \quad \psi = \psi(\phi), \quad \psi = \psi(\phi), \quad z = z(\phi), \quad \phi = 0$$

solutions:

$$\varphi = \frac{1}{3} \ln \left| \frac{t - t_0}{T} \right| + \varphi_0$$

$$\varphi = p_{\varphi,i} \ln \left| \frac{t - t_0}{T} \right| + (p_{\varphi,f} - p_{\varphi,i}) \ln \left| \frac{t - t_0}{T} \right|^{\frac{1}{3}} + 1 \left| \frac{t - t_0}{T} \right|^{\frac{1}{3}} + \varphi_0$$

$$\varphi = p_{\varphi,i} \ln \left| \frac{t - t_0}{T} \right| + (p_{\varphi,f} - p_{\varphi,i}) \ln \left| \frac{t - t_0}{T} \right|^{\frac{1}{3}} + 1 \left| \frac{t - t_0}{T} \right|^{\frac{1}{3}} + \varphi_0$$

$$z = d \left| \frac{t - t_0}{T} \right|^{\frac{1}{3}} + \left| \frac{t - t_0}{T} \right|^{\frac{1}{3}} + z_0$$

$t - t_0$ (+) branch
 $t - t_0$ (-) branch

Constraints on integration const.

$\varphi > 0$ (+) branch; $\varphi < 0$ (-) branch

$$3p_{\varphi,n}^2 + p_{\varphi,n}^2 = \frac{4}{3} \quad \text{for } n = i, f$$

$$\varphi = p_{\varphi,i} - p_{\varphi,i}$$

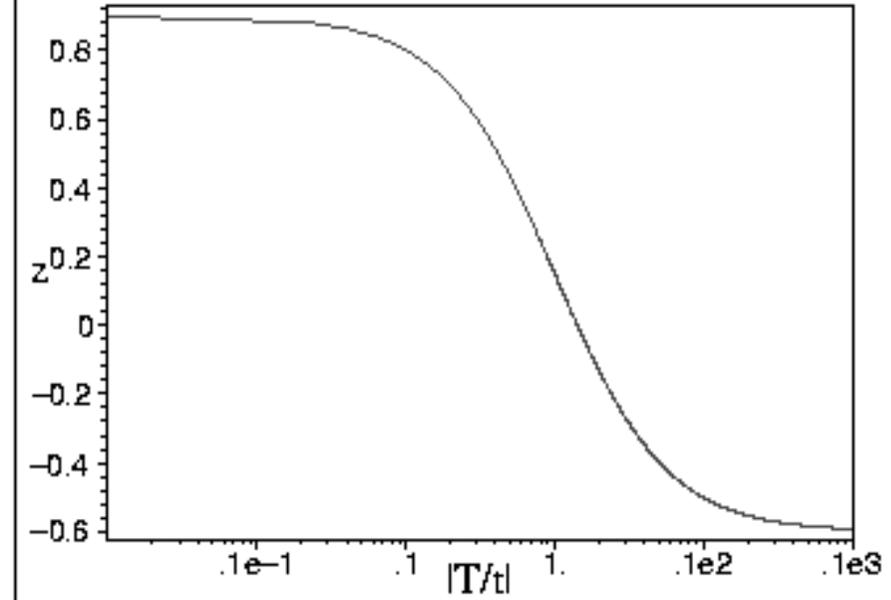
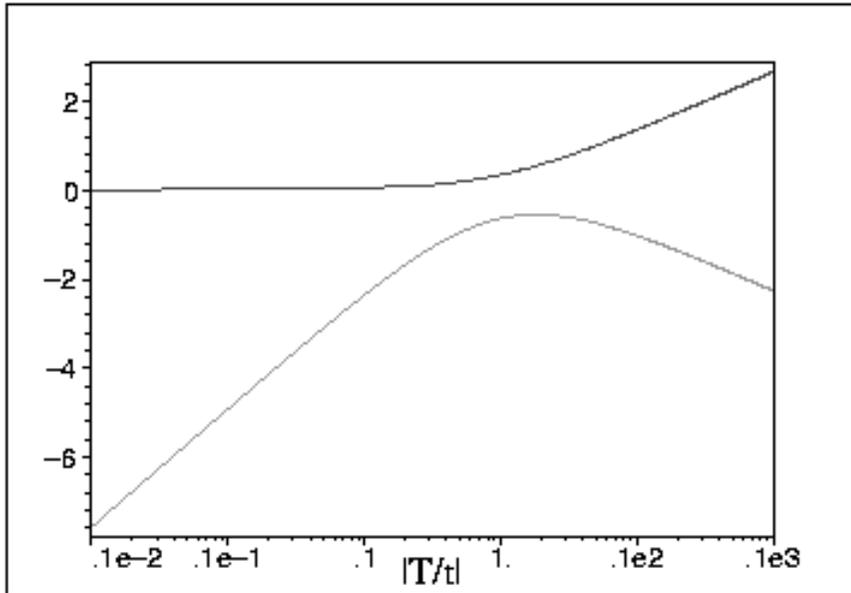
$$\begin{pmatrix} p_{\varphi,f} \\ p_{\varphi,f} \end{pmatrix} = P \begin{pmatrix} p_{\varphi,i} \\ p_{\varphi,i} \end{pmatrix} \quad P = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$$

$$\varphi_0 - \varphi_0 = \ln \left(\frac{2q_5 d}{3} \right)$$

An example: 1. Negative time branch. 2.

$$p_{\square,i} = 0 \quad \square \quad p_{\square,f} = \square \frac{1}{\sqrt{3}}$$

$$p_{\square,i} = \square \frac{2}{\sqrt{3}} \quad \square \quad p_{\square,f} = \frac{1}{\sqrt{3}}$$



$\square \square$ upper; $\square \square$ lower

$z \square$ evolution.

Strong coupling expansion
given by $e^{\square\square\square}$

Become large asymp.

Boundaries at $z=0,1$. Five
brane collides with $z=0$
boundary at $\left| \frac{t}{T} \right| \square 1$

5D Brane scenario

(Binetruy et al (2000); Shiromizu et al (2000))

$$H^2 = \frac{\Lambda^2}{3} + \frac{\Lambda}{2a} + \frac{\Lambda_4}{3} + \frac{\Lambda}{a^4} \quad ; \quad \ddot{a} + 3H\dot{a} + V(a) = 0$$

Λ_4

4D Cosm const – assume 0

Λ

Contrib of bulk gravitons on brane

$$\Lambda : M_4 = \sqrt{\frac{3}{4\Lambda}} \frac{M_5^2}{\sqrt{\Lambda}} M_5$$

Brane tension

Note: Modified Friedmann eqn → increased Hubble parameter → increased friction

Example: Inflation with steep potentials that ends without need for minima and reheats due to gravitational particle production.

$$V(\phi) = V_0 e^{\alpha \phi / M_{\text{pl}}}$$

Standard inflation requires $\alpha^2 < 2$

Here can have $\alpha^2 \gg 2$

Inf when $\phi / M_{\text{pl}} \gg 1$

Ends naturally when $\phi / M_{\text{pl}} \sim 1$

Pertns generated, COBE \rightarrow

$$V_{\text{end}}^{1/4} = \frac{10^{15}}{\alpha} \text{ GeV} \quad \alpha^{1/4} = \frac{\alpha^{1/4} 10^{15}}{\alpha^{3/2}} \text{ GeV}$$

Spectral index: $n = 1 - 4/(N+1) = 0.92$ for $N=50$

$R=0.4$: ratio tensor/scalar amplitude – observable with Planck.

Reheat with grav particle production.

Offers possibility of using moduli fields, dilaton and also

Quintessential inflation

Cyclic Universe

Steinhardt and
Turok

No Big Bang – sequences of bangs and crunches.

Example – brane collisions, radion field potential vital.

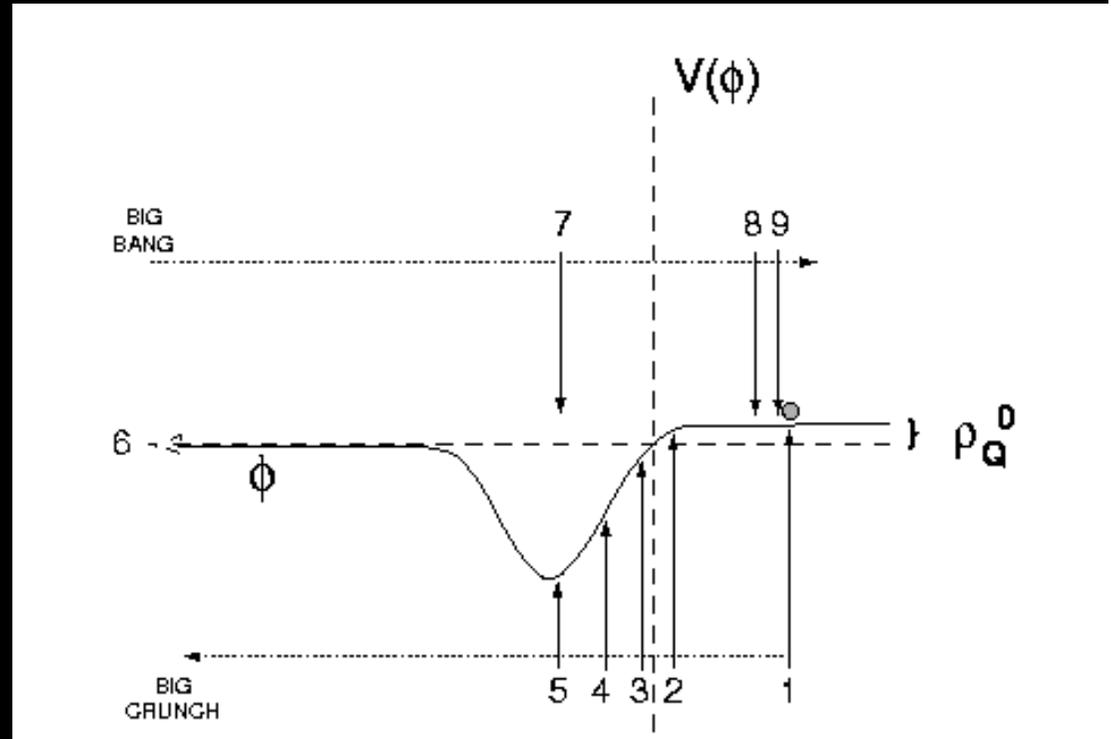
Described by effective 4D field theory involving radion and non-trivial coupling to radiation.

Claim: scale invariant density perturbations generated during contraction phase.

At crunch, temp and density remain finite and small because of special coupling.

Negligible production of gravitational waves.

1. Quintessence (trillion years)
2. Decelerated expansion (billion yrs)
3. $H=0$, contraction begins.
4. Density fluctuations on observed scales.
5. KE dom
6. Bounce and reversal.
7. End of KE dom
8. RD begins (10^{-25} s)
9. MD begins (10^{10} s)
10. Pot dominates, field turns round, back to (1)



$$S = \int d^4x a^4 \left[\frac{1}{2} \dot{\phi}^2 - V(\phi) + \frac{1}{2} \dot{\phi}^2 \right]$$

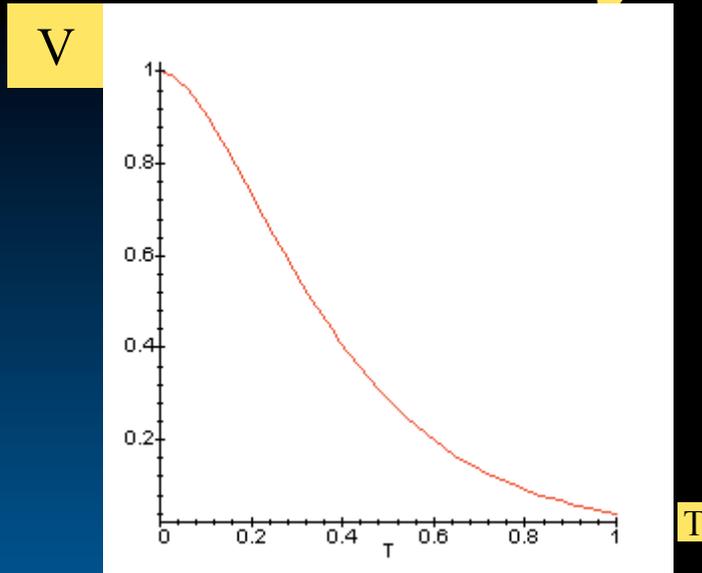
Require: $a \dot{\phi} = \text{const}$, as $a \rightarrow 0$

for finite energy density at crunch.
Require exponential potential for scale-invariant fluctuations during contraction²³

Tachyon cosmology

Sen; Gibbons;
Kutsaov et al;

Alexander;
Mazumdar et al;
Fairbairn and
Tytgat



Idea: Open string vacuum unstable ($T=0$), condensation of closed strings stable ($T=1$).

$$L_m = \int a^3 V \sqrt{1 - \dot{T}^2}$$

$$p = -\int V \sqrt{1 - \dot{T}^2} \equiv -\int \Pi(T) (1 - \dot{T}^2)$$

Condensate smoothly interpolates between:

$$p = -\int \Pi \quad \Pi = \int 1 \text{ for } \dot{T} = 0;$$

$$p = 0 \quad \Pi = 0 \text{ for } \dot{T} = 1$$

Possible role in
inflation at early times
and as new dark matter
at late times ?

Summary

1. Low energy string action admits inflating rolling radii solns.
2. PBB one such example -- weak coupling regime in infinite past, enters strong coupling epoch as approaches singularity -- issues over fine tuning.
3. Graceful exit to FRW phase requires higher order loop corrections generally arising in strong coupling regime.
4. Density fluctuations generated too steep for structure formation (debateable). Isocurvature axion fluctuations can be scale inv.
5. Including moving bulk branes possible -- full solution requires all fields and drives the system into strong coupling asymptotically.
6. Cyclic universe, strong claims, results based on effective field theories. Could have same problems as PBB models.

Not mentioned many areas (string statistical mechanics, multi-brane models, warped geometries...)

In the words of The Spontaneous
Harmony Breakers:

When doing string cosmology you
need to

Think with lots of fantasy

Think a new world