Hadronic vacuum polarization effects in $\alpha_{\rm em}(M_Z)$

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Abstract

Recent evaluations of the hadronic vacuum polarization contributions to the effective fine-structure constant $\alpha_{\rm em}(M_Z)$ are summarized and commented. A new update based on corrected CMD-2 data is presented. My new estimates are $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027773 \pm 0.000354$ (e^+e^-–data based) and $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027664 \pm 0.000173$ (via the Adler-function and extended use of pQCD). Prospects of further possible progress is discussed.

1 Introduction

Precision physics requires appropriate inclusion of higher order effects and the knowledge of very precise input parameters of the electroweak Standard Model SM. One of the basic input parameters is the fine structure constant which depends logarithmically on the energy scale. Vacuum polarization effects lead to a partial screening of the charge in the low energy limit (Thomson limit) while at higher energies the strength of the electromagnetic interaction grows. Non-perturbative strong interaction effects (virtual hadron fluctuations) lead to a partially non-perturbative relationship between the very precisely known classical fine structure constant α and its effective value $\alpha(\mu^2)$ at nonzero energy scales μ. Presently, the only save way to evaluate the non-perturbative hadronic contributions is via a dispersion integral over experimental $e^+e^- \to$ hadrons data (see e.g. [1, 2]). The required relationship derives from analyticity (as a consequence of causality) and the optical theorem (unitarity). A drawback is that the experimental errors allow us to calculate the shift in the fine structure constant only at limited accuracy. In fact, at present, the corresponding uncertainty imposes one of the limiting factors in making precise SM predictions. While $\delta\Delta\alpha$ is dominating the uncertainty $\delta \sin^2\Theta_f$ the experimental error of the top mass measurement δm_t is the main uncertainty contributing to δM_W. Both measurements $\sin^2\Theta_f$ and M_W yield constraints on the as yet unknown Higgs mass M_H.

The photon propagator provides a simple way to derive the concept of an effective charge. Including a factor e^2 and considering the renormalized (multiplied by the appropriate wave function renormalization factor Z) full photon propagator we have

$$e^2 D_{\mu\nu}(q) = \frac{g_{\mu\nu}\, e^2\, Z}{q^2\, \left(1 + \Pi'_\gamma(q^2)\right)} + \text{ gauge terms} \tag{1.1}$$

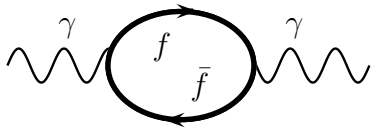
which in effect means that the charge has to be replaced by a *running charge*

$$e^2 \to e^2(q^2) = \frac{e^2 Z}{1 + \Pi'_\gamma(q^2)} \quad . \tag{1.2}$$

The wave function renormalization factor Z is fixed by the condition that for $q^2 \to 0$ one obtains the classical charge (charge renormalization in the Thomson limit). Thus the renormalized charge is

$$e^2 \to e^2(q^2) = \frac{e^2}{1 + (\Pi'_\gamma(q^2) - \Pi'_\gamma(0))} \tag{1.3}$$

where, in perturbation theory, the lowest order diagram which contributes to $\Pi'_\gamma(q^2)$ is



which describes virtual creation and reabsorption of fermion pairs

$$\gamma^* \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, u\bar{u}, d\bar{d}, \cdots\to \ \gamma^*$$

in leading order. In terms of the fine structure constant $\alpha = e^2/4\pi$ Eq. (1.3) reads

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha} \quad ; \quad \Delta\alpha = -\mathrm{Re}\,\left(\Pi'_\gamma(q^2) - \Pi'_\gamma(0)\right) \quad . \tag{1.4}$$

The shift $\Delta\alpha$ is large due to the large change in scale going from zero momentum to the Z-mass scale $\mu = M_Z$ and due to the many species of fermions contributing. Zero momentum more precisely means the light fermion mass thresholds.

The various contributions to the shift in the fine structure constant come from the leptons (lep = e, μ and τ) the 5 light quarks (u, b, s, c, and b and the corresponding hadrons = had) and from the top quark:

$$\Delta\alpha = \Delta\alpha_{\rm lep} + \Delta^{(5)}\alpha_{\rm had} + \Delta\alpha_{\rm top} + \cdots \tag{1.5}$$

Also W–pairs contribute at $q^2 > 2M_W^2$ (see [3, 4]). The leptonic contributions are calculable in perturbation theory where at leading order the free lepton loops yield

$$\begin{aligned} \Delta\alpha_{\rm lep}(s) = & \\ &= \sum_{\ell=e,\mu,\tau} \tfrac{\alpha}{3\pi}\left[-\tfrac{8}{3} + \beta_\ell^2 - \tfrac{1}{2}\beta_\ell(3-\beta_\ell^2)\ln\left(\tfrac{1-\beta_\ell}{1+\beta_\ell}\right)\right] \\ &= \sum_{\ell=e,\mu,\tau} \tfrac{\alpha}{3\pi}\left[\ln\left(s/m_\ell^2\right) - \tfrac{5}{3} + O\left(m_\ell^2/s\right)\right] \ \text{for}\ |s| \gg m_\ell^2 \\ &\simeq \ 0.03142 \ \text{for}\ s = M_Z^2 \end{aligned} \tag{1.6}$$

where $\beta_\ell = \sqrt{1 - 4m_\ell^2/s}$. This leading contribution is affected by small electromagnetic corrections only in the next to leading order. The leptonic contribution is actually known to three loops [5, 6] at which it takes the value ($M_Z \sim 91.19$ GeV)[1]

$$\Delta\alpha_{\rm lep}(M_Z^2) \ \simeq \ 314.98 \ \times \ 10^{-4}. \tag{1.7}$$

[1]For $m_t \sim 174.3$ GeV we have $\Delta\alpha_{\rm top}(M_Z^2) \simeq -\frac{\alpha}{3\pi}\frac{4}{15}\frac{M_Z^2}{m_t^2} \simeq -6 \times 10^{-5}$.

In contrast, the corresponding free quark loop contribution gets substantially modified by low energy strong interaction effects, which cannot be obtained by perturbative QCD (pQCD). As already mentioned, fortunately, one can evaluate this hadronic term $\Delta\alpha^{(5)}_{\rm had}$ from hadronic e^+e^-- annihilation data by using a dispersion relation. The relevant once subtracted vacuum polarization amplitude (1.4) satisfies a convergent dispersion relation and correspondingly the shift of the fine structure constant α is given by

$$\Delta^{(5)}\alpha_{\rm had} = -\frac{\alpha s}{3\pi}\left(\mathcal{P}\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R_\gamma^{\rm data}(s')}{s'(s'-s)} + \mathcal{P}\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R_\gamma^{\rm QCD}(s')}{s'(s'-s)}\right) \tag{1.8}$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi {\rm Im}\Pi'_{\rm had}(s) \ . \tag{1.9}$$

Accordingly, the one particle irreducible (1pi) blob

$\Pi'_{\rm had}(q^2) \quad =$

which is the relevant building block in our context and is given by diagrams which cannot be cut into two disconnected parts by cutting a single photon line, at low energies exhibits intermediate states like $\pi^0\gamma, \rho, \omega, \phi, \cdots, \pi\pi, 3\pi, 4\pi, \cdots, \pi\pi\gamma, \pi\pi Z, \ \cdots, \pi\pi H, \cdots, KK, \cdots$ (at least one hadron plus any strong, electromagnetic or weak interaction contribution) and the corresponding contributions are to be calculated via a dispersion relation from the imaginary parts which are given by the production of the corresponding intermediate states in e^+e^-–annihilation via virtual photons (at energies sufficiently below the point where $\gamma - Z$ interference comes into play).

A direct evaluation of the $R(s)$–data up to $\sqrt{s} = E_{cut} = 5$ GeV and for the Υ resonance–region between 9.6 and 13 GeV and applying perturbative QCD from 5.0 to 9.6 GeV and for the high energy tail above 13 GeV at $M_Z = 91.19$ GeV yields[2]:

$$\begin{aligned} \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027773 \pm 0.000354 \\ \alpha^{-1}(M_Z^2) &= 128.922 \pm 0.049 \ . \end{aligned} \tag{1.10}$$

The contributions from different energy ranges are shown in Tab. 1[3].

Our analysis is as close to the experimental results as possible by utilizing the trapezoidal rule together with PDG rules for taking weighted averages between different experiments as described in detail in [2]

[2]pQCD for calculating $R(s)$, as worked out to high accuracy in Refs. [7]–[9], is used here only where it has been checked to work and converge well: in non–resonant regions at sufficiently high energies and sufficiently far from resonances and thresholds. I have further checked that results obtained with my own routines agree very well with the ones obtained via the recently published program `rhad-1.00` [10].

[3]Table 1 also specifies largely details of the error handling. The different energy ranges mark typical generation of experiments within which systematic errors are considered to be 100% correlated, while all errors are treated as independent for all entries of the table.

final state	energy range (GeV)	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ (stat) (syst)	$\Delta\alpha^{(5)}_{\rm had}(-s_0)$ (stat) (syst)
χPT	(0.28, 0.32)	0.04 (0.00) (0.00)	0.03 (0.00) (0.00)
ρ	(0.28, 0.81)	26.16 (0.24) (0.27)	24.26 (0.23) (0.25)
ω	(0.42, 0.81)	3.02 (0.04) (0.08)	2.75 (0.03) (0.07)
ϕ	(1.00, 1.04)	4.74 (0.07) (0.11)	4.07 (0.06) (0.09)
J/ψ		11.50 (0.56) (0.61)	4.06 (0.19) (0.19)
Υ		1.27 (0.05) (0.07)	0.07 (0.00) (0.00)
hadrons	(0.81, 1.40)	12.92 (0.13) (0.52)	11.05 (0.11) (0.43)
hadrons	(1.40, 3.10)	27.13 (0.11) (0.60)	15.75 (0.06) (0.37)
hadrons	(3.10, 3.60)	5.31 (0.11) (0.10)	1.90 (0.04) (0.04)
hadrons	(3.60, 9.46)	51.49 (0.25) (3.00)	8.41 (0.04) (0.44)
hadrons	(9.46,13.00)	18.59 (0.25) (1.36)	0.90 (0.01) (0.07)
perturb	(13.0,∞)	115.59 (0.00) (0.12)	1.09 (0.00) (0.00)
data	(0.28,13.00)	162.14 (0.74) (3.46)	73.21 (0.33) (0.80)
total		277.73 (0.74) (3.46)	74.30 (0.33) (0.80)

Table 1: Results for $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)^{\rm data}$ and $\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$ ($\sqrt{s_0} = 2.5$ GeV).

The most important ingredient of our analysis are the e^+e^-–data which we described in detail in [2] (see also [11]) and the new data which have become available since then. The developments concerning the experimental data as well as some theoretical aspects are the following:

• The updated results of the precise measurements of the processes $e^+e^- \to \rho \to \pi^+\pi^-$, $e^+e^- \to \omega \to \pi^+\pi^-\pi^0$ and $e^+e^- \to \phi \to K_L K_S$ performed by the CMD-2 collaboration which have just been presented [12]. The update appeared necessary due to an overestimate of the integrated luminosity in previous analyzes[4]. The latter was published in 2002 [13]. A more progressive error estimate (improving on radiative corrections, in particular) allowed a reduction of the systematic error from 1.4% to 0.6 % . Also some other CMD-2 and SND data at energies $E < 1.4$ GeV have become available and have been included.

• Before in 2001 BES-II published their final R–data which, in the region 2.0 GeV to 5.0 GeV, allowed to reduce the previously huge systematic errors of about 20% to 7% [14].

• After 1997 precise τ–spectral functions became available [15, 16, 17] which, to the extent that flavor $SU(2)_{\rm f}$ in the light hadron sector is a symmetry, allows to obtain the iso–vector part of the e^+e^-–cross section [18, 19]. This possibility has first been exploited in the present context in [11].

• With increasing precision of the low energy data it more and more turned out that we are confronted with a serious obstacle to further progress: in the region just above the ω–resonance, the iso-spin rotated τ–data, corrected for the known iso-spin violating effects, do not agree with the e^+e^-–data at the 10% level [20]. Before the origin of this discrepancy is found it will be hard to make further progress in pinning down theoretical uncertainties.

[4]This affects in particular the leading hadronic contribution to the anomalous magnetic moment of the muon. I now obtain $a_\mu^{\rm had(1)} = (695.5 \pm 8.6) \times 10^{-10}$ (e^+e^-–data based).

• In this context iso-spin breaking effects in the relationship between the τ– and the e^+e^-–data have been extensively investigated in [21]. Whatever uncertainties of the estimated iso-spin violations might remain, it is very unlikely that they can be made responsible for the observed discrepancies.

• New results for hadronic e^+e^- cross–sections are expected soon from KLOE, BABAR and BELLE. These experiments, running at fixed energies, are able to perform measurements via the radiative return method [22, 23, 24]. Results presented recently by KLOE seem to agree very well with the final CMD-2 e^+e^-–data.

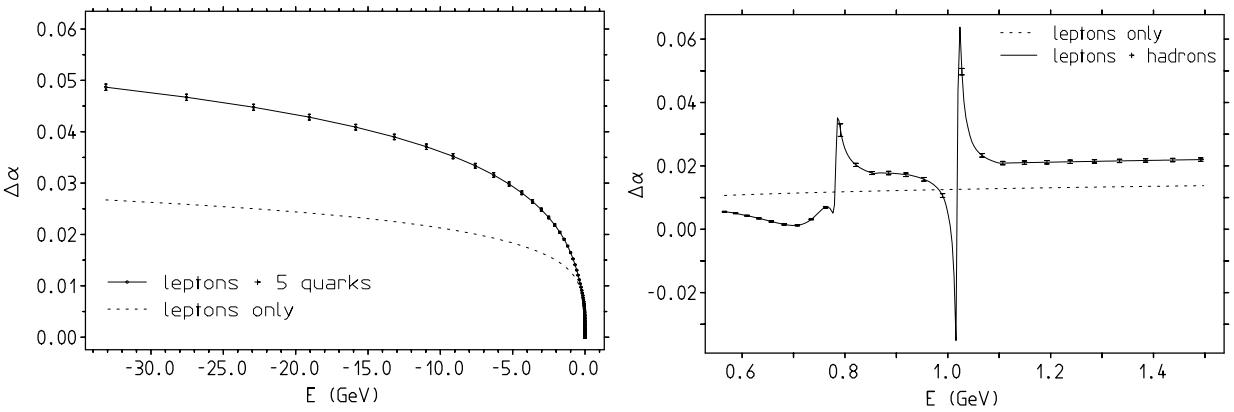


Figure 1: The running of α. The "negative" E axis is chosen to indicate space-like momentum transfer. The vertical bars at selected points indicate the uncertainty. In the time-like region the resonances lead to pronounced variations of the effective charge (shown in the $\rho - \omega$ and ϕ region).

Fig. 1 illustrates the running of the effective charge at lower energies in the space-like region. Typical values are $\Delta\alpha(5\text{GeV}) \sim 3\%$ and $\Delta\alpha(M_Z) \sim 6\%$, where about $\sim 50\%$ of the contribution comes from leptons and about $\sim 50\%$ from hadrons.

An analysis similar to ours, however, using piecewise linear approximants to the non–resonant $R(s)$, which are then integrated analytically, yields [25]

$$\begin{aligned} \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027680 \pm 0.000360 \\ \alpha^{-1}(M_Z^2) &= 128.935 \pm 0.049\ . \end{aligned} \tag{1.11}$$

The precise choice of the lattice used for the linearization remains somewhat unclear in this method.

An evaluation via the Adler-function which allows to utilize safely perturbative QCD for the latter at Euclidean energies above 2.5 GeV (and data at lower energies) yields (see below):

$$\begin{aligned} \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027664 \pm 0.000173[0.000137] \\ \alpha^{-1}(M_Z^2) &= 128.937 \pm 0.024\ . \end{aligned} \tag{1.12}$$

The errors here are dominated by the QCD parameter uncertainties and are given for the worst case correlation [uncorrelated] case. In contrast to the so called *theory–driven*

approaches, the Adler function methods is more elaborate (because it requires to transform data and theory to the space–like region) but allows for a dramatically better control of the validity of pQCD.

There are also theory-driven approaches which utilize perturbative QCD directly for the evaluation of $R(s)$ (and/or for rescaling of the normalization of the data) and assuming some local version of quark–hadron duality. A recent result is [26, 27] (see also [28]–[34])

$$\begin{aligned} \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027690 \pm 0.000180 \\ \alpha^{-1}(M_Z^2) &= 128.933 \pm 0.025\ . \end{aligned} \tag{1.13}$$

2 $\alpha_{\rm em}(s)$ in precision physics

A major drawback of the partially non-perturbative relationship between $\alpha(0)$ and $\alpha(M_Z)$ is that one has to rely on experimental data exhibiting systematic and statistical errors which implies a non-negligible uncertainty in our knowledge of the effective fine structure constant. In precision predictions of gauge boson properties this has become a limiting factor. Since α, G_μ and M_Z are the most precisely measured parameters, they are used as input parameters for accurate predictions of observables like the effective weak mixing parameter $\sin^2\Theta_f$, the vector v_f and axial-vector a_f neutral current couplings, the W mass M_W the widths Γ_Z and Γ_W of the Z and the W, respectively, etc. However, for physics at higher energies, we have to use the effective couplings at the appropriate scale, for physics at the Z–resonance, for example, $\alpha(M_Z)$ is more adequate to use than $\alpha(0)$. Of course this just means that part of the higher order corrections may be absorbed into an effective parameter. If we compare the precision of the basic parameters

$$\begin{array}{lcl@{\quad\quad}lcl} \frac{\delta\alpha}{\alpha} & \sim & 3.6 \times 10^{-9} & \frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 1.6 \div 6.8 \times 10^{-4} \\ \frac{\delta G_\mu}{G_\mu} & \sim & 8.6 \times 10^{-6} & \frac{\delta M_Z}{M_Z} & \sim & 2.4 \times 10^{-5} \end{array} \tag{2.1}$$

we observe that the uncertainty in $\alpha(M_Z)$ is roughly an order of magnitude worse than the next best, which is the Z–mass. Future TESLA requirements are $\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5.3 \times 10^{-5}$ (based on the assumption that a precision $\delta \sin^2\Theta^\ell_{\rm eff} \simeq 0.000013$ (GigaZ option) and $\delta M_W \sim 6$MeV (MegaW) may be reached) [35].

Let me remind the reader that $\Delta\alpha$ enters in electroweak precision physics typically when calculating versions of the weak mixing parameter $\sin^2\Theta_i$ from α, G_μ and M_Z via

$$\sin^2\Theta_i\ \cos^2\Theta_i\ = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i} \tag{2.2}$$

where $\Delta r_i = \Delta r_i(\alpha,\ G_\mu,\ M_Z, m_H,\ m_{f\neq t},\ m_t)$ includes the higher order corrections, which can be calculated in the SM or in alternative models. Δr has been calculated for the first time by A. Sirlin in 1980 [36]. In the SM, today, the Higgs boson mass m_H is the only relevant unknown parameter and by confronting the calculated with the experimentally determined value of $\sin^2\Theta_i$ one obtains important indirect constraints on the Higgs mass. Δr_i depends on the definition of $\sin^2\Theta_i$. The various definitions coincide at tree level and

hence only differ by quantum effects. From the weak gauge boson masses, the electroweak gauge couplings and the neutral current couplings of the charged fermions we obtain

$$\sin^2\Theta_W = 1 - \frac{M_W^2}{M_Z^2} \tag{2.3}$$

$$\sin^2\Theta_g = e^2/g^2 = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_W^2} \tag{2.4}$$

$$\sin^2\Theta_f = \frac{1}{4|Q_f|}\left(1 - \frac{v_f}{a_f}\right), \quad f \neq \nu\,, \tag{2.5}$$

for the most important cases. Δr_i usually is written in the form

$$\Delta r_i = \Delta\alpha - f_i(\sin^2\Theta_i)\,\Delta\rho + \Delta r_{i\,\text{reminder}} \tag{2.6}$$

with a universal term $\Delta\alpha$ which affects the predictions of M_W, A_{LR}, A_{FB}^f, Γ_f, etc. The uncertainty $\delta\Delta\alpha$ implies uncertainties δM_W, $\delta\sin^2\Theta_i$ given by[5]

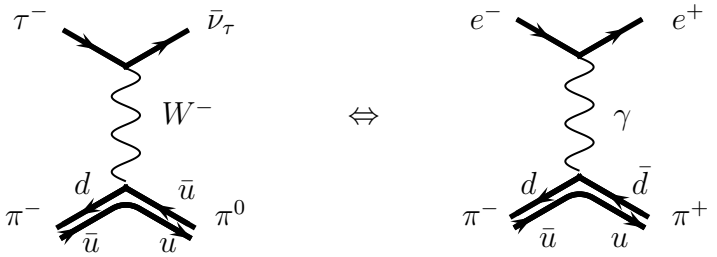
$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\Theta_W}{\cos^2\Theta_W - \sin^2\Theta_W}\,\delta\Delta\alpha \sim 0.213\,\delta\Delta\alpha \tag{2.7}$$

$$\frac{\delta\sin^2\Theta_f}{\sin^2\Theta_f} \sim \frac{\cos^2\Theta_f}{\cos^2\Theta_f - \sin^2\Theta_f}\,\delta\Delta\alpha \sim 1.54\,\delta\Delta\alpha\,. \tag{2.8}$$

The present indirect Higgs mass "measurement" reads $m_H = 96^{+60}_{-38}$ GeV. The discrepancy between e^+e^-– and τ–data based evaluations amounts to $\delta m_H \sim -19$ GeV [20] (the direct lower bound is $m_H > 114$ GeV at 95% CL while the indirect upper bound reads $m_H < 219$ GeV at 95% CL (1-sided). For more details we refer to [37] (in these proceedings).

3 The τ vs. e^+e^- problem

The iso-vector part of $\sigma(e^+e^- \to \text{hadrons})$ may be calculated by an iso-spin rotation from τ–decay spectra, to the extend that the conserved vector current is conserved (CVC). The relation may be derived by comparing diagrams like:



[5]This compares with the second major source of uncertainty coming from the top mass [currently $\delta m_t/m_t \sim 2.9\times10^{-2}$]

$$\frac{\delta M_W}{M_W} \sim \frac{\cos^2\Theta_W}{\cos^2\Theta_W - \sin^2\Theta_W}\,\frac{\delta m_t}{m_t} \sim 1.426\,\frac{\delta m_t}{m_t}$$

$$\frac{\delta\sin^2\Theta_f}{\sin^2\Theta_f} \sim -2\,\frac{\cos^2\Theta_f}{\cos^2\Theta_f - \sin^2\Theta_f}\,\frac{\delta m_t}{m_t} \sim -2.852\,\frac{\delta m_t}{m_t}\,.$$

Thus comparing $\tau^- \to X^- \nu_\tau$ with $e^+e \to X^0$ the hadronic states X^- and X^0 are approximately related by an iso-spin rotation if the states are $I = 1$ iso-vector states. The e^+e^- cross–section is then given by

$$\sigma^{I=1}_{e^+e^- \to X^0} = \frac{4\pi\alpha^2}{s} v_{X^-} \quad , \quad \sqrt{s} \leq m_\tau \tag{3.1}$$

in terms of the τ spectral function v_V. The τ spectral function $v_V(s)$ for a given vector hadronic state V is defined by

$$v_V(s) \equiv \frac{m_\tau^2}{6\,|V_{ud}|^2\,S_{\rm EW}} \frac{B(\tau^- \to \nu_\tau\, V^-)}{B(\tau^- \to \nu_\tau\, e^-\, \bar{\nu}_e)} \frac{1}{N_V} \frac{dN_V}{ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1} , \tag{3.2}$$

where $|V_{ud}| = 0.9752 \pm 0.0007$ [38] denotes the CKM weak mixing matrix element and $S_{\rm EW} = 1.0233 \pm 0.0006$ accounts for electroweak radiative corrections. The spectral functions are obtained from the corresponding invariant mass distributions.

Before a precise comparison is possible all kind of iso-spin breaking effects have to be taken into account. As mentioned earlier, this has been investigated in [21] for the most relevant $\pi\pi$ channel. Writing

$$\sigma^{(0)}_{\pi\pi} = \left[\frac{K_\sigma(s)}{K_\Gamma(s)}\right] \frac{d\Gamma_{\pi\pi[\gamma]}}{ds} \times \frac{R_{\rm IB}(s)}{S_{\rm EW}} \tag{3.3}$$

with

$$K_\Gamma(s) = \frac{G_F^2\,|V_{ud}|^2\,m_\tau^3}{384\pi^3} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2\,\frac{s}{m_\tau^2}\right) \quad ; \quad K_\sigma(s) = \frac{\pi\alpha^2}{3s} , \tag{3.4}$$

the iso-spin breaking correction

$$R_{\rm IB}(s) = \frac{1}{G_{\rm EM}(s)} \frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^+\pi^0}} \left|\frac{F_V(s)}{f_+(s)}\right|^2 \tag{3.5}$$

include the photonic corrections (based on scalar QED), phase space corrections due to the $\pi^\pm - \pi^0$ mass difference and the form–factor corrections which are dominated by the $\rho - \omega$ mixing effects. These corrections were applied in [20] and were not able to resolve the puzzle of the observed discrepancy (see [20] for details)[6].

[6]The only large effect I am aware of (of order 10%) which is in the game of the comparison is a possible shift of the invariant mass of the pion-pairs in the ρ resonance region. An idea one gets if one is looking at the experimental ρ–mass values, shown in the particle data tables [38] ("dipole shape"). If the energy calibration of the $\pi\pi$–system would be to low in e^+e^- measurements or to high in τ measurements by 1% one could easily get a 10% decrease or increase in the tail, respectively. Since the $\rho^\pm - \rho^0$ mass difference as well as the difference in the widths $\Gamma^{\pm,0}(\rho \to \pi\pi, \pi\pi\gamma)$ are neither experimentally nor theoretically established, corresponding iso-spin violations cannot be corrected for appropriately. Note that the subtraction of the large and strongly energy dependent vacuum polarization effects (see Fig. 1) necessary for the e^+e^-–data, which seems to worsen the problem, is properly treated in the analysis.

4 Controlling pQCD via the Adler function

In view of the increasing precision LEP experiments have achieved during the last few years, more accurate theoretical prediction became desirable. As elaborated in the introduction, one of the limiting factors is the hadronic uncertainty of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$. Because of the large uncertainties in the data, many authors advocated to extend the use of perturbative QCD in place of data [28]–[34]. The assumption that pQCD may be reliable to calculate (1.9) down to energies as low as 1.8 GeV seems to be supported by

• the apparent applicability of pQCD to τ physics. In fact the running of $\alpha_s(m_\tau) \to \alpha_s(M_Z)$ from the τ mass up to LEP energies agrees well with the LEP value. The estimated uncertainty may be debated, however.

• the smallness [29] (see also: [1]) of non–perturbative (NP) effects if parameterized as prescribed by the operator product expansion (OPE) of the electromagnetic current correlator [39]

$$\begin{aligned}
\Pi'^{\rm NP}_\gamma(Q^2) &= \frac{4\pi\alpha}{3}\sum_{q=u,d,s} Q_q^2 N_{cq}\cdot\left[\frac{1}{12}\left(1-\frac{11}{18}a\right)\frac{<\frac{\alpha_s}{\pi}GG>}{Q^4}\right.\\
&+ 2\left(1+\frac{a}{3}+\left(\frac{11}{2}-\frac{3}{4}l_{q\mu}\right)a^2\right)\frac{<m_q\bar{q}q>}{Q^4} \qquad (4.1)\\
&+ \left.\left(\frac{4}{27}a+\left(\frac{4}{3}\zeta_3-\frac{257}{486}-\frac{1}{3}l_{q\mu}\right)a^2\right)\sum_{q'=u,d,s}\frac{<m_{q'}\bar{q'}q'>}{Q^4}\right]\\
&+ \cdots
\end{aligned}$$

where $a \equiv \alpha_s(\mu^2)/\pi$ and $l_{q\mu} \equiv \ln(Q^2/\mu^2)$. $<\frac{\alpha_s}{\pi}GG>$ and $<m_q\bar{q}q>$ are the scale-invariantly defined condensates.

Progress in pQCD here comes mainly from [40]. In addition an exact two–loop calculation of the renormalization group (RG) in the background field MOM scheme (BF-MOM) is available [41]. This allows us to treat "threshold effects" closer to physics than in the $\overline{\rm MS}$ scheme. The BF-MOM scheme respects the QCD Slavnov-Taylor identities (non-Abelian gauge symmetry) but in spite of that is gauge parameter (ξ) dependent[7].

In Ref. [42] a different approach of pQCD improvement was proposed, which relies on the fact that the vacuum polarization amplitude $\Pi(q^2)$ is an analytic function in q^2 with a cut in the s–channel $q^2 = s \geq 0$ at $s \geq 4m_\pi^2$ and a smooth behavior in the t–channel (space-like or Euclidean region). Thus, instead of trying to calculate the complicated function $R(s)$, which obviously exhibits non-perturbative features like resonances, one considers the simpler Adler function in the Euclidean region. In [42] the Adler function was investigated and pQCD was found to work very well above 2.5 GeV, provided the exact three–loop mass dependence was used (in conjunction with the background field MOM scheme). The Adler function may be defined as a derivative

$$D(-s) = -(12\pi^2)\,s\,\frac{d\Pi'_\gamma(s)}{ds} = \frac{3\pi}{\alpha}s\frac{d}{ds}\Delta\alpha_{\rm had}(s) \qquad (4.2)$$

[7] In applications considered below all numerical calculations have been performed in the "Landau gauge" $\xi = 0$.

of (1.9) which is the hadronic contribution to the shift of the fine structure constant. It is represented by

$$D(Q^2) = Q^2 \left(\int_{4m_\pi^2}^{E_{\rm cut}^2} \frac{R^{\rm data}(s)}{(s+Q^2)^2} ds \; + \; \int_{E_{\rm cut}^2}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2} ds \right) \tag{4.3}$$

in terms of the experimental e^+e^-–data. The standard evaluation ([2]) of (4.3) then yields the non–perturbative "experimental" Adler function, as displayed in Fig. 2 for the lower energies where it becomes non–perturbative.

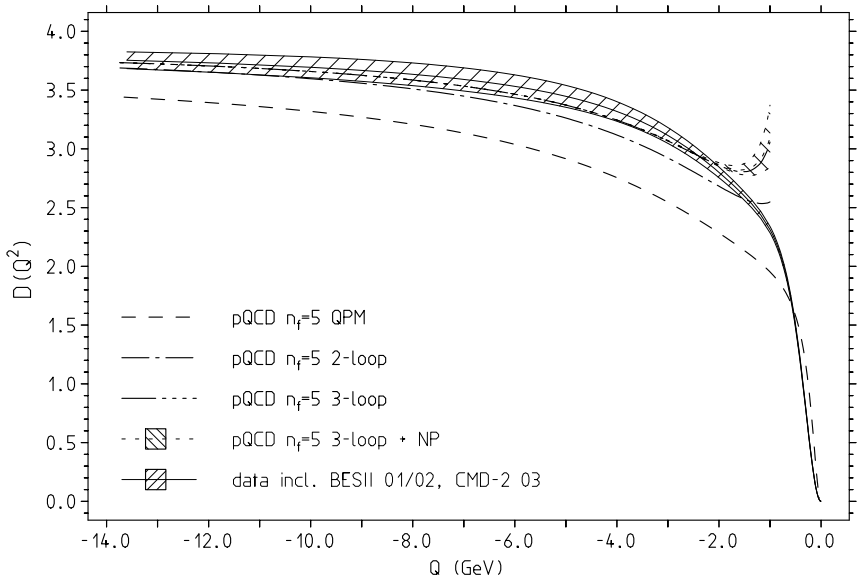


Figure 2: Adler function: theory vs. experiment [42].

For the pQCD evaluation it is mandatory to utilize the calculations with massive quarks which are available up to three–loops [40]. The four-loop corrections are known in the approximation of massless quarks [7]. The outcome of this analysis is pretty surprising and is shown in Fig. 2. For a discussion we refer to the original paper [42]. The result was obtained using the background–field MOM renormalization scheme, mentioned before. In the transition from the $\overline{\rm MS}$ to the MOM scheme we adapt the rescaling procedure described in [41], such that for large μ

$$\overline{\alpha}_s((x_0\mu)^2) = \alpha_s(\mu^2) + 0 + O(\alpha_s^3) \; .$$

This means that x_0 is chosen such that the couplings coincide to leading and next–to–leading order at asymptotically large scales. Numerically we find $x_0 \simeq 2.0144$. Due to this normalization by rescaling the coefficients of the Adler–function remain the same in both schemes up to three–loops. In the MOM scheme we automatically have the correct mass dependence of full QCD, i.e., we have automatic decoupling and do not need decoupling by hand and matching conditions like in the $\overline{\rm MS}$ scheme. For the numerical evaluation we use the pole quark masses [38] $m_c = 1.55{\rm GeV}$, $m_b = 4.70{\rm GeV}$, $m_t = 173.80{\rm GeV}$ and the strong interaction coupling $\alpha_{s\,\overline{\rm MS}}^{(5)}(M_Z) = 0.120 \pm 0.003$. For further details we refer to [42].

According to (4.2), we may compute the hadronic vacuum polarization contribution to the shift in the fine structure constant by integrating the Adler function. In the region where pQCD works fine we integrate the pQCD prediction, in place of the data. We thus calculate in the Euclidean region [43]

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} \quad . \tag{4.4}$$

A save choice is $s_0 = (2.5\,{\rm GeV})^2$ where we obtain

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007430 \pm 0.000087 \tag{4.5}$$

from the evaluation of the dispersion integral (1.9). With the results presented above we find

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027626 \pm 0.000087 \pm 0.000149[0.000101] \tag{4.6}$$

for the Euclidean (t–channel) effective fine structure constant. The second error comes from the variation of the pQCD parameters. In square brackets the error if we assume the uncertainties from different parameters to be uncorrelated. The uncertainties coming from individual parameters are listed in the following table (masses are the pole masses):

parameter	range	pQCD uncertainty	total error
α_s	0.117 ... 0.123	0.000051	0.000155
m_c	1.550 ... 1.750	0.000087	0.000170
m_b	4.600 ... 4.800	0.000011	0.000146
m_t	170.0 ... 180.0	0.000000	0.000146
all correlated		0.000149	0.000209
all uncorrelated		0.000101	0.000178

The largest uncertainty is due to the poor knowledge of the charm mass. I have taken errors to be 100% correlated. The uncorrelated error is also given in the table.

Comments:

• Contributions to the Adler function up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An error for missing higher order terms is not included. The scheme dependence $\overline{\rm MS}$ versus background field MOM has been discussed in Ref. [41].

• The effective fine structure constant in the time–like region (s–channel), as required for e^+e^-–collider physics, may be obtained from the Euclidean one by adding the difference

$$\Delta = \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.000038 \pm 0.000005\,, \tag{4.7}$$

which may be calculated perturbatively or directly from the "non–perturbative"[8] dispersion integral. It accounts for the $i\pi$–terms

$$\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$$

[8]Since we utilize pQCD for the high energy tail in the dispersion integral, $\Delta(s)$ for large s is dominated by the tail and thus in fact is perturbative.

from the logs.

• One may ask the question whether these terms should be resummed at all, i.e., included in the running coupling. Usually such terms tend to cancel against constant rational terms which are not included in the renormalization group (RG) evolution. It should be stressed that the Dyson summation (propagator bubble summation) in general is not a systematic resummation of leading, sub-leading etc. terms as the RG resummation is.

It is worthwhile to stress here that the running coupling is **not** a true function of q^2 (or even an analytic function of q^2) but a function of the RG scale μ^2. The coupling as it appears in the Lagrangian in any case must be a constant, albeit a μ^2–dependent one, if we do not want to end up in conflict with basic principles of quantum field theory. The effective identification of μ^2 with a particular value of q^2 must be understood as a subtraction (reference) point.

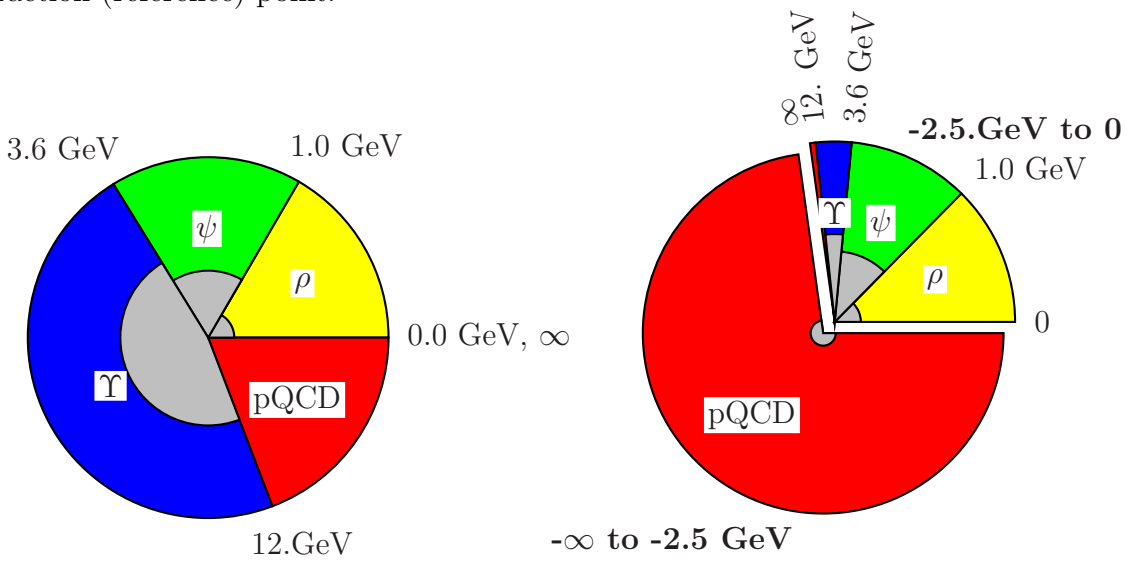


Figure 3: Comparison of the distribution of contributions and errors (shaded areas scaled up by 10) in the standard (left) and the Adler function based approach (right), respectively.

Since Δ Eq. (4.7) is small we may include it in the resummation without further worrying and thus obtain

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027664 \pm 0.000173[0.000137] \ . \tag{4.8}$$

The alternative evaluation by the Euclidean approach is compared with the standard evaluation in Tab. 1. The two methods (standard vs. Euclidean) of evaluating $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ are also compared in Fig. 3

Our alternative procedure to evaluate $\Delta\alpha^{(5)}_{\rm had}(-M_Z^2)$ in the Euclidean region has several advantages as compared to other approaches used so far: The virtues of our analysis are the following:

• no problems with the physical threshold and resonances

• pQCD is used only in the Euclidean region and not below 2.5 GeV. For lower scales pQCD ceases to describe properly the functional dependence of the Adler function [42] (although the pQCD answer remains within error bands down to about 1.6 GeV).
• no manipulation of data must be applied and we need not refer to global or even local duality. That power corrections of the type Eq. (4.1) are negligible has been known for a long time. This, however, does not proof the absence of other kind of non-perturbative effects. Therefore our conservative choice of the minimum Euclidean energy seems to be necessary.
• According to Tab. 1 our non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data, which changes the chances of possible future experimental improvement dramatically, as illustrated in Fig. 3.

While the uncertainties to $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ in the standard approach are coming essentially from everywhere below M_Υ, which would make a new scan over all energies for a precision measurement of $\sigma_{\rm had} \equiv \sigma(e^+e^- \to \gamma^* \to \text{hadrons})$ unavoidable, the new approach leads to a very different situation. The uncertainty of $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is completely dominated by the uncertainties of data below $M_{J/\psi}$ and thus new data on $\sigma_{\rm had}$ are only needed below about 3.6 GeV which could be covered by a tunable "τ–charm facility".

5 Status and outlook

Recent result obtained by different authors for the hadronic contributions to $\alpha_{\rm em}(M_Z)$ are in fairly good agreement. The estimated uncertainties vary substantially, depending mainly on additional theoretical assumptions made in the analyzes. Table 2 compares our results with results obtained by other authors which obtain smaller errors because they are using pQCD in a less controlled manner.

$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	$\delta\Delta\alpha$	$\delta \sin^2\Theta_f$	δM_W	Method	Ref.
0.02800	0.00065	0.000232	11.1	data $<$ 12. GeV	[2]
0.027773	0.000354	0.000126	6.1	[2] + new data CMD & BES	(1.10)
0.027664	0.000173	0.000062	3.0	Euclidean $>$ 2.5 GeV	(1.12)
0.027680	0.000360	0.000128	6.2	data $<$ 12. GeV	[25]
0.02777	0.00017	0.000061	2.9	data $<$ 1.8 GeV	[30]
0.02763	0.00016	0.000057	2.7	data $<$ 1.8 GeV	[32]
0.027690	0.000180	0.000064	3.1	scaled data, pQCD 2.8-3.7, 5-∞	[26, 27]
-	0.00007	0.000025	1.2	$\delta\sigma \lesssim 1\%$ up to J/ψ	
-	0.00005	0.000018	0.9	$\delta\sigma \lesssim 1\%$ up to Υ	
world average		0.000170	23.0	LEPEWWG 2003	

Table 2: $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ and its uncertainties in different evaluations and their contribution to the errors of $\sin^2\Theta_f$ and M_W according to (2.8). Two entries show what can be reached by increasing the precision of cross section measurements to 1%. δM_W in MeV.

The reduction of theoretical and experimental uncertainties must go on to cope with the increased energy and luminosity at future colliders (like TESLA). Ideally a reduction of the errors in $\alpha_{\rm em}(M_Z)$ by about a factor 5 should be achieved. Such progress is equally important for future precision experiments at lower energies.

In order to be able to rely more on pQCD the required reduction of errors can be achieved only if the precision of QCD parameters improves accordingly. On the one hand this proceeds along the traditional perturbative QCD vs. experimental data line (see [44] and references therein), on the other hand, lattice QCD calculations will become of increasing importance, in this context. A lot has been achieved in this direction already in recent years [45]-[49].

However, equally important, experimental efforts must go on in measuring $\sigma(e^+e^- \to hadrons)$ at the 1% level up to energies 3.6 GeV. As most of the existing facilities which are able to measure $\sigma(e^+e^- \to hadrons)$ have approved or discuss upgrade programs, we are confident that the progress needed actually will take place.

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