Δr and Z boson observables – electroweak aspects

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Abstract

An overview is given on the status of Standard Model calculations for the electroweak precision observables from the Z resonance and for the vector-boson mass interdependence, as far as the electroweak loop corrections are concerned. The various theoretical improvements over the recent years are summarized and an estimate of the theoretical uncertainty is given.

1 Introduction

High-precision experiments at electron-positron and hadron colliders impose stringent tests on the Standard Model and delimit the allowed range of the mass of the Higgs boson quite substantially. The experimental accuracy in the electroweak observables has reached sensitivity to the quantum effects of the Standard Model and requires the highest standards on the theoretical side as well. A sizeable amount of work has continuously contributed over the last two decades to a steadily rising improvement of the Standard Model predictions, pinning down the theoretical uncertainties to the level required for the proper interpretation of the precision data. Theoretical uncertainties, besides the parametric uncertainties from the experimental values of the input parameters, arise from unknown higher-order terms. Their estimate is an important entry in the global electroweak fits and in the derivation of the blue-band-plot for the mass M_H of the Higgs boson [1]. A comprehensive documentation of the set of electroweak precision observables has been given in the proceedings of the Workshop on Precision Calculations for the Z Resonance [2] in 1995, where also the theoretical uncertainties have been allocated in a systematic way. Since then, quite a few important improvements from higher-order calculations have become available, stabilizing the Standard Model predictions and reducing the theoretical uncertainty of the precision observables.

2 The vector-boson mass correlation and Δr

The basic physical quantity for the M_W–M_Z correlation is the muon lifetime τ_μ, which defines the Fermi constant G_F according to

$$\frac{1}{\tau_\mu} = \frac{G_F^2\, m_\mu^5}{192\pi^3}\, F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1+\frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1+\Delta_{\rm QED})\,, \tag{2.1}$$

with $F(x) = 1-8x-12x^2\ln x+8x^3-x^4$. By convention, the QED corrections within the Fermi Model, $\Delta_{\rm QED}$, are included in this defining equation for G_F. The one-loop result for $\Delta_{\rm QED}$ [3], has already been known for several decades; it has recently been supplemented by the two-loop correction [4], yielding

$$\Delta_{\rm QED} = 1 - 1.81\,\frac{\alpha(m_\mu)}{\pi} + 6.7\left(\frac{\alpha(m_\mu)}{\pi}\right)^2, \quad \text{with} \quad \alpha(m_\mu) \simeq \frac{1}{135.90}\ . \tag{2.2}$$

The tree-level W-propagator effect giving rise to the (numerically insignificant) term $3m_\mu^2/(5M_W^2)$ in (2.1), is conventionally also included in the definition of G_F, although not part of the Fermi Model prediction. From the precisely measured muon-decay width the value [5] $G_F = (1.16637 \pm 0.00001)\,10^{-5}$ GeV^{-2} for the Fermi constant is derived.

Calculating the muon lifetime within the standard model or the MSSM and comparing the result with (2.1) yields the relation

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}\,(1+\Delta r)\,, \tag{2.3}$$

where the radiative corrections are summarized in the quantity Δr. Thereby, a set of infrared-divergent QED-correction graphs has to be removed, which reproduce the QED-correction factor of the Fermi-model result in (2.1). They have no influence on the relation between G_F and the model parameters.

In the standard model the quantum correction Δr was calculated for the first time by Sirlin [6] at the one-loop level. The one-loop result has been improved over the last two decades by numerically important QCD and electroweak higher-order terms.

The photon vacuum polarization is part of Δr; in general, it is a basic entry in the predictions for electroweak precision observables. The subtracted finite quantity

$$\mathrm{Re}\,\hat\Pi^\gamma(M_Z^2) = \mathrm{Re}\,\Pi^\gamma(M_Z^2) - \Pi^\gamma(0) \tag{2.4}$$

in its purely fermionic part corresponds to standard QED and does not depend on the details of the electroweak theory. It can be split into a leptonic and a hadronic contribution, yielding

$$\Delta\alpha = \Delta\alpha_{\rm lept} + \Delta\alpha_{\rm had} = -\,\mathrm{Re}\,\hat\Pi^\gamma_{\rm lept}(M_Z^2) - \mathrm{Re}\,\hat\Pi^\gamma_{\rm had}(M_Z^2)\,, \tag{2.5}$$

which represents a QED-induced shift in the electromagnetic fine structure constant

$$\alpha \;\to\; \alpha(1+\Delta\alpha)\,. \tag{2.6}$$

It can be resummed [7] according to the renormalization group, accommodating all the leading logarithms of the type $\alpha^n \log^n(M_Z/m_f)$, to give an effective fine-structure constant at the Z mass scale,

$$\alpha(M_Z^2) \;=\; \frac{\alpha}{1-\Delta\alpha}\,. \tag{2.7}$$

The leptonic content of $\Delta\alpha$ can be directly evaluated in terms of the known lepton masses, yielding at three-loop order [8]

$$\Delta\alpha_{\rm lept} \;=\; 314.97687\cdot 10^{-4}\,. \tag{2.8}$$

For the light hadronic part, perturbative QCD is not applicable and quark masses are no reasonable input parameters. Instead, the 5-flavour contribution to $\hat{\Pi}^{\gamma}_{\rm had}$ can be derived from experimental data with the help of a dispersion relation

$$\Delta\alpha_{\rm had} = \,-\,\frac{\alpha}{3\pi}\,M_Z^2\,{\rm Re}\int_{4m_\pi^2}^{\infty}{\rm d}s'\,\frac{R^\gamma(s')}{s'(s'-M_Z^2-i\varepsilon)} \tag{2.9}$$

where

$$R^\gamma(s) = \frac{\sigma(e^+e^-\to\gamma^*\to{\rm hadrons})}{\sigma(e^+e^-\to\gamma^*\to\mu^+\mu^-)}$$

is an experimental input quantity for the low energy range. A recent update including new data points from BES [9] and CMD [10] yields the value [11] $\Delta\alpha = 0.027572 \pm 0.000359$, or $\alpha^{-1}(M_Z^2) = 128.952 \pm 0.049$, respectively. For more details, see [12].
The heavy quark doublet (t,b) contributes predominantly via the ρ parameter to Δr, at the one-loop level given by [13]

$$\Delta\rho = \frac{3G_F m_t^2}{8\pi^2\sqrt{2}} \equiv 3\,X_t. \tag{2.10}$$

Δr thus has a simple form in the light- and heavy-fermion terms at one-loop order,

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\,\Delta\rho + \Delta r_{\rm rem}\,. \tag{2.11}$$

$\Delta\alpha$ contains the large logarithmic corrections from the light fermions and $\Delta\rho$ the quadratic correction from m_t. All other terms are collected in the remainder $\Delta r_{\rm rem}$, which has a typical size of the order ~ 0.01.

Beyond the one-loop order, higher-order 1-particle reducible and irreducible 2- and 3-loop contributions to the ρ parameter have been obtained [14]–[19], with electroweak and QCD terms. QCD corrections to Δr beyond the contribution via $\Delta\rho$ are known at $\mathcal{O}(\alpha\alpha_s)$ [20] and $\mathcal{O}(\alpha\alpha_s^2)$ [21]. First approximative electroweak two-loop calculations were performed based on expansions for asymptotically large values of M_H [22] and m_t [23].

In the meantime, the complete electroweak two-loop result in the standard model has become available: the fermionic two-loop terms [24, 25] with all two-loop diagrams for the muon-decay amplitude containing at least one closed fermion loop, and the residual class of the two-loop purely bosonic diagrams [26]. Their influence is displayed in Fig. 1 for the fermionic and in Fig. 2 for the bosonic contributions, in terms of Δr and M_W.

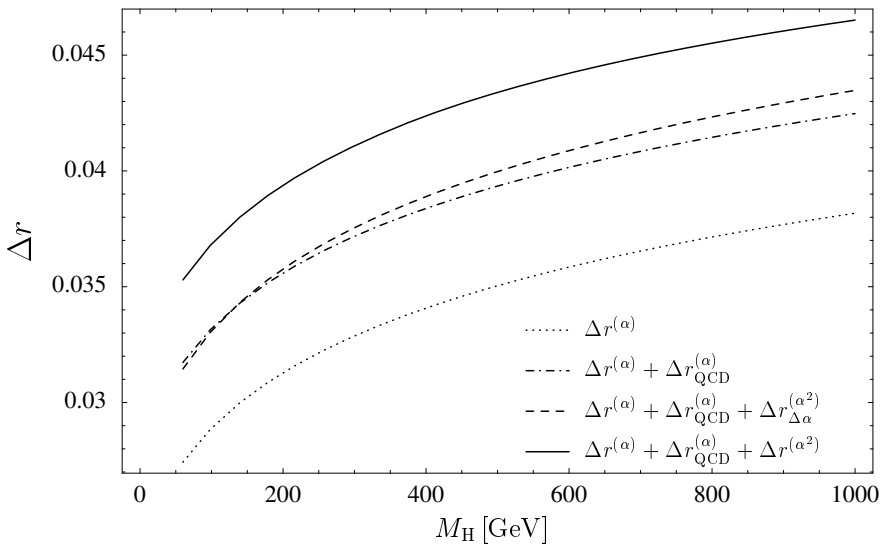

Figure 1: Various stages of Δr, as a function of M_H. The one-loop contribution, $\Delta r^{(\alpha)}$, is supplemented by the two-loop and three-loop QCD corrections, $\Delta r^{(\alpha)}_{\rm QCD} \equiv \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)}$, and the fermionic electroweak two-loop contributions, $\Delta r^{(\alpha^2)} \equiv \Delta r^{(N_{\rm f}\alpha^2)} + \Delta r^{(N_{\rm f}^2\alpha^2)}$. For comparison, the effect of the two-loop corrections induced by a resummation of $\Delta\alpha$, $\Delta r^{(\alpha^2)}_{\Delta\alpha}$, is shown separately.

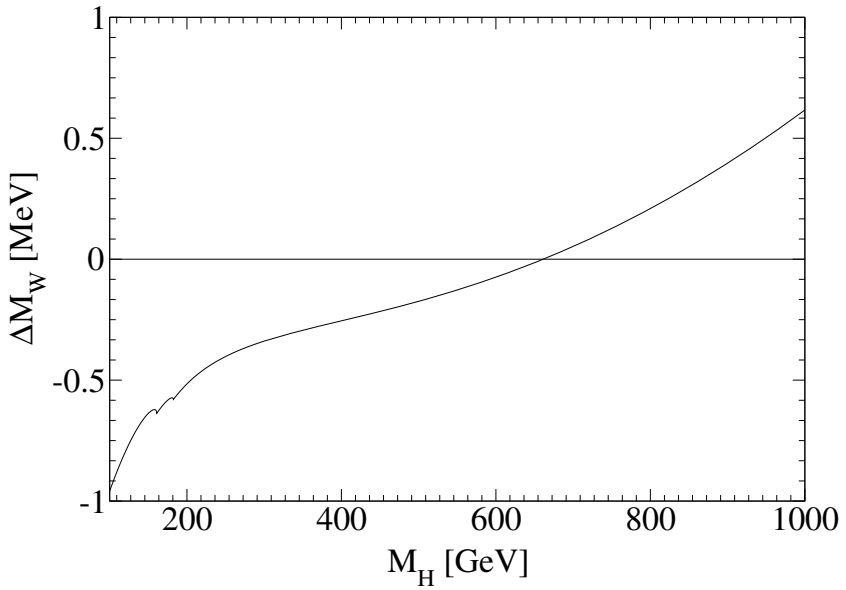

Figure 2: The shift in M_W from the two-loop bosonic contributions to Δr (from [26]).

The recently computed 3-loop top-quark contributions of $O(\alpha_s X_t^2)$ and $O(X_t^3)$ to the ρ parameter for general values of M_H [19], extending the previous result of [18] for $M_H = 0$, lead to a further shift in the calculated value of M_W of 2 MeV for a light and 5 MeV for a heavy Higgs boson.

The availablity of both the complete electroweak 2-loop calculation for Δr and the leading 3-loop terms via $\Delta\rho$ reduces the remaining theoretical uncertainty quite sizeably. The estimate of the theoretical error on M_W, before these new items, was 6 MeV [24, 27]; it can now be estimated to be of the order 3 MeV.

A final note concerns the definition of the vector-boson masses. The 2-loop W and Z mass counterterms determine the 2-loop counterterm δs_W^2 for the mixing angle $s_W^2 = 1 - M_W^2/M_Z^2$, which has to be gauge invariant since it is an observable quantity. With the use of a general R_ξ gauge it has explicitly been checked [24] that δs_W^2 is gauge-parameter independent for the complex-pole mass definition. The mass definition via the complex pole corresponds to a Breit-Wigner parameterization of the resonance shape with a constant width. For the experimental determination of the gauge-boson masses, however, a Breit-Wigner ansatz with a running width is used. This has to be accounted for by a shift of the values for the complex pole masses,

$$\overline{M} = M - \frac{\Gamma^2}{2M}\,, \tag{2.12}$$

which yields the relations

$$\begin{array}{lcl} \overline{M_Z} &=& M_Z - 34.1 \text{ MeV}, \\ \overline{M_W} &=& M_W - 27.4 \text{ MeV} \quad \text{for} \quad M_W = 80.4 \text{ GeV}. \end{array} \tag{2.13}$$

For M_Z and Γ_Z the experimental numbers are taken. The W mass is a calculated quantity, and therefore also a theoretical value for the W-boson width should be applied here. An approximate, but sufficiently accurate, expression for the W width is given by

$$\Gamma_W = 3\frac{G_F M_W^3}{2\sqrt{2}\pi}\left(1 + \frac{2\alpha_s}{3\pi}\right). \tag{2.14}$$

3 *Z* boson observables

3.1 Amplitudes and effective couplings

In lowest order, the processes $e^+e^- \to f\bar{f}$ are described by diagrams with photon and Z exchange. The 1-loop diagrams with virtual photon insertions (virtual QED corrections) form a gauge invariant subset of the full 1-loop terms. The sum of the virtual photon loop graphs is UV finite but IR divergent. The IR-divergence is cancelled by adding the cross section with real photon bremsstrahlung (after integrating over the photon phase space), which always accompanies realistic scattering processes.

The residual set of 1-loop diagrams, without any real or virtual photon, is commonly called non-QED or weak corrections. This class is free of IR-singularities but sensitive to the details beyond the lowest order amplitudes. The UV-singular terms associated with the loop diagrams are cancelled by the counterterms, as a consequence of renormalizability.

The 1-loop amplitude for $e^+e^- \to f\bar{f}$ contains the sum of the individual contributions to the self-energies and vertex corrections, including the external fermion self-energies via wave function renormalization, and box graphs. The total amplitude around the Z pole can be cast into a form close to the lowest order amplitude

$$A(e^+e^- \to f\bar{f}) = A_\gamma + A_Z + (box)$$

as the sum of a dressed photon and a dressed Z exchange amplitude plus the contribution from the box diagrams which are numerically not significant around the peak (relative contribution $< 10^{-4}$). For theoretical consistency (gauge invariance) they have to be retained; for practical purposes they can be neglected in Z physics. Resummation of the iterated self-energy insertions in the photon and Z propagators brings the finite Z decay width into the denominator, and treats the higher order leading terms in the proper way. Since the leading terms arise from fermion loops only, we do not have problems with gauge invariance; the bosonic loop terms have to be understood as expanded to 1-loop order. Numerically their resummation yields small differences which give an impression of the possible size of the yet unknown next order bosonic terms.

The factorized Z amplitude allows us to define neutral current (NC) vertices at the Z resonance with effective coupling constants $g^f_{V,A}$ [28, 29], synonymously to the use of ρ_f, κ_f,

$$\begin{aligned} J^{NC}_\mu &= \left(\sqrt{2}G_\mu M_Z^2 \rho_f\right)^{1/2} \left[(I_3^f - 2Q_f s_W^2 \kappa_f)\gamma_\mu - I_3^f \gamma_\mu\gamma_5\right] \\ &= \left(\sqrt{2}G_\mu M_Z^2\right)^{1/2} \left[g_V^f\,\gamma_\mu - g_A^f\,\gamma_\mu\gamma_5\right]. \end{aligned} \tag{3.1}$$

We can define effective mixing angles for a given fermion species f according to

$$s_f^2 \equiv \sin^2\theta_f = s_W^2\,\mathrm{Re}\,\kappa_f = \frac{1}{4\mid Q_f \mid}\left(1 - \frac{\mathrm{Re}\,g_V^f}{\mathrm{Re}\,g_A^f}\right). \tag{3.2}$$

A special role plays the leptonic effective mixing angle (for $\ell = e$), which commonly is also denoted by $\sin^2\theta_{\mathrm{eff}}$,

$$\sin^2\theta_{\mathrm{eff}} = s_\ell^2\,. \tag{3.3}$$

Beyond the leading 2-loop corrections $\sim m_t^4$, which are related to the ρ-parameter, the subleading electroweak 2-loop corrections $\sim G_\mu^2 m_t^2 M_Z^2$ in the top mass expansion for the leptonic mixing angle [23] s_ℓ^2 are available, as well as for ρ_ℓ [30]. More complete electroweak 2-loop calculations are still missing.

Exact results were also derived for the Higgs-mass dependence of the fermionic 2-loop corrections in s_ℓ^2 [31, 32], and comparisons were performed with those obtained via the top mass expansion [31]. Differences in the values of s_ℓ^2 can amount to $8 \cdot 10^{-5}$ when M_H is varied over the range from 100 GeV to 1 TeV.

Figure 3 shows the Higgs-mass dependence of the 2-loop corrections to s_ℓ^2 associated with the t/b doublet, with $\Delta\alpha$, and with the light fermion terms not in $\Delta\alpha$. As can be seen, the M_H-dependence of the light fermions yields contributions to s_ℓ^2 up to $2 \cdot 10^{-5}$. For ρ_ℓ or equivalently the leptonic Z widths, the subleading 2-loop effects are small, and differences with the results from the top mass expansion are irrelevant.

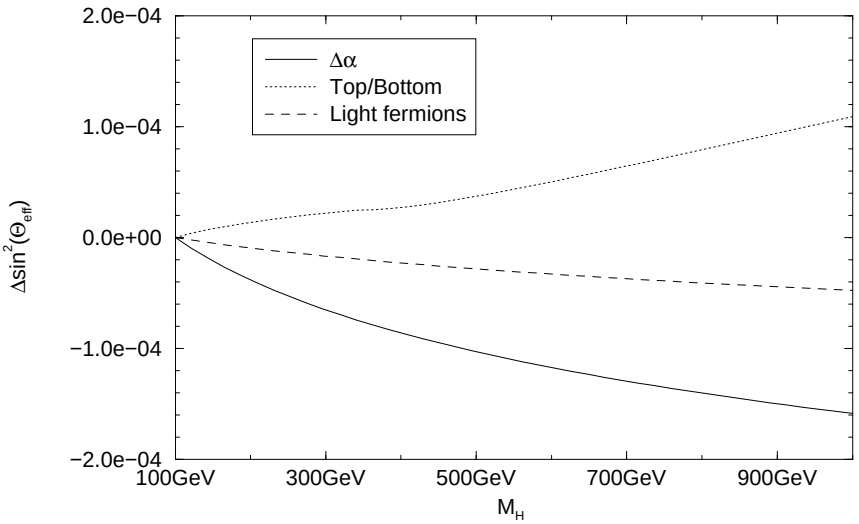

Figure 3: Higgs mass dependence of the various fermionic contributions at the two-loop level to the effective leptonic mixing angle s^2_ℓ at the Z peak: light fermion contribution via $\Delta\alpha$, light fermion contribution not in $\Delta\alpha$, and the contribution from the (tb) doublet. Shown in each case is the difference $s^2_\ell(M_H) - s^2_\ell(100\,\mathrm{GeV})$.

The 3-loop top-quark contributions to the ρ parameter obtained in [19] also have an influence on s^2_ℓ, with a positive shift of $1 \cdot 10^{-5}$ for a light and $4 \cdot 10^{-5}$ for a heavy Higgs boson.

The effective mixing angle is an observable most sensitive to the mass M_H of the Higgs boson. A light Higgs boson corresponds to a low value of s^2_ℓ. The inclusion of the two-loop electroweak corrections $\sim m_t^2$ from [23] yields a sizeable positive contribution to s^2_ℓ, see Figure 4. The inclusion of this term, hence, has strengthened the upper bound on M_H considerably. On the other hand, the unknown higher-order terms for s^2_ℓ induce a rather large uncertainty in the Higgs-boson mass determination. The theoretical uncertainty of s^2_ℓ is estimated to be $6 \cdot 10^{-5}$. This estimate follows from the shift in s^2_ℓ when calculated with the complete 2-loop calculation for Δr versus the value obtained from the 2-loop expansion in m_t^2 for Δr.

The Z boson coupling to b quarks is a particular case, with a specific non-universal part showing a strong dependence on the top-quark mass m_t, resulting from the virtual top quark in the vertex corrections. The difference between the d and b couplings can be parametrized in the following way,

$$\rho_b = \rho_d(1+\tau_b)^2, \quad s_b^2 = s_d^2(1+\tau_b)^{-1} \tag{3.4}$$

with the quantity

$$\tau_b = \Delta\tau^{(1)} + \Delta\tau^{(2)}$$

calculated perturbatively, comprising the complete 1-loop order term [36] and the leading electroweak 2-loop contribution of $O(G_\mu^2 m_t^4)$ [16, 37]. The 2-loop terms are rather small.

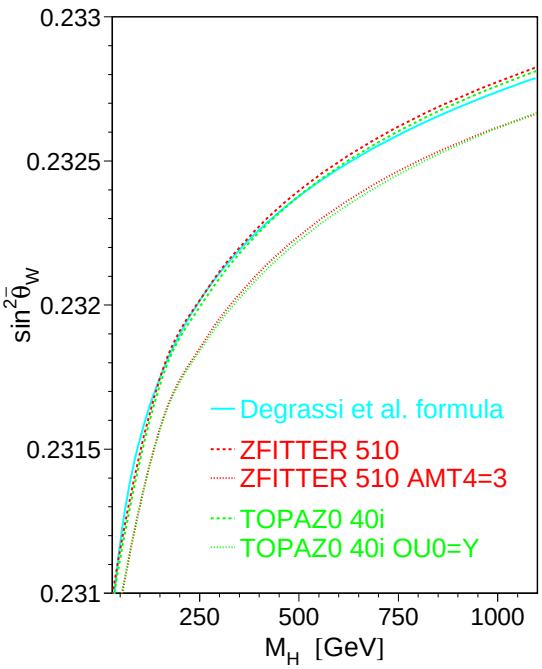

Figure 4: Higgs mass dependence of s^2_ℓ with and without the electroweak 2-loop term $\sim m_t^2$, comparison of ZFITTER [33, 35] and TOPAZ0 [34, 35] codes. The lower sample of curves is without, the upper sample with the 2-loop term.

3.2 The Z line shape

The integrated cross section $\sigma(s)$ for $e^+e^- \to f\bar{f}$ around the Z can be expressed in terms of the effective vector and axial vector coupling constants. It is, however, convenient to rewrite $\sigma(s)$ in terms of the Z width and the partial widths Γ_e, Γ_f in order to have a more model independent parametrization. The following form [38, 39] includes final state photon radiation and QCD corrections in case of quark final states: [1]

$$\sigma(s) = \frac{12\pi\Gamma_e\Gamma_f}{\mid s - M_Z^2 + \mathrm{i}M_Z\Gamma_Z(s) \mid^2}\left\{\frac{s}{M_Z^2} + R_f\,\frac{s-M_Z^2}{M_Z^2} + I_f\,\frac{\Gamma_Z}{M_Z} + \cdots\right\} + \frac{4\pi\,\alpha(s)^2}{3s}\,Q_f^2 N_C^f\,K_{\mathrm{QCD}}\,(1+\delta_{\mathrm{QED}}) \tag{3.5}$$

with

$$\Gamma_Z(s) = \Gamma_Z\left\{\frac{s}{M_Z^2} + \epsilon\,\frac{s-M_Z^2}{M_Z^2} + \cdots\right\} \tag{3.6}$$

[1] Since initial state photon radiation is treated separately the QED correction factor in Eq. (3.5) has to be removed in Γ_e.

and $N_C^f = 1$ for leptons and = 3 for quarks. The terms R_f, I_f, ϵ are small quantities calculable in terms of the basic parameters. R_f and I_f describe the γ-Z interference

$$\begin{aligned} R_f &= \frac{2\,Q_e Q_f\,\mathrm{Re}(g_V^e g_V^f)}{[(g_V^e|^2+|g_A^e|^2((|g_V^f|^2+|g_A^f|^2)}\,\frac{4\pi\alpha(s)}{\sqrt{2}G_\mu M_Z^2}\,, \\ I_f &= \frac{2\,Q_e Q_f\,\mathrm{Re}(g_V^e g_V^f)}{(|g_V^e|^2+|g_A^e|^2)(|g_V^f|^2+|g_A^f|^2)}\,\frac{4\pi\alpha(s)}{\sqrt{2}G_\mu M_Z^2}\cdot\frac{s}{M_Z^2}\,\mathrm{Im}\,\hat{\Pi}^\gamma\,, \end{aligned} \tag{3.7}$$

and the last term is the QED background from pure photon exchange with

$$\alpha(s) = \frac{\alpha}{1+\mathrm{Re}\,\hat{\Pi}^\gamma_{\mathrm{ferm}}(s)} \tag{3.8}$$

involving the fermionic vacuum polarization in the running coupling. The small correction

$$\epsilon = \sum_f \epsilon_f, \quad \epsilon_f \simeq \frac{6m_f^2}{M_Z^2}\frac{\Gamma_f}{\Gamma_Z}\frac{|g_A^f|^2}{|g_V^f|^2+|g_A^f|^2} \tag{3.9}$$

is due to finite fermion-mass effects. I_f and ϵ have very little influence on the line shape. In electroweak fits, the quantities R_f, I_f, ϵ_f are taken from the Standard Model. Model-independent fits leaving these parameters free are compatible with the restricted fits, with a slight increase on the errors of the Z mass and width.

QED corrections:

The observed cross section is the result of convoluting Eq. (3.5) with the initial state QED corrections consisting of virtual photon and real photon bremsstrahlung contributions,

$$\sigma_{\mathrm{obs}}(s) = \int_0^{x_{\max}} \mathrm{d}x\, H(x)\,\sigma(s(1-x))\,. \tag{3.10}$$

$x_{\max}$ denotes a cut to the radiated energy, $x = 2k/\sqrt{s}$. Kinematically it is limited by either $1-4m_\ell^2/s$ for leptons or $1-4m_\pi^2/s$ for hadrons, respectively. For the required accuracy, multiple photon radiation has to be included. The radiator function $H(x)$ is known with soft-photon resummation to all orders, with the exact $O(\alpha^2)$ result [40, 38], and with the leading logarithmic $O(\alpha^3)$ terms [41].

3.3 Width and partial widths

The total Z width Γ_Z can be calculated as the sum over the partial decay widths

$$\Gamma_Z = \sum_f \Gamma_f\,, \quad \Gamma_f = \Gamma(Z\to f\bar{f}) \tag{3.11}$$

(other decay channels are not significant). The fermionic partial widths, expressed in terms of the effective coupling constants as follows,

$$\Gamma_f = N_C^f\,\frac{\sqrt{2}G_\mu M_Z^3}{12\pi}\left(|\,g_V^f\,|^2+|\,g_A^f\,|^2\right)\cdot(1+\delta_{\mathrm{QED}}+\delta_{\mathrm{QCD}})\,, \tag{3.12}$$

contain QED corrections, at one-loop order given by

$$\delta_{\rm QED} = Q_f^2 \, \frac{3\alpha}{4\pi} \,, \tag{3.13}$$

which are small, maximum 0.17% for charged leptons. The factorizable QCD corrections for hadronic final states read [42]

$$\delta_{\rm QCD} \;\; = \;\; \frac{\alpha_s}{\pi} + 1.41 \left(\frac{\alpha_s}{\pi}\right)^2 - 12.8 \left(\frac{\alpha_s}{\pi}\right)^3 - \frac{Q_f^2}{4} \frac{\alpha\alpha_s}{\pi^2} \tag{3.14}$$

for the light quarks with $m_q \simeq 0$, with $\alpha_s = \alpha_s(M_Z^2)$.

For b quarks, $\delta_{\rm QCD}$ is different, due to the finite b-quark mass and to top quark dependent 2-loop diagrams. It is calculated up to third order in α_s, except for the m_b-dependent singlet terms, which are known to $O(\alpha_s^2)$ [43, 44]. For a review of the QCD corrections to the Z width see [45].

The partial decay rate into b-quarks is an observable of special sensitivity to the top quark mass. Therefore, beyond the pure QCD corrections, also the 2-loop contributions of the mixed QCD–electroweak type, are important. They were first derived for the leading term of $O(\alpha_s G_\mu m_t^2)$ [46] and were subsequently completed by the $O(\alpha_s)$ correction to the $\log m_t/M_W$ term [47] and the residual terms of $O(\alpha\alpha_s)$ [48].

At the same time, also the complete 2-loop $O(\alpha\alpha_s)$ to the partial widths into the light quarks have been obtained [49], which are beyond those that are already contained in the factorized expression (3.12). These "non-factorizable" corrections yield an extra negative contribution of $-0.55(3)$ MeV to the total hadronic Z width. In summary, the 2-loop corrections of $O(\alpha\alpha_s)$ to the Z widths are by now completely under control and the remaining theoretical uncertainty is insignificant.

Radiation of secondary fermions through photons from the primary final state fermions can yield another sizeable contribution to the partial Z widths; however, this is compensated by the corresponding virtual contribution through the dressed photon propagator in the final-state vertex correction for sufficiently inclusive final states, i.e. for loose cuts to the invariant mass of the secondary fermions [50].

3.4 Asymmetries

The on-resonance asymmetries contain specific combinations of the effective vector and axialvector coupling constants (taking only the real parts of the couplings),

$$A_f = \frac{2 g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2} \,, \tag{3.15}$$

in the following way for the forward-backward asymmetries, the final-state τ polarisation, or the left-right asymmetry from longitudinally polarized intitial-state electrons:

$$A_{\rm FB} = \frac{3}{4} \, A_e A_f, \quad A_\tau^{\rm pol} = A_\tau, \quad A_{\rm LR} = A_e \,. \tag{3.16}$$

Measurements of the asymmetries hence are measurements of the ratios g_V^f/g_A^f and thus of the effective mixing angles.

A special role is played by the forward-backward asymmetries for b-quarks, $A_{\rm FB}^b$. The factor A_b, owing accidentally to the assignment of quantum numbers, is practically a constant in the standard model, $A_b = 0.935 \pm 0.001$. $A_{\rm FB}^b$ is therefore essentially determined by A_e and is hence an observable very sensitive to the leptonic mixing angle. A precise measurement of $A_{\rm FB}^b$ is thus a precise measurement of s_ℓ^2.

There are also other contributions, besides the terms in (3.16), entering the on-resonance asymmetries, originating from finite-mass terms, the imaginary parts of the couplings, photon-Z interference, and pure photon exchange. They are small, and the present accuracy of the calculation is satisfactory.

On the other side, QED corrections from real and virtual photons are sizeable and have to be treated properly. Initial-state photonic corrections have a strong impact on $A_{\rm FB}$ due to its steep variation with the energy. As in the case of the line shape, the observed forward-backward asymmetry results from convoluting the s-dependent asymmetric cross section $\sigma_{FB}(s)$ with the photon spectrum $\tilde{H}(z)$ from initial-state radiation,

$$A_{\rm FB}(s) = \frac{1}{\sigma(s)} \int_{z_0}^1 {\rm d}z \, \frac{4z}{(1+z)^2} \, \tilde{H}(z) \, \sigma_{FB}(zs), \quad z_0 \geq \frac{4m_f^2}{s}\,, \tag{3.17}$$

with the integrated cross section $\sigma(s)$ from (3.10). In terms of the effective complex couplings the asymmetric cross section reads:

$$\begin{aligned} \sigma_{FB}(s) &= 2\,Q_e Q_f\,(\sqrt{2}G_\mu M_Z^2)\,{\rm Re}\left(g_A^e g_A^f\,\chi(s)\frac{e^2}{1+\hat{\Pi}^\gamma_{\rm ferm}(s)^*}\right) \\ &\quad + 4\,{\rm Re}(g_V^e g_A^{e\,*})\,{\rm Re}(g_V^f g_A^{f\,*})\,(\sqrt{2}G_\mu M_Z^2)^2\,|\chi(s)|^2 \end{aligned} \tag{3.18}$$

with

$$\chi(s) = \frac{s}{s - M_Z^2 + {\rm i}\frac{s}{M_Z}\Gamma_Z}\,. \tag{3.19}$$

The radiator function $\tilde{H}(z)$ is different from the radiator H for the symmetric cross section (3.10) in the hard photon terms. According to the present status of the calculation, $\tilde{H}(z)$ contains the exact $O(\alpha)$ contribution [51], the $O(\alpha^2)$ contribution in the leading logarithmic approximation [52] and the resummation of soft photons to all orders [53].

Final state corrections in the absence of cuts are at one-loop given by [54, 51]

$$A_{\rm FB} \to A_{\rm FB} \cdot \left(1 - \frac{3\alpha(M_Z^2)}{4\pi}\right)\,. \tag{3.20}$$

Differently from the integrated cross section, also initial-final state interference yields a contribution to the asymmetry, which, however, in case of loose cuts is very small. The application of more severe cuts, as it might be necessary for the experimental analysis, increases both the final state and the interference contributions and makes their proper treatment indispensable. They are included in the program packages used for the data analysis and are not described here in more detail. An overview can be found in [53].

In case of quark pair production, final-state QCD corrections to the asymmetries have to be considered. At 1-loop order they have a similar structure as the corresponding QED corrections. Finite mass effects have to taken into account for b-quarks only; they are discussed in [54]. Two-loop QCD corrections have also become available [55]. For more details see [56].

3.5 Pseudo-observables versus realistic observables

The electroweak quantities discussed in the previous subsections, like widths, mixing angles s_f^2, on-resonance asymmetries, are obtained from experiment only after applying a series of manipulations: deconvoluting the QED corrections, subtracting photon-exchange and interference terms, subtracting box-diagram contributions, etc. In the terminology of [2], the primarily measured quantities like cross sections, line shape, asymmetries, are denoted as REALISTIC OBSERVABLES, whereas the secondary electroweak quantities are labeled as PSEUDO-OBSERVABLES. The procedure to get from realistic to pseudo-observables involves in several steps the use of the Standard Model to calculate the subtraction terms. Since the pseudo-obserables are conventionally used for the global electroweak fits, one may wonder how consistent the extraction of the basic parameters like M_H is with respect to a more direct Standard Model fit to the realistic observables themselves. Such direct fits are hence a crucial check and they confirm the validity of the results [57], e.g. for the blue band for the Higgs-boson mass.

4 Conclusions

For electroweak precision tests it is indispensable to have accurate and reliable predictions for the electroweak observables from the Standard Model in terms of the basic parameters, including m_t and M_H. Two of the most-precisely measured quantities with the biggest impact on the determination of limits to the Higgs-boson mass are the effective leptonic mixing angle at the Z peak, $\sin^2\theta_{\rm eff}$, and the mass of the W boson, M_W. M_W is correlated with the Z mass via the Fermi constant and the quantum correction Δr. This correlation is known meanwhile with the complete set of electroweak 2-loop contributions and part of the 3-loop terms. The theoretical uncertainty on M_W from unknown higher-order terms has thus considerably decreased and can by now be estimated to be of the order 3 MeV, which is sufficient for the analysis of present days' data and even for future experiments with envisaged higher precision [27, 58]. For the Z-boson observables, no complete electroweak 2-loop calculation is available as yet. The partial 2-loop terms known so far have little influence on the widths and the $Zb\bar{b}$ vertex, but they are important for the effective mixing angle and thus for the upper limit of M_H, shifting M_H to lower values. The theoretical uncertainty in $\sin^2\theta_{\rm eff}$, estimated to be $6 \cdot 10^{-5}$, is relatively large, nearly half of the experimental error, and improvement for the future is required.

References

[1] M. Grünewald, these proceedings, and hep-ex/0304023

[2] D. Bardin et al., *Reports of the Working Group on Precision Calculations for the Z Resonance*, CERN 95-03 (1995), eds. D. Bardin, W. Hollik, G. Passarino;

[3] R.E. Behrends, R.J. Finkelstein and A. Sirlin, *Phys. Rev.* **101**, 866 (1956); S.M. Berman, *Phys. Rev.* **112**, 267 (1958); T. Kinoshita and A. Sirlin, *Phys. Rev.* **113**, 1652 (1959)

[4] T. van Ritbergen and R.G. Stuart, *Phys. Rev. Lett.* **82**, 488 (1999); *Phys. Lett.* B **564**, 343 (2000); M. Steinhauser and T. Seidensticker, *Phys. Lett.* B **467**, 271 (1999)

[5] K. Hagiwara et al. [Particle Data Group], *Phys. Rev.* D **66**, 010001 (2002)

[6] A. Sirlin, *Phys. Rev.* D **22**, 971 (1980); W.J. Marciano, A. Sirlin, *Phys. Rev.* D **22**, 2695 (1980)

[7] W.J. Marciano, *Phys. Rev.* D **20**, 274 (1979)

[8] G. Källén, A. Sabry, *K. Dan. Vidensk. Selsk. Mat.-Fys. Medd.* **29** (1955) No. 17; M. Steinhauser, *Phys. Lett.* B **429**, 158 (1998)

[9] BES Collaboration, J.Z. Bai et al., *Phys. Rev. Lett.* **84**, 594 (2000); **88**, 101802 (2002)

[10] CMD-2 Collaboration, R.R. Akhmetshin et al., *Phys. Lett.* B **527**, 161 (2002)

[11] F. Jegerlehner, *J. Phys.* G **29**, 101 (2003)

[12] F. Jegerlehner, these proceedings

[13] D. Ross, M. Veltman, *Nucl. Phys.* B **95**, 135 (1975); M. Veltman, *Nucl. Phys.* B **123**, 89 (1977); M.S. Chanowitz, M.A. Furman, I. Hinchliffe, *Phys. Lett.* B **78**, 285 (1978)

[14] A. Djouadi, C. Verzegnassi, *Phys. Lett.* B **195**, 265 (1987)

[15] M. Consoli, W. Hollik, F. Jegerlehner, *Phys. Lett.* B **227**, 167 (1989); J. van der Bij, F. Hoogeveen, *Nucl. Phys.* B **283**, 477 (1987

[16] R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere, *Phys. Lett.* B **288**, 95 (1992); *Nucl. Phys.* B **409**, 105 (1993); J. Fleischer, O.V. Tarasov, F. Jegerlehner, *Phys. Lett.* B **319**, 249 (1993); *Phys. Rev.* D **51**, 3820 (1995)

[17] L. Avdeev, J. Fleischer, S.M. Mikhailov, O. Tarasov, *Phys. Lett.* B **336**, 560 (1994); E: **349**, 597 (1995); K. Chetyrkin, J. Kühn, M. Steinhauser, *Phys. Lett.* B **351**, 331 (1995)

[18] J. van der Bij, K. Chetyrkin, M. Faisst, G. Jikia, T. Seidensticker, *Phys. Lett.* B **498** (2001) 156

[19] M. Faisst, J. Kühn, T. Seidensticker, O. Veretin, hep-ph/0302275

[20] B.A. Kniehl, *Nucl. Phys.* B **347**, 89 (1990); F. Halzen, B.A. Kniehl, *Nucl. Phys.* B **353**, 567 (1991); B.A. Kniehl, A. Sirlin, *Nucl. Phys.* B **371**, 141 (1992), *Phys. Rev.* D **47**, 883 (1993); A. Djouadi, P. Gambino, *Phys. Rev.* D **49**, 3499 (1994);

[21] K. Chetyrkin, J. Kühn, M. Steinhauser, *Phys. Rev. Lett.* **75**, 3394 (1995)

[22] J. van der Bij, M. Veltman, *Nucl. Phys.* B **231**, 205 (1984)

[23] G. Degrassi, P. Gambino, A. Vicini, *Phys. Lett.* B **383**, 219 (1996); G. Degrassi, P. Gambino, M. Passera, A. Sirlin, *Phys. Lett.* B **394**, 188 (1997); G. Degrassi, P. Gambino, A. Sirlin, *Nucl. Phys.* B **418**, 209 (1998)

[24] A. Freitas, W. Hollik, W. Walter, G. Weiglein, Phys. Lett. **B495**, 338 (2000); *Nucl. Phys.* B **632**, 189 (2002)

[25] M. Awramik, M. Czakon, hep-ph/0305248

[26] M. Awramik, M. Czakon, *Phys. Rev. Lett.* **89** (2002) 241801; A. Onishchenko, O. Veretin, *Phys. Lett.* B **551** (2003) 111; M. Awramik, M. Czakon, A. Onishchenko, O. Veretin, hep-ph/0209084

[27] U. Baur et al., hep-ph/0111314

[28] W. Hollik, *Fortschr. Phys.* **38** (1990) 165; *Precision Tests of the Standard Model*, pp. 37 and 117, ed. P. Langacker, World Scientific, Singapore 1995

[29] D. Bardin et al., *Reports of the Working Group on Precision Calculations for the Z Resonance*, p. 7, CERN 95-03 (1995), eds. D. Bardin, W. Hollik, G. Passarino; hep-ph/9709229

[30] G. Degrassi, P. Gambino, *Nucl. Phys.* B **567**, 3 (2000)

[31] G. Weiglein, *Acta Phys. Polon. B* **29**, 2735 (1998); hep-ph/9901317

[32] A. Stremplat, Diploma Thesis (Karlsruhe 1998)

[33] D. Bardin et al., hep-ph/9412201

[34] G. Montagna, O. Nicrosini, F. Piccinini, G. Passarino, hep-ph/9804211

[35] D. Bardin, G. Passarino, hep-ph/9803425; D. Bardin, M. Grünewald, G. Passarino, hep-ph/9902452

[36] A.A. Akhundov, D. Bardin, T. Riemann, *Nucl. Phys.* B **276** (1986) 1; W. Beenakker, W. Hollik, *Z. Phys.* C **40** (1988) 141; J. Bernabeu, A. Pich, A. Santamaria, *Phys. Lett.* B **200** (1988) 569; *Nucl. Phys.* B **363** (1991) 326

[37] A. Denner, W. Hollik, B. Lampe, *Z. Phys.* C **60** (1993) 93

[38] F.A. Berends et al., in: *Z Physics at LEP 1*, CERN 89-08 (1989), eds. G. Altarelli, R. Kleiss, C. Verzegnassi, Vol. I, p. 89; W. Beenakker, F.A. Berends, S.C. van der Marck, *Z. Phys.* C **46** (1990) 687

[39] A. Borelli, M. Consoli, L. Maiani, R. Sisto, *Nucl. Phys.* B **333** (1990) 357

[40] G. Burgers, F.A. Berends, W.L.van Neerven, *Nucl. Phys.* B **297** (1988) 429; E: *Nucl. Phys.* B **304** (1988) 921

[41] S. Jadach, M. Skrzypek, B.F.L. Ward, *Phys. Lett.* B **257**, 173 (1991); S. Jadach, M. Skrzypek, *Z. Phys.* C **49**, 584 (1991); G. Montagna, O. Nicrosini, F. Piccinini, *Phys. Lett.* B **406**, 243 (1997)

[42] K.G. Chetyrkin, A.L. Kataev, F.V. Tkachov, *Phys. Lett.* B **85**, 277 (1979); M. Dine, J. Sapirstein, *Phys. Rev. Lett.* **43**, 668 (1979); W. Celmaster, R. Gonsalves, *Phys. Rev. Lett.* **44**, 560 (1980); S.G. Gorishny, A.L. Kataev, S.A. Larin, *Phys. Lett.* B **259**, 144 (1991); L.R. Surguladze, M.A. Samuel, *Phys. Rev. Lett.* **66**, 560 (1991); A. Kataev, *Phys. Lett.* B **287**, 209 (1992)

[43] K.G. Chetyrkin, J.H. Kühn, *Phys. Lett.* B **248**, 359 (1990) and B **406**, 102 (1997); K.G. Chetyrkin, J.H. Kühn, A. Kwiatkowski, *Phys. Lett.* B **282**, 221 (1992); K.G. Chetyrkin, A. Kwiatkowski, *Phys. Lett.* B **305**, 285 (1993) and B **319**, 307 (1993)

[44] B.A. Kniehl, J.H. Kühn, *Phys. Lett.* B **224**, 229 (1990); *Nucl. Phys.* B **329**, 547 (1990); K.G. Chetyrkin, J.H. Kühn, *Phys. Lett.* B **307**, 127 (1993); S. Larin, T. van Ritbergen, J.A.M. Vermaseren, *Phys. Lett.* B **320**, 159 (1994); K.G. Chetyrkin, O.V. Tarasov, *Phys. Lett.* B **327**, 114 (1994)

[45] K.G. Chetyrkin, J.H. Kühn, A. Kwiatkowski, in *Reports of the Working Group on Precision Calculations for the Z Resonance*, p. 175, CERN 95-03 (1995), eds. D. Bardin, W. Hollik, G. Passarino; *Phys. Rep.* **277**, 189 (1996)

[46] J. Fleischer, F. Jegerlehner, P. Rączka, O.V. Tarasov, *Phys. Lett.* B **293**, 437 (1992); G. Buchalla, A.J. Buras, *Nucl. Phys.* B **398**, 285 (1993); G. Degrassi, *Nucl. Phys.* B **407**, 271 (1993); K.G. Chetyrkin, A. Kwiatkowski, M. Steinhauser, *Mod. Phys. Lett.* A **8**, 2785 (1993)

[47] A. Kwiatkowski, M. Steinhauser, *Phys. Lett.* B **344**, 359 (1995); S. Peris, A. Santamaria, *Nucl. Phys.* B **445**, 252 (1995)

[48] R. Harlander, T. Seidensticker, M. Steinhauser, *Phys. Lett.* B **426**, 125 (1998)

[49] A. Czarnecki, J.H. Kühn, *Phys. Rev. Lett.* **77**, 3955 (1996); E: **80**, 893 (1998)

[50] A. Hoang, J.H. Kühn, T. Teubner, *Nucl. Phys.* B **455**, 3 (1995); **452**, 173 (1995)

[51] D. Bardin, M.S. Bilenky, A. Chizhov, A. Sazonov, Yu. Sedykh, T. Riemann, M. Sachwitz, *Phys. Lett.* B **229** (1989) 405; D. Bardin, M.S. Bilenky, A. Chizhov, A. Sazonov, O. Fedorenko, T. Riemann, M. Sachwitz, *Nucl. Phys.* B **351** (1991) 1

[52] W. Beenakker, F.A. Berends, W.L. van Neerven, in: Proceedings of the 1989 Ringberg Workshop *Radiative Corrections for e^+e^- Collisions*, p. 3, ed. J.H. Kühn, Springer, Berlin - Heidelberg - New York 1989

[53] M. Böhm, W. Hollik et al., in: *Z Physics at LEP 1*, CERN 89-08 (1989), eds. G. Altarelli, R. Kleiss, C. Verzegnassi, Vol. I, p. 203; W. Beenakker, F.A. Berends, S.C. van der Marck, *Phys. Lett.* B **252** (1990) 299

[54] J. Jersak, E. Laerman, P.M. Zerwas, *Phys. Rev.* D **25** (1980) 1218; A. Djouadi, *Z. Phys.* C **39** (1988) 561; A. Djouadi, B. Lampe, P. Zerwas, *Z. Phys.* C **67** (1995) 123; J.H. Kühn, P.Zerwas et al., in: *Z Physics at LEP 1*, CERN 89-08 (1989), eds. G. Altarelli, R. Kleiss, C. Verzegnassi, Vol. I, p. 267

[55] G. Altarelli, B. Lampe, *Nucl. Phys.* B **391**, 3 (1993) V. Ravindran, W. van Neerven, *Phys. Lett.* B **445**, 214 (1998); S. Catani, M. Seymour, *JHEP* **9907**, 023 (1999)

[56] W. van Neerven, these proceedings

[57] G. Passarino, these proceedings

[58] J. Erler, S. Heinemeyer, W. Hollik, G. Weiglein, P.M. Zerwas, *Phys. Lett.* B **486**, 125 (2000)