Pseudo versus Realistic Observables:

all that theories can tell us is how the world could be

Giampiero Passarino

Dipartimento di Fisica Teorica, Università di Torino, Italy

INFN, Sezione di Torino, Italy

Abstract

Fine points in phenomenological tools for the evaluation of realistic observables at Lep1 and their connection with the analysis of high-precision electroweak pseudo-observables are shortly discussed.

1 Prolegomena, comments and – perhaps – conclusions

The goal of this mini-workshop is to stimulate a critical discussion aimed at improving the understanding of errors in the theoretical calculations and experimental analyses of the high-precision electroweak data that are exploited to derive estimates of the Higgs mass in the Standard Model and beyond. The antefact is that the standard model has to survive a residual 96% probability of being false [1] and, among other things, we must examine anew the error analysis of all phenomenological tools that have been used.

During the Lep era the – relatively small – community of theorists trying to bridge the gap towards the experimental side of particle physics has made a plenty of mistakes and many things could have been done better, but this hardly explains the present would-be-crisis and we may conclude that the parts of our codes [2] dealing with realistic observables (RO) have been working properly.

For many years the fits have been telling us that any new theory is smootly joined to the minimal standard model (MSM), no edge no wedge; you do something else for a while and, all of the sudden, it is only 4% probability. What's going on?

As usual many authors will take care of speculations while few of them will be left behind the enemy's line to make sure that the dirty stuff is under control.

2 Mistakes and incompleteness: the Babel's tower

Among the items that should appear in this list it is worth mentioning the definition of mass for unstable particles; we insisted on using on-mass-shell masses in a situation where we had to use complex poles [3]. This is, perhaps, not a big deal with $M_{_Z}$ (input parameter) but it is a mess, now that we start having two-loop results [4], for $M_{_W}$ (a derived quantity)

Firthermore, semi-analytical codes could now be replaced by Monte Carlos, most likely with the same speed.

Nowadays the treatment of QED corrections would be even better [5]: better treatment for structure functions, also in those processes that are not dominated by annihilation; for initial/final interference; etc.

Finally, the whole procedure of de-convolution could be changed or made more efficient like for the (in)famous and well-known t-channel subtraction in Bhabha scattering [6]. Possibly we could agree on a signal defined by cuts and not by diagrams.

3 Quintessence of fits

Fits are (usually) performed from pseudo-observables (PO) $\rightarrow$ lagrangian parameters and RO are basically used to operate the procedure of de-convolution so, before discussing New Physics (at the tree level?) we must make a point in proving that all the ingredients are correct, are the whole answer or, at least, the bulk of it and do reflect the status of art.

Technically, we are forced to go from distributions through the PO to a LEP-combined set of PO, and then we interpret these within the frame-work of the MSM: to what extend are the PO *model-independent* measurements, valid even in the case of the SM being not the correct theory? Many effects are indeed absorbed in the PO, e.g. the Γ's are largely independent of QED or QCD; these only enter when interpreting the widths within the SM to extract its parameters.

The quantities that have been measured, $M_{_Z}, \Gamma_{_Z}$ etc have a dependence on the MSM and cannot be understood without it. In a sence, the measured value of $M_{_Z}$ does depend on our present-day understanding of the theory.

4 Underground work

A painful but necessary work [7] was, towards the end of the Lep era, devoted to fixing all the conventions/definitions regarding PO, RO $\rightarrow$ PO, SM remnants, occurence of imaginary parts, Z/γ interference etc, with special emphasis to the error analysis of phenomenological tools.

The experimental strategy can be summarized by saying that, technically, each LEP experiment extracts a set of POs, from their measured σs and $A_{_{\rm FB}}$s. The 4 sets are combined, taking correlated errors into account producing a LEP-average set of POs, $< PO >_{\rm Lep}$ then interpreted, e.g. within the MSM. The practical attitude is to stay with a model-independent (MI) fit, from ROs $\rightarrow$ POs ($\oplus$ a SM remnant) for each experiment,

and these sets of POs are averaged. The extraction of $M_{_Z}, m_t, M_{_H}$, $\alpha_{_S}(M_{_Z}^2)$ and $\alpha(M_{_Z}^2)$, is based on $< PO >_{\rm Lep}$.

Within the context of the SM the ROs are described in terms of some set of amplitudes

$$A_{\rm SM} = A_\gamma + A_{_Z} + \text{non-factorizable}, \tag{4.1}$$

$$\sigma(s) = \int dz\ H_{\rm in}(z,s)\, H_{\rm fin}(z,s)\, \hat{\sigma}(zs), \tag{4.2}$$

One needs to specify $M_{_Z}$, the (remaining) relevant SM parameters for the SM-complement, RO = PO $\oplus$ $\overline{\rm SM}$. This part of the procedure is particularly cumbersome. However, one has to live with the fact that – for practical reasons – this is the procedure and one is left with the task of making sure that it is acceptable.

5 PO - ology: the anti-realistic approach

The explicit formulae for the $Zf\overline{f}$ vertex are

$$\rho_{_Z}^f \gamma_\mu \left[\left(I_f^{(3)} + i\, a_{_L}\right)\gamma_+ - 2\, Q_f \kappa_{_Z}^f s^2 + i\, a_{_Q}\right] = \gamma_\mu \left(\mathcal{G}_{_V}^f + \mathcal{G}_{_A}^f\, \gamma_5\right), \tag{5.1}$$

where $\gamma_+ = 1 + \gamma_5$ and $a_{Q,L}$ are the SM imaginary parts. By definition, the total and partial widths of the Z boson include also QED and QCD corrections. Therefore

$$\Gamma_f \equiv \Gamma\left(Z \to f\overline{f}\right) = 4\, c_f\, \Gamma_0 \left(|\mathcal{G}_{_V}^f|^2\, R_{_V}^f + |\mathcal{G}_{_A}^f|^2\, R_{_A}^f\right) + \Delta_{\rm EW/QCD}\ , \tag{5.2}$$

where $c_f = 1$ or 3 for leptons or quarks ($f = l, q$), and $R_{_V}^f$ and $R_{_A}^f$ describe the final state QED and QCD corrections and take into account the fermion mass m_f. The last term,

$$\Delta_{\rm EW/QCD} = \Gamma^{(2)}_{\rm EW/QCD} - \frac{\alpha_{_S}}{\pi}\, \Gamma^{(1)}_{\rm EW}\ , \tag{5.3}$$

accounts for the non-factorizable corrections.

The standard partial width, Γ_0, is

$$\Gamma_0 = \frac{G_{_F} M_{_Z}^3}{24\sqrt{2}\,\pi} = 82.945(7)\,\text{MeV}. \tag{5.4}$$

The peak hadronic and leptonic cross-sections are defined by

$$\sigma_h^0 = 12\pi\, \frac{\Gamma_e \Gamma_{_h}}{M_{_Z}^2 \Gamma_{_Z}^2} \qquad \sigma_\ell^0 = 12\pi\, \frac{\Gamma_e \Gamma_l}{M_{_Z}^2 \Gamma_{_Z}^2}\ , \tag{5.5}$$

where $\Gamma_{_Z}$ is the total decay width of the Z boson, i.e, the sum of all partial decay widths.

The effective electroweak mixing angles (*effective sinuses*) are always defined by

$$4\,|Q_f|\sin^2\theta^f_{\rm eff} = 1 - \frac{\text{Re}\ \mathcal{G}_{_V}^f}{\text{Re}\ \mathcal{G}_{_A}^f} = 1 - \frac{g_{_V}^f}{g_{_A}^f}\ , \tag{5.6}$$

where we define

$$g_{V}^{f} = \mathrm{Re}\,\mathcal{G}_{V}^{f}, \qquad g_{A}^{f} = \mathrm{Re}\,\mathcal{G}_{A}^{f}. \tag{5.7}$$

The forward-backward asymmetry A_{FB} is defined via

$$A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}\,, \qquad \sigma_{\mathrm{T}} = \sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}\,, \tag{5.8}$$

where σ_{F} and σ_{B} are the cross sections for forward and backward scattering, respectively.

Before analysing the forward-backward asymmetries we have to describe the inclusion of imaginary parts. A_{FB} is calculated as

$$A_{\mathrm{FB}} = \frac{3}{4}\frac{\sigma_{\mathrm{VA}}}{\sigma_{\mathrm{T}}}\,, \tag{5.9}$$

where

$$\begin{aligned}\sigma_{\mathrm{VA}} = {} & \frac{G_{F} M_{Z}^{2}}{\sqrt{2}}\,\sqrt{\rho_{e}\rho_{f}}\,Q_{e}Q_{f}\mathrm{Re}\Big[\alpha^{*}(M_{Z}^{2})\mathcal{G}_{V}^{e}\mathcal{G}_{A}^{f}\chi(s)\Big] \\ & + \frac{G_{F}^{2} M_{Z}^{4}}{8\,\pi}\,\rho_{e}\rho_{f}\mathrm{Re}\Big[\mathcal{G}_{V}^{e}\left(\mathcal{G}_{A}^{e}\right)^{*}\Big]\mathrm{Re}\Big[\mathcal{G}_{V}^{f}\left(\mathcal{G}_{A}^{f}\right)^{*}\Big]\,s\,|\chi(s)|^{2}.\end{aligned} \tag{5.10}$$

To summarise the MI Ansatz: one starts with the SM, which introduces complex-valued couplings, calculated to some order in perturbation theory. Next we define g_{V}^{f}, g_{A}^{f} as the real parts of the effective couplings and Γ_{f} as the physical partial width absorbing all radiative corrections including the imaginary parts of the couplings and fermion mass effects. Furthermore,

$$R_{q} = \frac{\Gamma_{q}}{\Gamma_{h}}\,, \qquad R_{l} = \frac{\Gamma_{h}}{\Gamma_{l}}\,, \tag{5.11}$$

for quarks and leptons, respectively. The experimental collaborations report POs for the following sets:

$$(R_{f}, A_{\mathrm{FB}}^{0,f}), \qquad (g_{V}^{f}, g_{A}^{f}), \qquad (\sin^{2}\theta_{\mathrm{eff}}^{f}, \rho_{f}). \tag{5.12}$$

In order to extract g_{V}^{f}, g_{A}^{f} from Γ_{f} one has to get the SM-remnant, all else is trivial. However, the parameter transformation cannot be completely MI, due to the residual SM dependence.

In conclusion, the flow of the calculation requested by the experimental Collaborations is: pick the Lagrangian parameters m_{t}, M_{H} etc. for the explicit calculation of the residual SM-dependent part; perform the SM initialisation of everything, such as imaginary parts etc. giving, among other things, the complement $\overline{\mathrm{SM}}$; select g_{V}^{f}, g_{A}^{f}; perform a SM-like calculation of Γ_{f}, but using arbitrary values for g_{V}^{f}, g_{A}^{f}, and only the rest, namely

$$R_{V}^{f}, \quad R_{A}^{f}\,, \qquad \Delta_{\mathrm{EW/QCD}}\,, \qquad \mathrm{Im}\,\mathcal{G}_{V}^{f}, \quad \mathrm{Im}\,\mathcal{G}_{A}^{f}\,, \tag{5.13}$$

from the SM. An example of the parameter transformations is the following: starting from $M_{_Z}, \Gamma_{_Z}, R_{e,\mu,\tau}$ and $A_{\rm FB}^{0,e,\mu,\tau}$ we first obtain

$$\Gamma_e = M_{_Z}\Gamma_{_Z}\left[\frac{\sigma_h^0}{12\,\pi R_e}\right]^{1/2}, \qquad \Gamma_{_h} = M_{_Z}\Gamma_{_Z}\left[\frac{\sigma_h^0 R_e}{12\,\pi}\right]^{1/2}. \tag{5.14}$$

With

$$\mathcal{A}_e = \frac{2}{\sqrt{3}}\sqrt{A_{\rm FB}^{0,e}}\,, \qquad \text{and} \quad \gamma = \frac{G_{_F} M_{_Z}^3}{6\,\sqrt{2}\,\pi}\,, \tag{5.15}$$

we subtract QED radiation,

$$\Gamma_e^0 = \frac{\Gamma_e}{1+\dfrac{3}{4}\,\dfrac{\alpha(M_{_Z}^2)}{\pi}}\,, \tag{5.16}$$

and get

$$\begin{aligned}
\sin^2\theta_{\rm eff}^e &= \frac{1}{4}\left(1+\frac{\sqrt{1-\mathcal{A}_e^2}-1}{\mathcal{A}_e}\right), \\
\rho_e &= \frac{\Gamma_e^0}{\gamma}\left[\left(\frac{1}{2}-2\,\sin^2\theta_{\rm eff}^e\right)^2+\frac{1}{4}+\left(\mathrm{Im}\;\mathcal{G}_{_V}^e\right)^2+\left(\mathrm{Im}\;\mathcal{G}_{_A}^e\right)^2\right]^{-1}.
\end{aligned} \tag{5.17}$$

With

$$\mathcal{A}_f = \frac{4}{3}\,\frac{A_{\rm FB}^{0,f}}{\mathcal{A}_e}\,, \tag{5.18}$$

We further obtain

$$\sin^2\theta_{\rm eff}^f = \frac{1}{4\,|Q_f|}\left(1+\frac{\sqrt{1-\mathcal{A}_f^2}-1}{\mathcal{A}_f}\right), \tag{5.19}$$

$$\rho_f = \frac{\Gamma_f^0}{\gamma}\left[\left(\frac{1}{2}-2|Q_f|\,\sin^2\theta_{\rm eff}^f\right)^2+\frac{1}{4}+\left(\mathrm{Im}\;\mathcal{G}_{_V}^f\right)^2+\left(\mathrm{Im}\;\mathcal{G}_{_A}^f\right)^2\right]^{-1}, \tag{5.20}$$

where $f=\mu,\tau$. For quarks one should remember to subtract first non-factorizable terms and then to distinguish between $R_{_V}^f$ and $R_{_A}^f$.

6 RO - ology: the structuralist approach

RO are notoriously more difficult to compute than PO and we further distinguish between $RO_{\rm ext}$, corresponding to an (idealized) extrapolated setup and $RO_{\rm real}$ corresponding to some (simplified) realistic set of kinematical cuts.

The ROs are computed in the context of the SM. Thus the comparison between `TOPAZ0` and `ZFITTER` is mainly a SM comparison, however one of the goals was to pin down the

definition of POs; the calculation of ROs in terms of the defined POs for the purpose of MI fits, showing that for POs with values as calculated in the SM, the ROs are by construction identical to the full SM RO calculation.

The last point requires expressing ρ's and effective mixing angles in terms of POs, assuming the validity of the SM. After this transformation the ROs will be given as a function of the POs at their SM values.

This is not at all a trivial affair because of gauge invariance and one should remember that gauge invariance at the Z pole (on-shell gauge invariance) is entirely another story from gauge invariance at any arbitrary scale (off-shell gauge invariance). Some of the re-summations that are allowed at the pole and that heavily influence the definition of effective Z couplings are not trivially extendible to the off-shell case.

Therefore, the expression for RO=RO(PO), at arbitrary s, requires a careful examination and should be better understood as RO=RO(PO,$\overline{\text{SM}}$), that is, for example:

$$\sigma_{\text{MI}} = \sigma_{\text{SM}} \left(R_l, A_{\text{FB}}^{0,l}, \ldots \to g_V^f, g_A^f \to \rho_f, \sin^2\theta_{\text{eff}}^f; \text{residual SM} \right) . \tag{6.1}$$

As long as the procedure does not violate gauge invariance and the POs are given SM values, there is nothing wrong with the calculations. It is clear that in this case the SM ROs coincide with the MI ROs.

7 Ingredients for ROs: next-to-leading and mixed corrections

The inclusion of mixed two-loop correction for RO, at $s \neq M_Z^2$, can only represent an approximation to the real answer. One should realize that s' is not equivalent to the invariant mass of the final-state $f\overline{f}$ system, $M^2(f\overline{f})$, due to final state QED and QCD radiation.

For an s'-cut both QED and QCD final-state radiation are included through an inclusive correction factor. For M^2-cut the result remains perfectly defined for leptons, however for hadronic final states there is a problem. This has to do with QCD final-state corrections.

Indeed we face the following situation [8]:

- for $e^+e^- \to f\overline{f}\gamma$ the exact correction factor is known at $\mathcal{O}(\alpha)$ even in the presence of a $M^2(f\overline{f})$ cut;
- the complete set of final-state QCD corrections are known up to $\mathcal{O}\left(\alpha_s^3\right)$ only for the fully inclusive setup, i.e., no cut on the $f\overline{f}$ invariant mass;
- the mixed two-loop QED/QCD final-state corrections are also known only for a fully inclusive setup;
- at $\mathcal{O}(\alpha_s)$ QCD final-state corrections in presence of a M^2-cut follow from the analogous QED calculation.

The ideal thing would be to have QED $\oplus$ QCD final-state *radiator factors* R, with a kinematical cut imposed on $M^2(f\overline{f})$. Missing this calculation, we have three options

$$R^{\text{FS}} = 1 + R^{\text{FS}}_{\text{QED,cut}} + R^{\text{FS}}_{\text{QCD,ext}} ,$$

$$R^{\rm FS} = \left(1 + R^{\rm FS}_{\rm QCD,ext}\right)\,\left(1 + R^{\rm FS}_{\rm QED,cut}\right)\,,$$
$$R^{\rm FS} = 1 + R^{\rm FS}_{\rm QED,cut} + R^{\rm FS}_{\rm QCD,cut}\,. \tag{7.1}$$

The (QCD,ext) corrections are understood up to $\mathcal{O}\left(\alpha_{_S}^3\right)$, while those corresponding to the (QCD,cut) setup are only computed at $\mathcal{O}\left(\alpha_{_S}\right)$.

8 Deconvolution: a matter of public anxiety

There are several levels of de-convolution and the most important are: single-de-convolution (SD), giving the kernel cross-sections without initial state QED radiation, but including all final-state correction factors; double-de-convolution (DD), giving the kernel cross-sections without initial- and final-state QED radiation and without any final-state QCD radiation.

By comparing SD with DD quantities we are able to disentangle the effect of initial-state QED radiation from final-state QED $\oplus$ QCD radiation. Note the following relation between the PO R_b and the ratio of DD cross-sections (exp $= 0.21644 \pm 0.00065$)

$$R_b = \frac{\sigma_b}{\sigma_{\rm had}}\Big|_{\sqrt{s}=M_{_Z}} - \begin{cases} 0.00146 \ \texttt{TOPAZ0} \\ 0.00146 \ \texttt{ZFITTER}\,. \end{cases} \tag{8.1}$$

The difference reflects the $\overline{SM}$-remnant effect, since the ratio of RO cross-sections has γ-exchange, imaginary parts, ..., and (substantially negligible) weak boxes.

9 Rating the realistic observables

$\star\star\star$ final state QED

$\star\star\star$ $M_{_H}$-dependence of deconvoluted observables

$\star\star$ convoluted observables, extrapolated setup

$\star\star$ $M_{_H}$-dependence of convoluted observables

$\star$ convoluted observables, realistic setup

$\star$ convoluted observables, model independent approach

In the previous list rating must be understood as compared to the experimental error.

10 Ontology: the Blue Band

The most celebrate figure of the LEP era is the blue-band. I remember a meeting at Cern where I proposed to produce theoretical results with an *error* band, reflecting our lack of knowledge of missing higher order corrections, instead of dimensionless points. There was an immediate consensus in the community [9]. This is the progenitor of the blue-band.

This band was intended to honestly show our degree of ignorance and, several times, it was repeated that it should be used and interpreted with great care. Actually there is no definition of *theoretical error* (only of theoretical stupidity) and one should not attach to it any meaning more deep than modelling and selecting a set of options and see how large is the band, If it is too large then we better do a new calculation in that direction. If it is small yet it does not mean that we should take it as a rigorous bound.

From this point of view I disagree *ab initio* on having any discussion of this kind, one should even remember that some of the options in the codes have been buried and forgotten simply because they were not following the orthodoxy giving a too large band. We defintely need a complete two-loop calculation for some or all the PO and the fact that these calculations are not coming yet is a sign that they are not easy at all. I have no objection in having right now a larger blue-band (why not a safety factor?) but I refuse to accept incomplete arguments as the rationale for rigorously justifying the enlargement.

10.1 Logic of illogicality: chop the band?

Should a theoretical calculation which is running over the Higgs mass and assumes the validity of the SM include everything that is allowed despite experimental evidence? No, absence of a very light Higgs should be used and the $\Delta\chi^2$ curve should be replaced with another one where a penalty function is included and we never move into the forbidden region.

11 Autopsy: should we retreat from a metaphysics of fits entirely?

Another approach exists, extraction of lagrangian parameters directly from the RO, which are not (of course) raw data but rather educated manipulations of raw data, e.g. distributions defined for some simplified setup. I have nothing to say about this first part, theorists should do theory and experimentalists should do experiments. I have been asking so many times to produce this kind of fits that at the end I managed to get some answer. From the 2001 line-shape analysis [10] we obtain

	A	D	L	O
$\chi^2/N_{\rm df}$	174/180	184/172	168/170	161/198

Δ LP	% of error
$\Delta M_{_Z}$ [MeV]	1
$\Delta \log_{10}(M_{_H}/\text{GeV})$	4
$\Delta\alpha_{_S}$	4
$\Delta(\Delta^{(5)}_{\rm had})$	2

This is some shift but hardly explains the present paroxysmal attack of distress and tell us that for all what we know the RO - branches of our codes have been working properly. In the line-shape sector we have a reasonable $\chi^2/N_{\rm df}$ for each Lep experiment and shifts on lagrangian parameters correspond to few % of the experimental error (although $M_{_H}$ ...). It would be interesting to have the possibility of performing the same study for all remaining data where $\chi^2/N_{\rm df} = 27.7/13$. The comment is that we have Lep line-shape data under control, but what about ROs for non-Lep data? What about A_{LR} realistic and its $\overline{\rm SM}$?

Finally, as a short comment on theoretical accuracy we recall that cross sections generated with TOPAZ0 at several energy points, with errors assigned such as to represent the experimental statistics, plus correlated luminosity errors, were run through an MI fit with ZFITTER. The agreement of the parameters is remarkable.

12 Conclusion

There are few comments that can be made, some of then positive and some negative. Undoutedly the level of accuracy and the architectural complexity of theoretical calculations has no comparison with the past. However, two-loop (heavy top) next-to-leading corrections are missing in the b - quark sector; furthermore, the available two-loop expression for $M_{_W}$ cannot be directly used for $\sin^2\theta^l_{\rm eff}$ which remains unavailable.

In general we may conclude that ROs are pretty well known around the Z peak, reasonably well up to LEP 2 energies; we have been constantly aware of our mistakes and limitations and they have been under control at the required level of precision, at least for Lep processes. Examples are given in the following short list: $t-t$ and $s-t$ components in Bhabha scattering are included in $\overline{\rm SM}$; in model independent fits $\{m_t, \alpha_{_S}, M_{_H}\}_{\overline{\rm SM}}$ are fixed and by default $RO = RO({\rm Re}PO)$.

Negative aspects are mostly represented by two minuses: one-loop ROs have not been generated beyond the two fermion final state during the LEP cycle and the building of the full two-loop set of electroweak corrections is still in its early infancy.

Finally, another negative aspect derives from the feeling that here we are to bury theory and not to prize it.

References

[1] M. Grünewald, these Proceedings.

[2] G. Montagna, F. Piccinini, O. Nicrosini, G. Passarino and R. Pittau, Comput. Phys. Commun. **76** (1993) 328;
G. Montagna, F. Piccinini, O. Nicrosini, G. Passarino and R. Pittau, Nucl. Phys. B **401** (1993) 3;
D. Y. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann and T. Riemann, Comput. Phys. Commun. **133** (2001) 229 [arXiv:hep-ph/9908433].

[3] R. G. Stuart, Nucl. Phys. B **498** (1997) 28 [arXiv:hep-ph/9504215];
A. Sirlin, Phys. Rev. Lett. **67** (1991) 2127;
W. Beenakker *et al.*, Nucl. Phys. B **500** (1997) 255 [arXiv:hep-ph/9612260].

[4] A. Freitas, W. Hollik, W. Walter and G. Weiglein, Nucl. Phys. B **632** (2002) 189 [arXiv:hep-ph/0202131].

[5] G. Passarino, Nucl. Phys. B **619** (2001) 313 [arXiv:hep-ph/0108255];
Z. Was, S. Jadach and B. F. Ward, in *Proc. of the 5th International Symposium on Radiative Corrections (RADCOR 2000)* ed. Howard E. Haber, arXiv:hep-ph/0101246;
S. Jadach, B. F. Ward and Z. Was, Nucl. Phys. Proc. Suppl. **89** (2000) 106 [arXiv:hep-ph/0012124].

[6] W. Beenakker and G. Passarino, Phys. Lett. B **425** (1998) 199 [arXiv:hep-ph/9710376].

[7] D. Y. Bardin, M. Grunewald and G. Passarino, arXiv:hep-ph/9902452;
D. Y. Bardin and G. Passarino, arXiv:hep-ph/9803425.

[8] K. G. Chetyrkin et al. in 'Reports of the Working Group on Precision Calculations for the Z Resonance', D. Bardin, W. Hollik and G. Passarino eds., CERN-95-03, p. 175.

[9] D. Y. Bardin *et al.*, arXiv:hep-ph/9709229.

[10] [LEP Collaborations], arXiv:hep-ex/0101027.