Measurement of the W mass at LEP

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Abstract

The measurement of the W boson mass at LEP is discussed in detail, paying particular attention to the systematic errors which currently limit the precision and the prospects for improvement. An outlook for the future is also given.

1 Introduction

The precise measurement of m_{W}, the mass of the W boson, is of great importance in precision tests of the Standard Model of electroweak interactions [1], and is one of the major goals of the CERN LEP2 physics program. At the LEP $\mathrm{e^+e^-}$ collider, W bosons are mainly produced in pairs, via t-channel electron-neutrino exchange or s-channel $\mathrm{Z^0}/\gamma$ exchange. The subsequent decay of the W bosons into quark pairs ($\mathrm{q\bar{q}}$) 68 % and lepton pairs ($\mathrm{e}\nu_{\mathrm{e}}$, $\mu\nu_\mu$, $\tau\nu_\tau$) 3×11 % of the time leads to three distinct final state topologies: the four quark ($\mathrm{q\bar{q}q\bar{q}}$) channel in which both W bosons decay hadronically, the semileptonic ($\mathrm{q\bar{q}}\ell\nu$) channel, and the fully leptonic ($\ell\nu\ell\nu$) channel, in which both decay leptonically. Most of the experimental information on the W mass comes from the first two channels, as the two undetected neutrinos produced in $\mathrm{WW} \to \ell\nu\ell\nu$ events mean there is not enough information to reconstruct the W mass without making additional assumptions.

Backgrounds to WW production are dominated by other four-fermion final states, including those involving ZZ production, and particularly in the $\mathrm{q\bar{q}q\bar{q}}$ channel, $\mathrm{q\bar{q}}$ production via $\mathrm{Z^0}/\gamma$ exchange, where four-jet final states are produced through gluon radiation from the final state quarks. The experiments typically achieve signal purities of more than 95 % in the $\mathrm{q\bar{q}}\ell\nu$ channel and around 80 % in the $\mathrm{q\bar{q}q\bar{q}}$ channel.

2 Measuring the W mass

The first measurement of the W mass at LEP was obtained by measuring the WW production cross section close to threshold. Assuming the Standard Model dependence of the cross-section on the W mass, a result of

$$m_{\mathrm{W}} = 80.40 \pm 0.20(\mathrm{stat}) \pm 0.07(\mathrm{syst}) \pm 0.03(\mathrm{E_{beam}})\,\mathrm{GeV}$$

was obtained from approximately $10\,\mathrm{pb^{-1}}$ of data per experiment taken at a centre-of-mass energy ($\sqrt{s}$) of 161 GeV in 1996 [1]. The third error originates from the limited knowledge of the LEP beam energy.

After this initial running, each experiment recorded around 700 pb^{-1} of data at centre-of-mass energies ranging from 172 to 209 GeV, corresponding to around 10 000 W-pairs each. At these energies, the dependence of the WW production cross-section on the W mass is very limited, and a much more precise result can be obtained from direct reconstruction of the invariant mass of the W boson from its decay products on an event-by-event basis. The resolution is greatly improved by using a kinematic fit of the decay products, imposing the constraints of energy and momentum conservation and exploiting the precise knowledge of the LEP beam energy. Additionally, since the event-by-event resolution on the W mass is typically substantially larger than the natural W width (around 2 GeV), the two W masses in an event are often constrained to be equal.

In the $\mathrm{q\bar{q}}\ell\nu$ channel, the neutrino energy and direction is unknown, leaving one constraint (two if the two masses are assumed to be equal). In the $\mathrm{q\bar{q}}\mathrm{e}\nu$ and $\mathrm{q\bar{q}}\mu\nu$ subchannels, the background from non-WW final states is very low and the event-by-event resolution is typically around 3–5 GeV. In $\mathrm{q\bar{q}}\tau\nu$ events, the lepton energy cannot be precisely reconstructed due to the neutrino produced in the τ decay, and mass information is usually extracted from the hadronic W decay alone, leading to somewhat worse resolution and higher background.

The kinematics in the $\mathrm{q\bar{q}q\bar{q}}$ channel are in general more constrained due to the absence of neutrinos, and the event-by-event resolution is typically around 3 GeV. However, the decay of both W bosons to quarks means that at least four jets are usually reconstructed, leading to a three-fold pairing ambiguity over which jets to associate to which W decay. This choice cannot usually be made uniquely, leading to several possible assignments per event and a combinatorial background resulting from incorrect jet pairings. In a substantial fraction of $\mathrm{WW} \rightarrow \mathrm{q\bar{q}q\bar{q}}$ events, at least one quark radiates an energetic gluon, leading to topologies which are best reconstructed as five jet events, giving increased combinatorial background. These complications, together with the irreducible background from $\mathrm{ZZ} \rightarrow \mathrm{q\bar{q}q\bar{q}}$ events and four-jet Z^0/γ decays, largely negate the advantages from better intrinsic resolution, and the statistical power of the $\mathrm{q\bar{q}q\bar{q}}$ channel is in fact rather similar to that of the $\mathrm{q\bar{q}}\ell\nu$ channel.

Some typical reconstructed mass distributions are shown in Figure 1. In all channels, the widths of the reconstructed distributions are significantly wider than the natural W width due to detector resolution effects, and there are substantial distortions due to *e.g.* initial state radiation photons escaping detection along the beampipe, which are generally not accounted for by the kinematic fits. Sophisticated fit procedures, calibrated using large samples of Monte Carlo events, are therefore used to extract the fitted W mass from the raw distributions. Techniques include reweighting Monte Carlo 'template' samples such that they best describe the observed data distribution, describing the observed distribution in terms of a convolution of detector resolution and W lineshape physics functions and simple parameterised fits to the data distribution. In all cases, the data is normally broken up into different categories based on the estimated precision of the event-by-event mass in order to extract the maximum possible information from the data.

The four LEP experiments have presented preliminary results based on the entire LEP2 datasample (apart from the OPAL data taken during the last year of LEP running) [2, 3]. The results have been combined by the LEP electroweak working group, taking correlated systematic uncertainties into account [1, 4]. A breakdown of the current statistical and

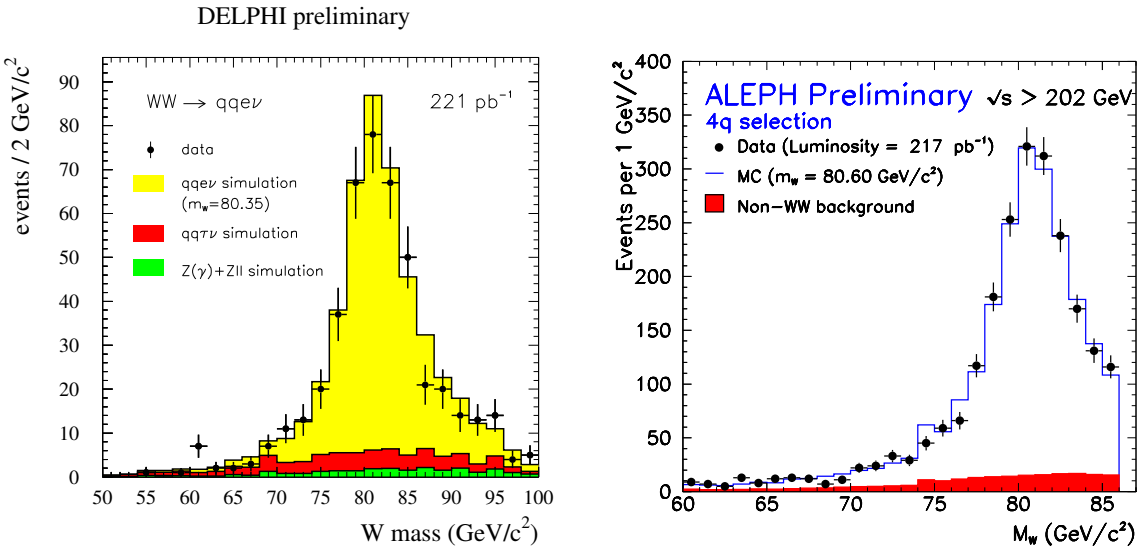

Figure 1: Reconstructed W mass distributions from DELPHI $q\bar{q}\ell\nu$ (left) and ALEPH $q\bar{q}q\bar{q}$ (right) events.

Source	$q\bar{q}\ell\nu$	$q\bar{q}q\bar{q}$	Combined
Hadronisation	19	18	18
Colour reconnection	-	90	9
Bose-Einstein correlations	-	35	3
Detector effects	14	10	14
Photon radiation	8	8	8
Beam energy	17	17	17
Other (uncorrelated)	4	5	4
Total systematic	31	101	31
Statistical	32	35	29
Total	44	107	43
Statistical (no systematics)	32	28	21

Table 1: Breakdown of systematic and statistical error sources (in MeV) for the LEP W mass measurement. The statistical error in the absence of all systematics is also shown.

systematic uncertainties is given in Table 1, for both the $q\bar{q}\ell\nu$ and $q\bar{q}q\bar{q}$ channels and their combination. For the $q\bar{q}\ell\nu$ channel, the systematic uncertainties are dominated by hadronisation, detector effects and the LEP beam energy. The $q\bar{q}q\bar{q}$ channel is dominated by the large uncertainties due to final state interactions between the decay products of the two W bosons, *i.e.* colour reconnection and Bose-Einstein correlation. This reduces the weight of the $q\bar{q}q\bar{q}$ channel in the combined average to only 0.10, even though the statistical precision is slightly better than that of the $q\bar{q}\ell\nu$ channel.

Ideally, the kinematic fits to extract the W mass would be performed directly with the four-vectors of the final state fermions, but the analyses have to be based on the reconstructed jets and leptons observed in the detector. The final state quarks undergo

gluon radiation and eventually hadronise into jets of particles, some of which decay before being observed in the detector. The reconstructed particles are then recombined into jets with calibrated energy scales which are finally used as input to the kinematic fit. For leptons, the most important uncertainties come from final state photon radiation and detector calibration. Monte Carlo simulations are used to understand all stages of the process, and have to be cross-checked using data wherever possible. The various systematic uncertainties that arise are discussed in more detail below.

3 Detector effects

In order to understand and calibrate the detector response to jets and leptons, the experiments use large samples of Z^0 resonance data (typically 50–100 k events) taken at various points during each year of LEP2 running. Jet energy scales and resolutions can be studied using $Z^0 \to q\bar{q}$ events, where the two jets from the Z^0 decay are typically produced back to back. Corrections are applied to Monte Carlo events so that they model the Z^0 data well, and these corrections are then propagated to the WW Monte Carlo samples used in the extraction of the W mass. Jet energy scales are typically understood to a precision of 0.3-0.5 %, limited by data statistics and residual differences between Z and W events (*e.g.* b flavour content).

Dependencies of the jet energy scale on the energy itself (so called 'non-linearities') are also important, and studied by means of $Z^0 \to q\bar{q}g$ 3-jet events, where the jet energies can be calculated from the inter-jet angles, and high energy $q\bar{q}$ events. Another significant source of uncertainty comes from polar angular biases in the detector, typically controlled by comparing angular reconstruction in the tracking chambers and calorimeters. Similar procedures employing leptonic Z^0 decays are used to understand the energy and angular reconstruction for leptons in the $q\bar{q}\ell\nu$ channel. Again, the final uncertainties are limited by residual differences between data and Monte Carlo, and Z^0 data statistics.

4 Hadronisation

Hadronisation uncertainties enter between the final state quarks in $W \to q\bar{q}$ decays and the subsequent production of jets of hadrons reconstructed in the detector. The experiments use different hadronisation models (namely Pythia, Ariadne and Herwig [5]) and different model tunes to estimate the uncertainties. All the models have been tuned separately by each of the experiments using Z^0 data, putting different emphasis on the description of various event properties (event shapes, individual particle rates, fragmentation functions *etc.*). Combined studies by the four experiments have demonstrated that these tunes predict W mass biases that differ by 10–50 MeV, and that these shifts are reproducible between the different experiments and analysis methods. It is therefore essential to understand which event properties most directly influence the reconstructed W mass.

Some of the differences have been understood as originating from differences in the kaon and baryon production rates in the various hadronisation models. Such particles are generally not reconstructed with the correct invariant mass, or in the case of neutral hadrons, with the correct energy. Monte Carlo corrections for these effects obviously

depend on the predicted rates, and the uncertainties in the measured data production rates and extrapolation from Z to W decay contribute to the uncertainty on the W mass.

An ingenious approach known as 'mixed Lorentz boosted Z' is employed by DELPHI to study both hadronisation and some detector effects. Two separate $Z^0 \to q\bar{q}$ events are boosted and combined to make a $WW \to q\bar{q}q\bar{q}$ event, with kinematics modelled on a simulated 'template' W-pair event. This procedure is applied consistently to both data and Monte Carlo Z^0 events, the standard W mass analysis applied, and any differences studied. The technique allows studies to be made directly on data, and also allows the influence of any differences seen in Z^0 data and Monte Carlo to be easily transported to W-pair events.

5 Final state interactions

Standard hadronisation models assume the decay products of the two W bosons in a $WW \to q\bar{q}q\bar{q}$ event do not interact, but the two bosons are typically separated in space by only 1 fm—a distance over which strong interactions are relevant. Two possible effects have been studied—colour reconnection (CR), in which colour is exchanged between the decay products of the two W bosons (either in the perturbative or non-perturbative hadronisation phases), and Bose-Einstein correlations (BEC) affecting pairs of identical bosons originating from different Ws. In both cases, the invariant mass spectra of the decay products of the two W bosons can be distorted, leading to a biased mass reconstruction in the $q\bar{q}q\bar{q}$ channel.

Several theoretical models of CR in W-pair events exist, of which the most commonly used are the SK1, Ariadne 2 (AR2) and Herwig models [5, 6]. The SK1 model is based on the Lund string fragmentation model as implemented in Pythia. Colour reconnection occurs when string segments overlap, with a probability dependent on the space-time overlap of the strings and an adjustable normalisation parameter, which varies the overall amount of colour reconnection in the model. In the Ariadne dipole cascade model, reconnection occurs when colour strings cross and the total string length would be reduced by the reconnection. In the commonly used AR2 variant, reconnection is only allowed between Ws for gluons with energies smaller than the W width. Finally, in the Herwig CR model, reconnection is allowed for favourable parton-cluster assignments, with a fixed probability of 1/9. These models predict W mass shifts of up to 300 MeV for the extreme version of the SK1 model with full reconnection, around 100 MeV for AR2 and 30 MeV for Herwig.

Initially, colour reconnection effects were searched for by comparing W decay properties such as particle multiplicities, fragmentation functions and event shapes in $q\bar{q}\ell\nu$ and $q\bar{q}q\bar{q}$ events. No significant effects were seen, but the sensitivity to the 'realistic' models discussed above is low. Recent analyses have focused on looking at the effects of CR on particle flow in the regions between the jet cores in $WW \to q\bar{q}q\bar{q}$ events. Using procedures similar to those employed for the W mass measurement, jets are assigned to the two W bosons, and the ratio of particle flow in 'intra-W' regions (*i.e.* between two jets assigned to the same W) to that in 'inter-W' regions (between jets assigned to different Ws) studied.

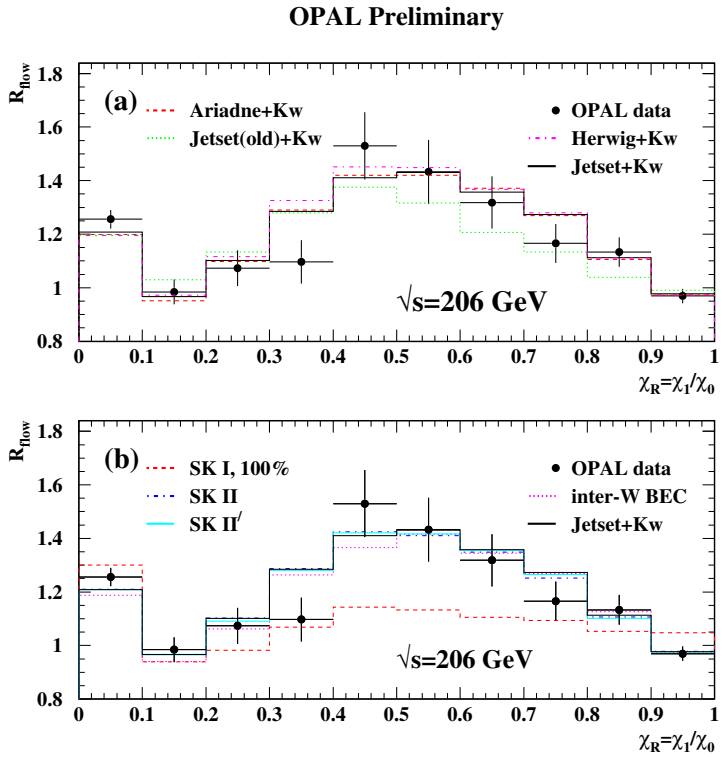

Figure 2: Distributions of the ratio of particle flow between jets in inter-W and intra-W regions. OPAL data is compared to the predictions of various models without (top) and with (bottom) colour reconnection.

An example of such an analysis from OPAL is shown in Figure 2 [7]. The ratio R of intra-W to inter-W particle flow is shown as a function of the 'reduced' inter-jet angle χ_R, where jet cores are located at $\chi_R = 0$ and 1. The top plot shows the data distribution compared to various hadronisation models without CR, and the bottom plot shows the same data compared to models with CR. The SK1 model with 100 % CR predicts a smaller value of R between the jets (*i.e.* more particle flow between jets from different Ws), and is clearly disfavoured. Similar results have been obtained by the other LEP experiments [8].

To quantify the difference between data and any Monte Carlo model, the R distribution is integrated over the range $0.2 < R < 0.8$ (to exclude the jet cores), and r defined as the ratio of data to Monte Carlo without CR. Values of r consistent with one indicate that the data is well described by models without CR, whilst CR models generally predict values smaller than 1 (the exact value depends on details of each experiment's analysis). The experimental results for r in the framework of the SK1 and AR2 models are shown in Figure 3, together with the LEP combined values taking correlated systematic effects into account [1]. The data disfavour the no-CR scenario at a level of around two standard deviations, but the 100 % SK1 scenario is excluded at more than five standard deviations. On the other hand, the method shows little sensitivity to the AR2 model, which predicts an r value very close to that of models without CR.

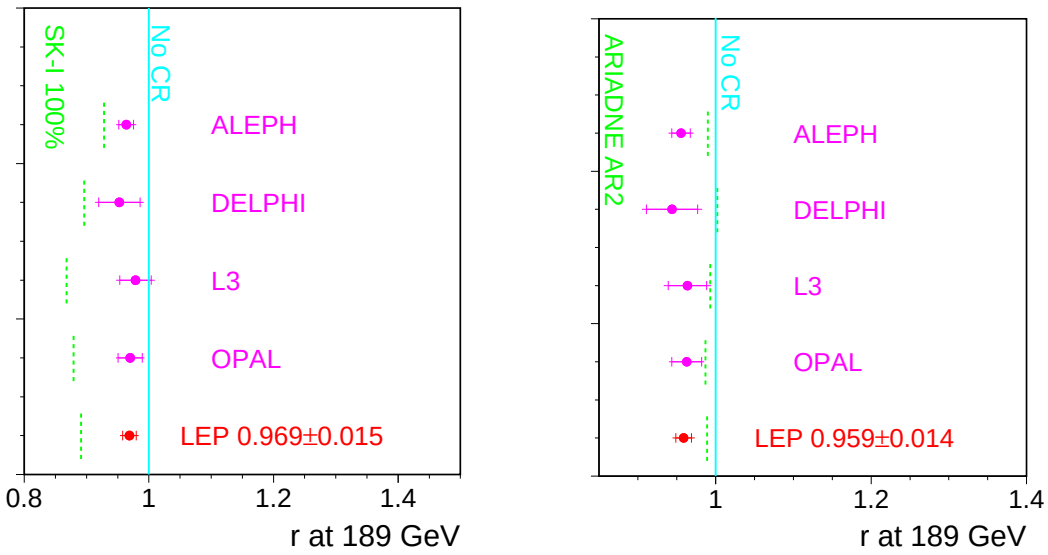

Figure 3: Measured and LEP combined particle flow ratio r values, compared to the predictions of the SK1 and AR2 colour reconnection models.

The SK1 model has a free parameter allowing the amount of CR to be adjusted to optimise the agreement with the measured r value. From the LEP combined results, the 68 % confidence limit on the colour reconnection probability p_{reco} is found to be $0.25 < p_{\mathrm{reco}} < 0.65$. The upper end of this range corresponds to a W mass shift in the $\mathrm{q\bar{q}q\bar{q}}$ channel of 90 MeV, and this is now used to define the systematic uncertainty due to CR in the $\mathrm{q\bar{q}q\bar{q}}$ channel (see Table 1). The AR2 model predicts a similar mass shift, whilst the Herwig CR model predicts a much smaller shift.

The $\mathrm{q\bar{q}q\bar{q}}$ channel is also affected by Bose-Einstein correlations, which lead to an enhancement of the production of pairs identical bosons close in phase space. This effect has been clearly observed in both Z and W decays, and may also correlate pairs of particles produced from different W bosons ('inter-W' BEC). Simulations using the Lund PYBOEI model [9] implemented in the framework of PYTHIA and tuned on Z^0 decays suggest W mass shifts of around 35 MeV—significant but much smaller than those from CR. The LEP experiments have searched for evidence of inter-W BEC using an event mixing technique—comparing $\mathrm{WW} \to \mathrm{q\bar{q}q\bar{q}}$ events with the hadronic parts of two $\mathrm{q\bar{q}}\ell\nu$ events appropriately combined together. A first LEP combination based on results from ALEPH and L3 gives a measurement of the strength of inter-W BEC of 0.03 ± 0.18 times than predicted by PYBOEI [1]. This suggests inter-W BEC is unlikely to be strong enough to significantly perturb the $\mathrm{q\bar{q}q\bar{q}}$ W mass, but more work, in particular including the results from DELPHI and OPAL, is required before drawing definite conclusions.

6 Theoretical uncertainties

The main theoretical uncertainties in the measurement of the W mass centre around the treatment of $O(\alpha)$ photon radiation from the initial state, the W bosons, the final state particles and the interference between them. The published W mass results are generally

based on the KoralW Monte Carlo program, but two more sophisticated treatments, KandY and RacoonWW using the leading- or double-pole approximations to handle photon radiation from the Ws have recently become available [10]. Interfacing these generators to the experimental simulation is non-trivial, in particular because photon radiation from final state quarks is generally handled as part of the hadronisation process, not the electroweak generator. The additional effects implemented in these generators give W mass shifts on the order of 10 MeV, but several possibly cancelling effects are involved, and detailed evaluations of the theoretical uncertainty are still ongoing.

7 LEP beam energy

The beam energy constraint used in the W mass kinematic fits considerably improves the event-by-event mass resolution. At LEP1, the beam energy was measured to 1–3 MeV using resonant depolarisation, but it was not possible to polarise the LEP beams above about 60 GeV. The LEP2 beam energy measurement therefore relies on precise polarisation-based measurements at 40–60 GeV extrapolated up to the running energy of around 100 GeV using the proportionality of the beam energy and the bending field. The latter is measured using a set of precise NMR probes installed in the LEP ring, with a flux loop measurement as a cross check. The current uncertainty on the beam energy is 20 MeV, dominated by uncertainties in the extrapolation. This gives a 17 MeV uncertainty on the W mass.

Several alternative methods have been developed to cross-check the beam energy derived from the polarisation/NMR extrapolation technique. A dedicated spectrometer, consisting of a precisely mapped steel dipole equipped with beam position monitors to measure the resulting beam deviation was inserted in the LEP ring for the last few years of running. The dependence of the synchrotron tune on the applied RF accelerating voltage provides another method to determine the beam energy, which was exploited during a series of dedicated machine experiments. Finally, the LEP detectors have themselves measured the beam energy using 'radiative return' events, $e^+e^- \to Z^0\gamma$. Kinematic fitting of the Z^0 decay products (quark or lepton pairs) enables the Z^0 mass to be determined, and comparing this with the precise Z^0 mass measurement from LEP1 allows a cross-check of the assumed beam energy. All these methods give alternative measurements of the LEP beam energy with useful precision, and agree with the standard technique. As the beam energy measurements are finalised, it is therefore possible that the current uncertainty on m_W of 17 MeV will be reduced.

8 Results

The individual and combined LEP W mass results in the $q\bar{q}\ell\nu$ and $q\bar{q}q\bar{q}$ channels are shown in Figure 4 [2, 3]. Compared to the results presented at the summer 2002 conferences, the only difference is due to an update of the ALEPH preliminary result [2], correcting an inadequacy in their calorimeter simulation which was not taken into account in their previous W mass analysis. This shifts the ALEPH $q\bar{q}\ell\nu$ and $q\bar{q}q\bar{q}$ mass values by -81 and -74 MeV.

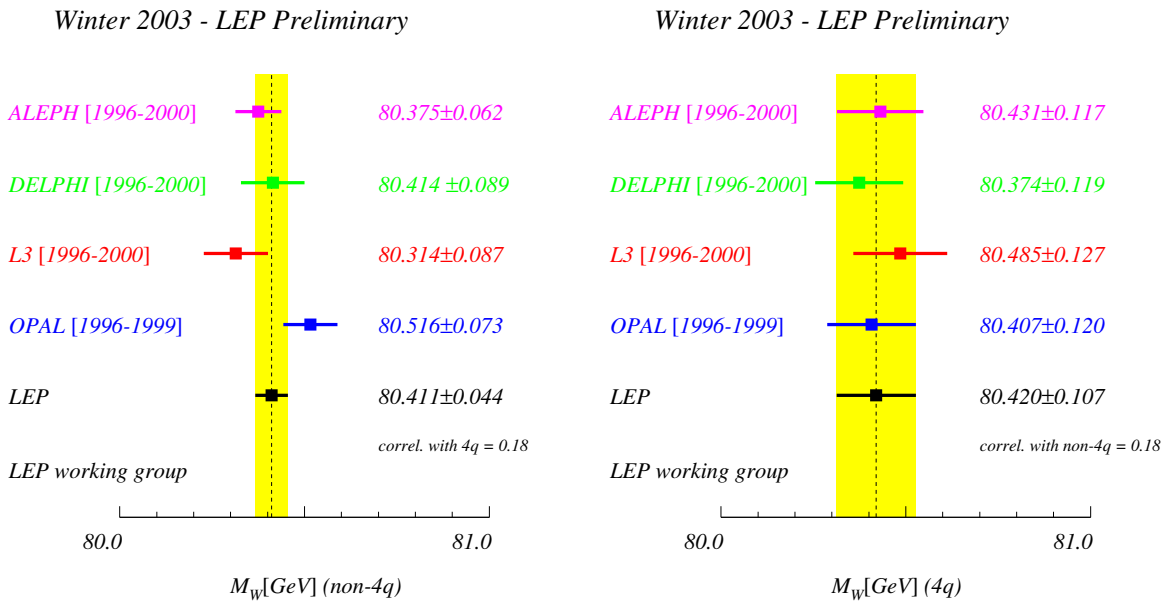

Figure 4: Individual and combined LEP W mass results in the $q\bar{q}\ell\nu$ and $q\bar{q}q\bar{q}$ channels.

Including the result from the threshold analysis at 161 GeV, the combined LEP W mass result is

$$m_{\mathrm{W}} = 80.412 \pm 0.029 \pm 0.031\,\mathrm{GeV}$$

where the first error is statistical and the second systematic [4]. The masses measured in the $q\bar{q}q\bar{q}$ and $q\bar{q}\ell\nu$ channels are consistent, giving no indication that the $q\bar{q}q\bar{q}$ mass is shifted due to final state interaction effects. If the systematic errors due to CR and BEC are set to zero, the mass difference $\Delta m_{\mathrm{W}}(q\bar{q}q\bar{q} - q\bar{q}\ell\nu)$ is 22 ± 43 MeV.

The result from LEP can be combined with the W mass measured at $p\bar{p}$ colliders, $m_{\mathrm{W}} = 80.454 \pm 0.059$ GeV [11] to give a world average value of

$$m_{\mathrm{W}} = 80.427 \pm 0.034\,\mathrm{GeV}\,.$$

This result is 22 MeV lower than the world average in summer 2002, again due to the updated value from ALEPH.

9 Outlook for the future

The current LEP W mass uncertainty from the preliminary results is 42 MeV, split approximately equally between statistical and systematic effects. The focus of the work is now to reduce the systematic uncertainties before finalising the results. Since the $q\bar{q}q\bar{q}$ channel is strongly affected by the possible presence of colour reconnection, one promising avenue is to reduce the analyses sensitivity to CR by removing soft particles and those far from the jet cores. If this effort is successful, theoretical uncertainties can be kept under control and the error on the LEP beam energy reduced, a final m_{W} uncertainty from LEP of 35 MeV may be feasible.

Beyond LEP, the current Tevatron uncertainty of 59 MeV should be substantially reduced by Run II, which will bring much larger data samples, improved detectors and better calibration. An error of around 20 MeV may be achievable. The LHC may improve this to 15 MeV, which is a useful complement to a 2 GeV uncertainty on the top quark mass, but this will be very challenging. Finally, a future 500–800 GeV linear collider has the potential to go below 10 MeV, using either a threshold scan or direct reconstruction.

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