Theoretical error of luminosity cross section at LEP*

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Abstract

The aim of this note is to characterize briefly main components of theoretical error of the small angle Bhabha measurement at LEP and to discuss critically how solid these estimates really are, from todays perspective. We conclude that the existing theoretical error of the LEP luminometer process (small angle Bhabha) is rather solid, and we add some new discussion concerning the remaining uncertainties and prospects of the future improvements toward the $\leq 0.025\%$ precision.

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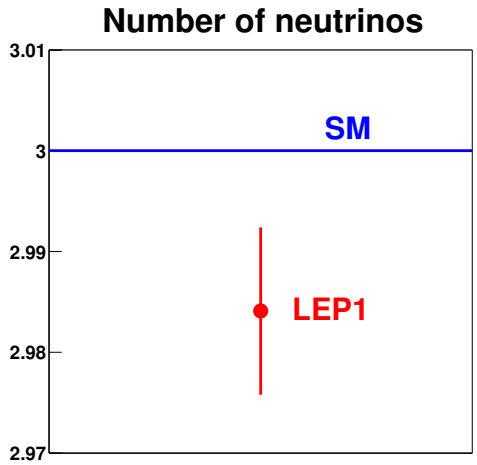

Figure 1: Numer neutrino from LEP experiments form ref...

1 The role of the luminosity measurement in the SM fits to LEP data

The luminosity measurement enters directly or indirectly into all LEP measurables. However, as pointed out in the introduction article of this collection, its role is not critical for the overall fit of the Standard Model to the data, except the measurement of the invisible width on the Z resonance, which is usually formulated as the measurement of the "neutrino" number N_ν and is presently quoted to be $N_\nu = 2.9841 \pm 0.0083$, see ADLO summary paper [1] and recent review [2]. This result is illustrated in Fig. 1. N_ν deviates 1.9σ from the SM value which is normalized to exactly three, taking into account radiative corrections to the partial widths. The two main ingredients in the measurement of N_ν are the cross section at the Z resonance peak (here the luminosity enters) and the strong coupling constant[1] α_s. The pure experimental error of the luminosity measurement at LEP is around 0.04%, the best for the OPAL experiment, a remarkable 0.034%, see ref. [3]. The total error of luminosity at LEP is dominated by the theoretical error, due to uncertainty in the theoretical prediction for the luminometer QED process, Bhabha scattering in the small angles 3°-6°, which is the main subject of this article.

1.1 Determination of N_ν

Let us sketch briefly how the invisible width and N_ν parameters are determined in the LEP experiments. Let us define the invisible decay rate of Z as follows

$$R_{\rm inv} \equiv \frac{Br(Z \to {\rm inv})}{Br(Z \to l^+l^-)}. \tag{1.1}$$

In the SM the invisible branching ratio $Br(Z \to {\rm inv})$ of Z is identified with its decay into three neutrinos. The pole cross section for $e^-e^+ \to Z \to f\bar{f}$ can be written in terms of the partial widths

$$\sigma_{ff}^{pole} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{e^+e^-}\Gamma_{f\bar{f}}}{\Gamma_Z^2} = \frac{12\pi}{m_Z^2} Br(Z \to e^+e^-)Br(Z \to f\bar{f}). \tag{1.2}$$

[1] Alternatively one may say that there are strong correlations between N_ν and α_s in the overall fit.

The invisible branching ratio is, of course, equal the total branching ratio (equal one) minus the branching ration for decay into hadrons and all charged leptons

$$Br(Z \to \text{inv}) = 1 - Br(Z \to \text{had}) - 3Br(Z \to l^+l^-). \tag{1.3}$$

The invisible decay rate is conveniently reformulated in terms of the visible leptonic pole cross sections $\sigma^{pole}_{l^+l^-}$ and the ratio of the hadronic to leptonic pole cross sections R_{had}

$$R_{\text{inv}} \equiv \frac{Br(Z \to \text{inv})}{Br(Z \to l^+l^-)} = \left(\frac{12\pi}{m_Z^2 \sigma^{pole}_{l^+l^-}}\right)^{1/2} - R_{\text{had}} - 3. \tag{1.4}$$

The Standard Model value of the invisible branching ratio, taking into account radiative corrections, is

$$R^{\text{SM}}_{\text{inv}} \equiv \frac{3Br(Z \to \nu\bar{\nu})}{Br(Z \to l^+l^-)} = 5.973 \pm 0.003, \quad \text{for} \quad \alpha_s = 0.119 \pm 0.003, \tag{1.5}$$

which we compare with the experimental value. Traditionally the ratio of the two is referred to as the experimentally measured "neutrino number"

$$N_\nu = 3\frac{R_{\text{inv}}}{R^{\text{SM}}_{\text{inv}}} = 2.9841 \pm 0.0083. \tag{1.6}$$

Note that the QED ISR theoretical error in $\sigma^{pole}_{l^+l^-}$ of $\pm 0.02\%$ is too small to contribute significantly to error of N_ν.

2 Theoretical prediction for the luminometer process

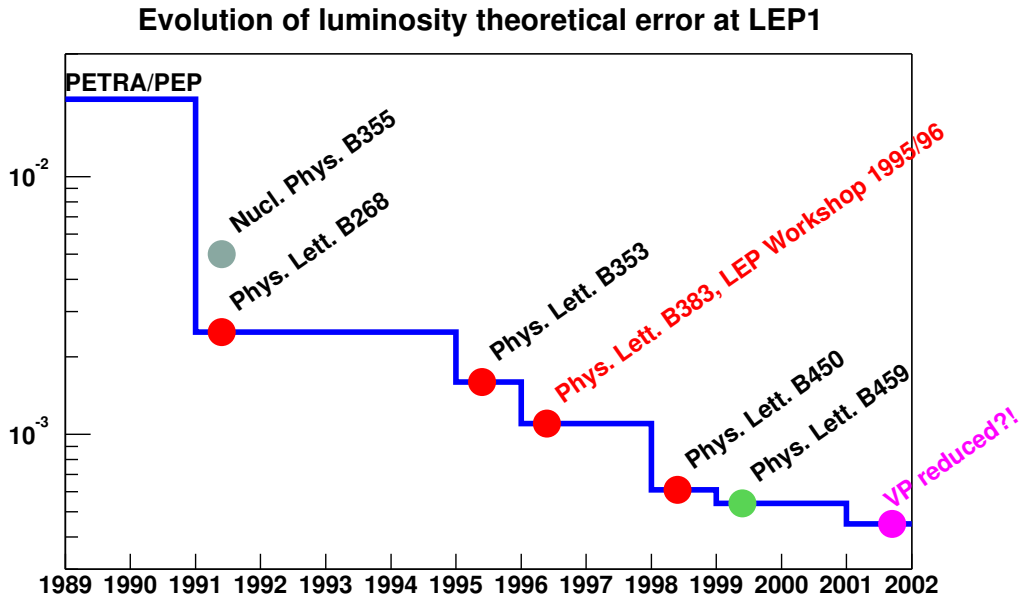

Figure 2: Time evolution of the theoretical luminosity error at LEP era.

All LEP collaborations calculate theoretical predictions for the luminometer small angle Bhabha process using BHLUMI Monte Carlo program version 4.04 of ref. [4]. Theoretical uncertainty of the BHLUMI prediction, mainly due to unaccounted higher order corrections, is however provided/estimated by a series of auxiliary works, see below.

The time evolution of the theoretical error in the LEP luminosity measurement is depicted in an approximate way in Fig. 2. This plot reflects error estimates from papers [5–11]. At the PEP/PETRA times the best prediction for the small angle Bhabha process was provided by the MC program of Kleiss and Berends of ref. [12] and its error was estimated to be $\sim 2\%$. The beginning of LEP was addressed by the improved calculation of ref. [5] with the precision tag of 0.5%. In the parallel works of refs. [6,7] the new precision 0.25% was achieved and this precision was referring for the first time to the prediction of the multiphoton BHLUMI Monte Carlo of refs. [13, 14]. The later improvements of the BHLUMI has led to its version of ref. [4] and its new theoretical precision 0.15% was established first in ref [8]. The LEP workshop 95/96 [15] was "The great consolidation", which has led to a new precision level[2] of 0.11%. It was based on the comparison of several calculations, revising all components of the theory error. The final result was published in the joint paper [9] of the workshop participants. After the 95/96 workshop there was a major reduction of the uncertainty due to photonic corrections in ref. [10], giving total precision 0.061%, and another one of ref. [11], where new estimation of the light pairs contribution has lead to an even lower error of 0.054%. The last entry Fig. 2 corresponds to "speculation" of the present note in which the recent reduction of the error of the hadronic vacuum polarization contribution δVP=0.40→0.025% provides for total theoretical error of 0.045%.

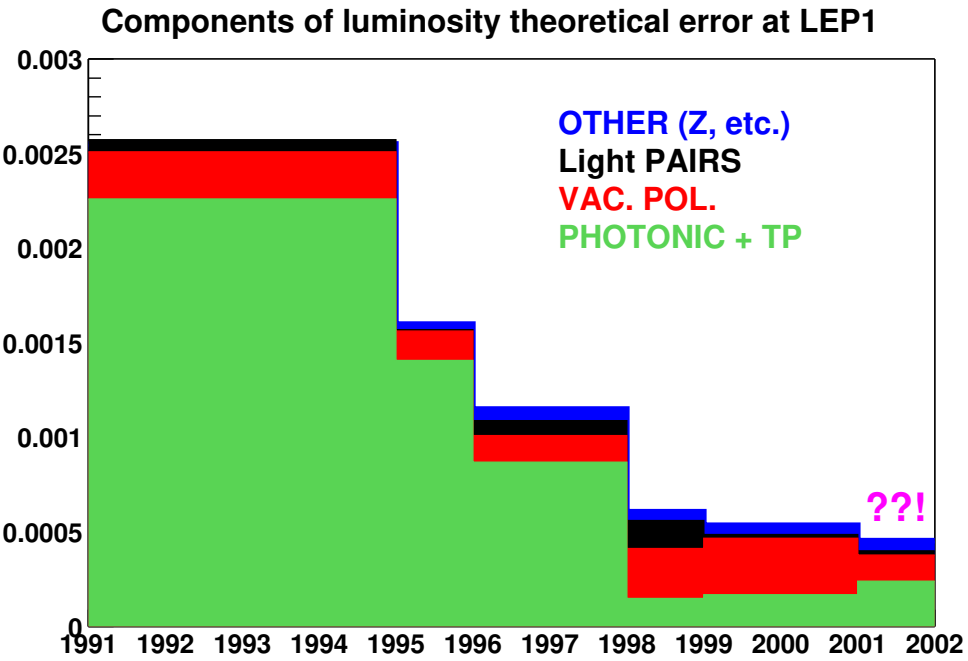

Figure 3: Components of the theoretical luminosity error and their time evolution.

[2]This precision is meant for the prediction of BHLUMI 4.04 [4].

2.1 Main components of the theoretical luminosity error

The main components of the theoretical luminosity error and their time evolution are depicted[3] in Fig. 3. As we see, the dominant component until work of ref. [10] was the missing photonic correction, mainly second order subleading. For the reasons discussed below, the *technical precision*[4] was in most cases combined together with the missing photonic corrections[5]. In the last two papers [10, 11], see Fig. 3, vacuum polarization contribution has become dominant, see below for discussion about its possible reduction, which is already indicated in Fig. 3.

2.2 Photonic corrections

Canonical coefficients in PHOTONIC corrections, $L = \ln(-t_{\min}/m_e^2)$					
		$\theta_{min} = 30$ mrad		$\theta_{min} = 60$ mrad	
		LEP1	LEP2	LEP1	LEP2
$\mathcal{O}(\alpha L)$	$\frac{\alpha}{\pi}4L$	137×10^{-3}	152×10^{-3}	150×10^{-3}	165×10^{-3}
$\mathcal{O}(\alpha)$	$2\frac{1}{2}\frac{\alpha}{\pi}$	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}
$\mathcal{O}(\alpha^2L^2)$	$\frac{1}{2}\left(\frac{\alpha}{\pi}4L\right)^2$	9.4×10^{-3}	11×10^{-3}	11×10^{-3}	14×10^{-3}
$\mathcal{O}(\alpha^2L)$	$\frac{\alpha}{\pi}\left(\frac{\alpha}{\pi}4L\right)$	0.31×10^{-3}	0.35×10^{-3}	0.35×10^{-3}	0.38×10^{-3}
$\mathcal{O}(\alpha^3L^3)$	$\frac{1}{3!}\left(\frac{\alpha}{\pi}4L\right)^3$	0.42×10^{-3}	0.58×10^{-3}	0.57×10^{-3}	0.74×10^{-3}

Table 1: Canonical coefficients determining size of the photonic corrections.

As we see from the values of the generic coefficient of the photonic corrections presented in first column of Table 1, we always expected the missing second order subleading and third order leading photonic corrections to be below 0.1%. The actual proof that it is true was provided in refs. [10, 16]. In particular results of ref. [10] were the real breakthrough. In Fig. 4 we present the principal results of this paper, which demonstrates that the second order subleading photonic correction in BHLUMI 4.04 is below[6] 0.03%. Unfortunately this correction is not included in the published BHLUMI code – as a consequence, the conclusion of ref. [10] is restricted to experimental selections considered in this paper.

[3]The plotted quantity is the total error which is split into components proportionally to the square of the component. This justified by the fact that components are combined in the quadrature.

[4]Technical precision summarizes all kind of numerical problems, including bugs in the programs and mistakes in numerical algorithms.

[5]Simply because there was no other MC of a comparable quality for establishing technical precision of BHLUMI in a solid and independent way.

[6]In fact the estimate of the total missing $\mathcal{O}(\alpha^2L)$ in this reference is an overestimate, because its three components are added in the quadrature and again combined with technical precision, see below for more discussion.

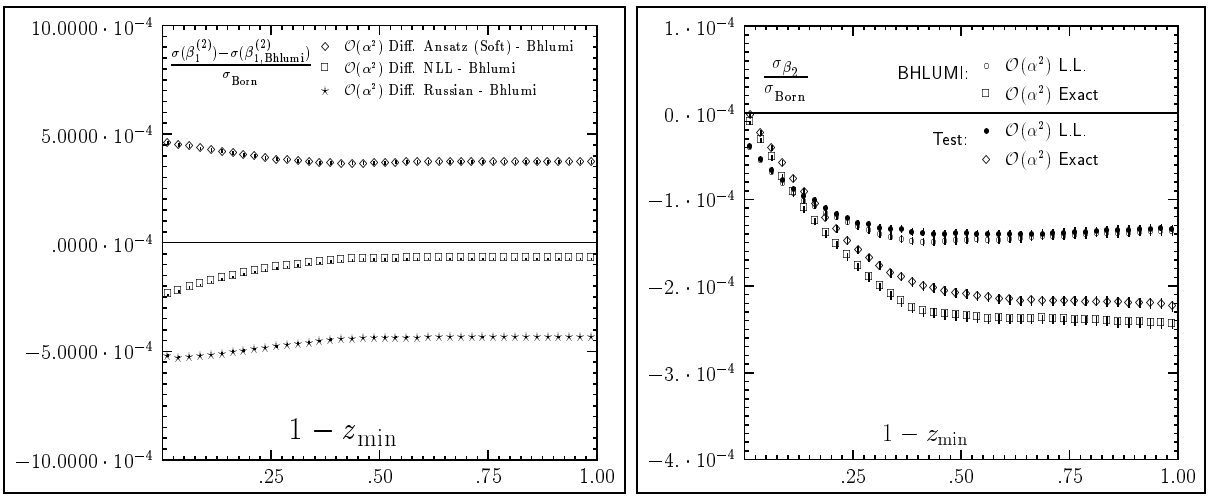

Figure 4: Missing second order subleading contributions in BHLUMI 4.04, according to calculation ref. [10], for realistic LEP event selections.

2.3 Fermion pairs

Main contribution is coming from the e^+e^- pairs. For a typical calorimetric LEP detector there are strong virtual-real cancellations and this is why this contribution is small for the inclusive event selections in all LEP luminometers. Contrary to some early claims that that this contribution is huge, $\sim 0.500\%$, in ref [17] the light pair corrections were found to be $-0.013\% \pm 0.020\%$, for the realistic LEP event selections. Using similar techniques, this contribution was calculated independently in refs. [11,18] and for typical LEP acceptance it was found there to be from $-.025\%$ to $-.030\%$, with the technical+physical precision $\simeq 0.010\%$. Both groups provided MC tools for correcting experimental data[7].

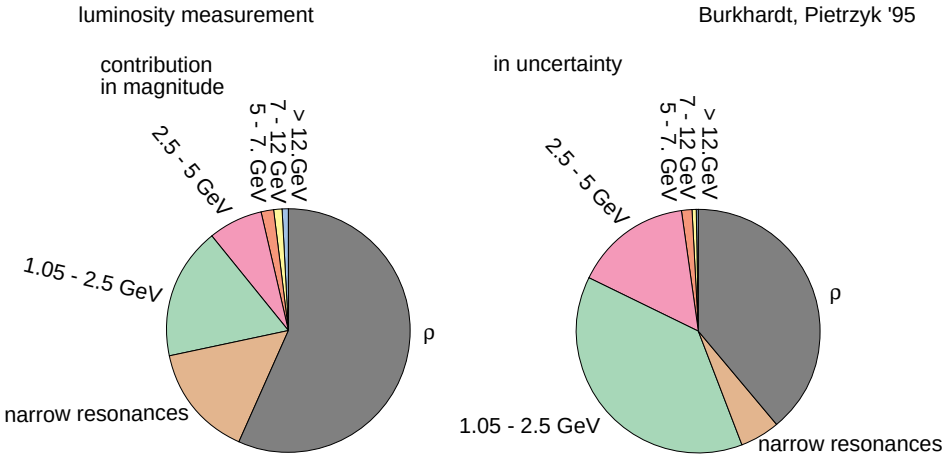

Figure 5: vacuum polarization, from ref [15].

[7]There exists unpublished version of BHLUMI which includes light pair production; the corresponding version 2.30 of BHLUMI is described in ref. [17].

2.4 Technical precision

In ref. [19] the technical precision (TP) of the unexponentiated $\mathcal{O}(\alpha)$ (no exponentiation) was determined in the MC calculation to be $0.013\% \pm 0.017\%$. It was calculated using difference between MC and semi-analytical codes, with $\pm 0.017\%$ MC statistical error. Since work of ref. [19] we had good reasons to believe that TP$\simeq 0.020\% - 0.030\%$ for the bulk of the $\mathcal{O}(\alpha)$ MC small angle Bhabha cross section.

New tests of LEP 95/96 workshop, see Fig. 15 in ref. [15], for realistic event selection provided TP$\simeq$ 0.030% using difference of two independent unexponentiated $\mathcal{O}(\alpha)$ MC programs. For the multiphoton BHLUMI, its TP was incorporated into the estimate of the photonic theoretical error 0.10%, which was estimated in ref. [15] by examining variety of the MC programs. The size of TP alone of the multiphoton BHLUMI was *guessed* to be $\simeq 0.20 - 0.040\%$, see Table 21 in this reference.

Let us stress that until the LEP 95/96 workshop the technical precision of BHLUMI was always tied up together with the missing photonic correction (mostly $\mathcal{O}(\alpha^2 L)$). An attempt of determining TP of the multiphoton BHLUMI was done in ref. [19] for the *quasi-realistic*[8] event selection by comparing the BHLUMI MC with the set of special high quality semi-analytical calculation. The agreement of $\simeq \pm 0.017\%$ was found, not only for the total cross section but also separately for each of many components of the multiphoton QED multiphoton matrix element.

Summarizing the TP$\simeq \pm 0.030\%$ seems to be a reasonable guesestimate of the technical precision for the multiphoton BHLUMI 4.x for a wide range of event selection, but we still miss some powerful test, which would definitely determine TP of the multiphoton BHLUMI 4.x at the level of 0.01%, *independently* of the missing higher orders.

2.5 Hadronic vacuum polarization

Until 1996-98 works of refs. [20, 21] were used as a source of hadronic vacuum corrections to small angle Bhabha. The error due to VP was 0.040%, its *relative* importance increased over the years. As seen from "pie-plot" of Fig. 5, it comes mainly from $R(s)$ in the ρ region. There, recent measurements of $R(s)$ are now much better and will be even better in the near future. Let us estimate how much can we profit from this progress. At the typical LEP luminometer average angle $\langle\vartheta\rangle = 0.034$, ie. $\langle\sqrt{|t|}\rangle = 1.54$GeV, using refs. [20] and [22] we estimate now $\Re\Pi_{1995} = 0.541 \pm 0.022$ and $\Re\Pi_{2001} = 0.535 \pm 0.014$[9]. Rescaling naively, we get the reduction of the error due to VP: $0.040\% \to 0.025\%$. Of course, a more systematic study should be done. Note, however, that since VP correction is rather weakly dependent on the details of the experimental event selection, it can be redone even the after LEP data analysis is finished[10].

[8] Event selections in this exercise were defined in terms of the final electrons not combined with FSR photons and in terms of the transfer t instead of the final electron angles.

[9] H. Burkhardt, private communication.

[10] This might be not necessarily true for the photonic corrections.

	LEP1		LEP2
Type of correction/error	Ref. [8]	Refs. [9, 15]	Refs. [9, 15]
(a) Missing photonic $\mathcal{O}(\alpha^2 L)$	0.15%	**0.10%**	0.20%
(a) Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.008%	**0.015%**	0.03%
(c) Vacuum polarization	0.05%	**0.04%**	0.10%
(d) Light pairs	0.04%	**0.03%**	0.05%
(e) Z-exchange	0.03%	**0.015%**	0.0%
Total	0.16%	**0.11%**	0.25%

Table 2: Error budget at LEP Workshop 95/96 [9, 15], "The Great Consolidation"

3 Total error budget

Let us now look into and discuss in a bit more details the budget of the total theoretical luminosity error, in the context of the last papers in this subject. In Table 2 we recapitulate and compare results of refs. [8,9,15]. The work of LEP 95/96 workshop [9,15] improves on almost all components, mainly on the photonic $\mathcal{O}(\alpha^2 L)$ and revises upward the $\mathcal{O}(\alpha^3 L^3)$ component, according to ref. [16]. Technical precision is a part of the $\mathcal{O}(\alpha^2 L)$ quoted as 0.11%. How big part? It is not specified, see discussion above. In ref. [8] there is a suggestion that technical precision alone of BHLUMI is $0.02\% - 0.04\%$.

Type of correction/error	Refs. [9, 15]	Refs. [10]		Refs. [11]		update
Technical precision	–	–	(0.03%)	–	(0.03%)	**0.03%**
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.10%	0.027%	(0.013%)	0.027%	(0.013%)	**0.013%**
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015%	(0.006%)	0.015%	(0.006%)	**0.006%**
Vacuum polarization	0.04%	0.04%		0.040%		**0.025%**
Light pairs	0.03%	0.03%		0.010%		**0.010%**
Z-exchange	0.015%	0.015%		0.015%		**0.015%**
Total	0.11%	0.061%	(0.062%)	0.054%	(0.055%)	**0.045%**

Table 3: My personal update of LEP1 theoretical error, Febr. 2003

In the second and third column of Table 3 we present the next two improvements of refs. [10, 11]. In the second column [10] we find the dramatic improvement of photonic part, while in the third column [11] we see an improved value of the light pairs. The other entries are unchanged with respect to Refs. [9, 15], and again both paper [10, 11] refer to corrections which are omitted or incomplete in BHLUMI 4.04 program.

Let us elaborate more on the estimate of the technical precision of BHLUMI adopted in refs. [10,11]. In ref. [10] a great deal of effort was done to get under control the TP of the $\mathcal{O}(\alpha^2 L)$ photonic contribution itself, which was calculated there and which *is not included* in BHLUMI 4.04. However, this does not directly address the question of the TP of the corrections which *are included* in BHLUMI 4.04. The estimate of the TP of BHLUMI 4.04 in refs. [10, 11] is essentially the same as in refs. [8, 9, 15], see also discussion in Sec. 2.4, except that we now know that most of 0.10%, which was attributed to both technical

precision and $\mathcal{O}(\alpha^2 L)$ missing photonic, is in fact not the $\mathcal{O}(\alpha^2 L)$ missing photonic! So how big TP is? The entire 0.10% is definitely too conservative as an etsimate of the technical precision of BHLUMI, because there is a number of partial tests which show that $\mathcal{O}(\alpha^1)$ part is under control to within 0.02% and that the other parts beyond $\mathcal{O}(\alpha^1)$ agree with a series of special semianalytical tests in ref. [19] to within 0.017%, albeit for special event selection, see Sec. 2.4. In the following we shall adopt 0.03% as a conservative estimate of the TP of BHLUMI 4.04, separately from any QED missing corrections.

In the 2-nd and 3-rd column of Table 3 we show in brackets the actual missing $\mathcal{O}(\alpha^3 L^3)$ and $\mathcal{O}(\alpha^2 L)$ contributions according to refs. [10,16], isolating the TP of 0.03% in a separate entry. The quoted value of the missing $\mathcal{O}(\alpha^2 L)$ of 0.013% corresponds to a coherent (not in quadrature) sum of contribution from β_0 and β_1, see Fig. 4, and from β_2 (with the dominant contribution from the vertex correction in β_0) – all that from ref. [10]. The $\mathcal{O}(\alpha^3 L^3)$ missing contribution of 0.006% is taken from Fig. 2 in ref. [16] for the SICAL detector, without any "safety factor" of two. The new total results 0.062% and 0.055% in the brackets in 2-nd and 3-rd column of Table 3 are almost the same as the original ones.

In the 4-th column of Table 3 we show also the result for the total theoretical error, assuming again the TP of 0.03%, and decreasing the vacuum polarization error to 0.025%, as suggested by the exercise in Sect.2.5. The total error is now 0.045%, see bottom row of the 4-th column in the Table 3. Obviously, the above exercise should be done in a more systematic way. Nevertheless, it illustrates very well *the remaining uncertainties* in the game of establishing theoretical error of the luminometer cross section for LEP1.

Last but not least let us try to answer whether it is feasible to reduce the total theoretical error down to $\simeq$ 0.025% and what is necessary to achieve this goal. The presently available CPU power allows one to compare two MC programs with statistical error below 0.01%. The toughest problem will be to reduce the technical error for the two multiphoton MCs, that is BHLUMI and another one, let us call it BHLUMI+, to the level of 0.01%. The hypothetical BHLUMI+ should feature a new QED matrix element, of the CEEX type of ref. [23] including the $\mathcal{O}(\alpha^2 L)$ of ref. [10] and augmented with the complete $\mathcal{O}(\alpha^3 L^3)$. It should necessarily feature an alternative parametrization of the multiphoton phase space [24]. The treatment of Z-exchange would automatically improve due to the use of CEEX. Hadronic VP should be updated. Light pair corrections are already under sufficient control. I believe that one could profit from this improvement even when LEP data analysis is finished, i.e. it will be possible to propagate it to N_ν and σ^{pole}, because the difference between BHLUMI+ and BHLUMI would be very small and weakly dependent on event selection. We would then see whether the 1.9σ discrepancy with the SM in N_ν disappears or becomes even larger[11]. The rest of the fits of the SM to the data would be unaffected.

4 Summary

- The present theoretical error of the small angle Bhabha $\simeq$0.06% seems rather solid.
- The room for an easy improvement exists (vacuum polarization).

[11] NB. there are models with the massive neutrino mixing [25, 26], which suggest the presence of a deficit in the experimental N_ν.

- Radical improvement of the TH precision to the level of $\leq 0.025\%$, i.e. below the best experimental error 0.034% is feasible.
- This will require reduction of the technical precision and absorbing the existing improvements of the photonic QED corrections in the MC. Hadronic VP will get reduced another factor 2 in the meantime.

Acknowledgments

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