Gauge dependence and renormalization of $\tan\beta$

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Abstract

Renormalization schemes for $\tan\beta$ are discussed in view of their gauge dependence. It is shown that several common renormalization schemes lead to a gauge-dependent definition of $\tan\beta$, whereas two classes of gauge-independent schemes show even worse disadvantages. We conclude that the $\overline{DR}$-scheme is the best compromise.

1 Introduction

The quantity $\tan\beta$ is one of the main input parameters of the minimal supersymmetric standard model (MSSM). At tree-level it is defined as the ratio of the two vacuum expectation values $v_{1,2}$ of the MSSM Higgs doublets,

$$\tan\beta = \frac{v_2}{v_1}. \tag{1}$$

Owing to its central appearance in the spontaneous symmetry breaking, $\tan\beta$ plays a crucial role in almost all sectors of the MSSM and has significant impact on most MSSM observables.

As all parameters in a quantum field theory, at higher orders $\tan\beta$ is actually defined by the choice of a renormalization scheme. Though in principle all renormalization schemes are equivalent, there are large practical differences. The renormalization scheme determines the relation of $\tan\beta$ to observables and thus its numerical value as well as its formal properties like gauge dependence and renormalization-scale dependence.

This talk is based on the analysis of [1], where several renormalization schemes for $\tan\beta$ were studied with the aim to find an optimal scheme. One important aspect we will consider is the gauge dependence induced by the renormalization schemes. Generally, the dependence on the gauge fixing always drops out in relations between different observables, but the relations between observable quantities and formal input parameters like $\tan\beta$ can be gauge dependent. As we will see, in some renormalization schemes, $\tan\beta$ indeed turns out to be gauge dependent.

Apart from gauge dependence there are two more desirable properties of renormalization schemes: numerical stability of the perturbative expansion, and process independence (in order not to spoil the intuition that $\tan\beta$ is a universal quantity of the Higgs sector). In the following we will discuss well-known and new schemes in view of these properties, providing arguments that no ideal scheme exists. The $\overline{DR}$-scheme will emerge as the best compromise.

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2   Gauge dependence of well-known schemes

Two classes of well-known and commonly used renormalization schemes are the $\overline{DR}$-scheme and the schemes introduced in [2, 3]. At the one-loop level they are defined by the conditions

$$\overline{DR} : \qquad\qquad\qquad \delta t_\beta \overset{!}{=} \text{pure divergence}, \qquad\qquad\qquad (2)$$

$$\text{DCPR} : \qquad\qquad\qquad \delta t_\beta \overset{!}{=} \frac{1}{2c_\beta^2 M_Z} \text{Re}\Sigma_{A^0 Z}(M_A^2) \qquad\qquad\qquad (3)$$

on the renormalization constant $\delta \tan \beta \equiv \delta t_\beta$. Here "pure divergence" denotes a term of the order $\Delta = \frac{2}{4-D} - \gamma_E + \log 4\pi$ in dimensional reduction and $\Sigma_{A^0 Z}$ is the unrenormalized $A^0 Z$ two-point function. These schemes have the advantage of defining $\tan \beta$ in a universal way in the Higgs sector and being technically very convenient. On the other hand, they do not imply any obvious relation between $\tan \beta$ and observable quantities; hence, they might lead to a gauge dependence of $\tan \beta$.

The gauge dependence of $\tan \beta$ can be computed by using an extended Slavnov-Taylor identity introduced in [4, 5]:

$$\tilde{S}(\Gamma) \equiv S(\Gamma) + \chi \, \partial_\xi \Gamma = 0. \qquad\qquad\qquad (4)$$

Here ξ denotes an arbitrary gauge parameter in the gauge-fixing term and χ is a fermionic variable acting as the BRS transformation of ξ, and $S(\Gamma)$ is the usual Slavnov-Taylor operator. The validity of $\tilde{S}(\Gamma) = 0$ is equivalent to the gauge independence of all input parameters, in particular of $\tan \beta$.

Consider the $\overline{DR}$-scheme as an example. In this scheme, the renormalization constant δt_β is obviously gauge independent:

$$\partial_\xi \delta t_\beta{}^{\text{fin}} = 0. \qquad\qquad\qquad (5)$$

But the validity of $\tilde{S}(\Gamma) = 0$ would imply a certain gauge-parameter dependence at the one-loop level:

$$\partial_\xi \delta t_\beta{}^{\text{fin}} \propto \left(- \sin \beta A_1 + \cos \beta A_2 \right), \qquad\qquad\qquad (6)$$

where $A_i = \Gamma^{(1),\text{reg}}_{\chi Y_{\phi_i}}$ denotes the unrenormalized one-loop Green function with the BRS transform χ of the gauge parameter and the source Y_{ϕ_i} of the BRS transformation of the Higgs field ϕ_i. The Green functions A_i can be calculated, but their results depend on the choice of the gauge fixing. In the class of R_ξ-gauges,

$$(A_1, A_2) \propto (\cos \beta, \sin \beta), \qquad\qquad\qquad (7)$$

and thus the r.h.s. of (6) yields zero and is compatible with (5). This shows that $\tan \beta$ is gauge independent in the $\overline{DR}$-scheme at the one-loop level and in the class of R_ξ-gauges. However, in a non-R_ξ-gauge, where the physical A^0 boson is introduced into the gauge-fixing function

$$\mathcal{F}^Z = \partial_\mu Z^\mu + M_Z(\xi G^0 + \zeta^{Z A^0} A^0), \qquad\qquad\qquad (8)$$

virtual A^0 bosons contribute to $A_{1,2}$ instead of virtual Goldstone bosons, and the results for $A_{1,2}$ are modified:

$$(A_1, A_2) \propto (-\sin\beta, \cos\beta). \tag{9}$$

Therefore, in this gauge the r.h.s. of (6) is non-vanishing and thus in contradiction with the $\overline{DR}$-condition (5). In other words, in general gauges, the $\overline{DR}$-scheme leads to a violation of $\tilde{S}(\Gamma) = 0$ and to a gauge dependence of $\tan\beta$ already at the one-loop level. In addition, in [6] it was found that the $\overline{DR}$-scheme leads to a gauge dependence at the two-loop level even in the R_ξ-gauges.

The gauge dependence of $\tan\beta$ in the DCPR-schemes can be studied in a similar way. It turns out that these schemes lead to a gauge dependence already in R_ξ-gauges at the one-loop level.

3   Gauge- and process-independent schemes and their drawbacks

As an alternative to the gauge-dependent schemes discussed in the previous section, we consider now a class of gauge-independent schemes. Three examples are

Tadpole scheme: $$\delta t_\beta^{\text{fin}} \overset{!}{=} \text{const.} \left(\frac{\delta t_1}{v_1} + \frac{\delta t_2}{v_2} \right), \tag{10}$$

m_3-scheme: $$\delta m_3^{\text{fin}} \overset{!}{=} 0, \tag{11}$$

HiggsMass-scheme: $$\cos^2(2\beta) \overset{!}{=} \frac{M_h^2 M_H^2}{M_A^2(M_h^2 + M_H^2 - M_A^2)}. \tag{12}$$

Each of these schemes has a different underlying intuition. In the first case, $\tan\beta$ is defined in a minimal gauge-independent way via a relation to tadpole counterterms, in the second case an indirect definition via the soft-breaking parameter m_3 is used, and in the third case $\cos(2\beta)$ and $\tan\beta$ are defined in terms of a ratio of physical Higgs masses.

It can be shown that these three schemes are in agreement with $\tilde{S}(\Gamma) = 0$ and thus with the gauge independence of $\tan\beta$. Furthermore, they define $\tan\beta$ in a universal way, using only quantities of the MSSM Higgs sector.

Unfortunately, in spite of these advantageous properties all three schemes are not useful in practice because they cause very large numerical uncertainties in loop corrections to quantities involving δt_β. This can be exemplified by the renormalization-scale dependence of $\tan\beta$ in the first two schemes, (10), (11). Table 1(a) shows that while the renormalization-scale dependence in the $\overline{DR}$-scheme is quite modest, the one in the schemes (10), (11) can be extremely large, and hence leads to an inacceptable numerical instability of loop calculations. Although $\tan\beta$ in the HiggsMass-scheme is renormalization-scale independent, it involves numerically very large contributions to observable quantities and also leads to numerical instabilities. As an example, table 1(b) shows the results for the one-loop corrected mass of the lightest MSSM Higgs boson, m_h. Obviously, the results obtained in the schemes (10–12) deviate strongly from the results obtained in the $\overline{DR}$- or DCPR-schemes.

	$\overline{DR}$	(10)	(11)
$\tan\beta = 3$	-0.1	4.5	0.8
$\tan\beta = 50$	-0.2	370.7	285.3

(a)

$\overline{DR}$	DCPR	(10)	(11)	(12)
134.6	134.4	173.5	143.2	119.6

(b)

Table 1: (a): The renormalization-scale $\bar{\mu}$-dependence $\partial\tan\beta/\partial\log\bar{\mu}$ of the schemes (10), (11) in comparison with the $\overline{DR}$-scheme. We have chosen $M_A = 500$ GeV, and the remaining parameter values are chosen according to the $M_h^{\max}$-scenario of [7]. (b): Results for the one-loop corrected lightest Higgs mass M_h using the same parameters and $\tan\beta = 3, \bar{\mu} = m_t$.

As shown in [1], one can generalize from these three particular schemes to the class of all schemes where $\tan\beta$ is defined via quantities of the MSSM Higgs sector (i.e. δt_β is composed of Higgs self energies and tadpoles). In all gauge-independent schemes of this class numerical instabilities like the ones shown in table 1 appear, so unfortunately all these schemes are useless in practice.

4   Process-dependent schemes

The results obtained up to now are negative. In the considered classes of schemes, there is no scheme that combines all three desirable properties

- gauge independence,

- numerical stability,

- process independence.

The $\overline{DR}$- and DCPR-schemes are gauge dependent, and the gauge- and process-independent schemes discussed in the previous section are numerically unstable. As an alternative we can think about dropping the requirement of process independence as advocated in [8]. Two possible schemes are given by

$$\tan^2\beta \stackrel{!}{=} \text{const.} \times \Gamma(A^0 \to \tau\tau), \tag{13}$$

$$\tan^2\beta \stackrel{!}{=} \text{const.} \times \Gamma(H^+ \to \tau^+\nu), \tag{14}$$

where "const." denotes the kinematical prefactors of the decay widths. Since $\tan\beta$ is directly related to observables in this way it is gauge independent, and these schemes do not induce numerical instabilities. However, these schemes have other disadvantages. At first they are technically complicated since the evaluation of δt_β requires the calculation of the full decay widths. In particular, the second process involves infrared divergent QED-corrections that cannot be split off from the definition of $\tan\beta$. Furthermore, such process-dependent schemes introduce a flavour dependence, which seems unnatural since these decays are just two examples amongst a variety of potential observables for the experimental determination of $\tan\beta$.

5 Conclusions

The gauge- and process-independent schemes presented are practically useless because of their numerical instabilities. The well-known $\overline{DR}$- and DCPR-schemes are gauge dependent already at the one-loop level, but they can be used very well in practice. Among these schemes the $\overline{DR}$-scheme is preferable for two reasons. It is gauge independent at the one-loop level in the class of R_ξ-gauges, and a recent study has shown that its numerical behaviour is particularly stable [9]. An alternative is provided by process-dependent schemes. Among these, the decay $A^0 \to \tau\tau$ is advantageous since there the infrared divergent QED-corrections can be split off. However, as all process-dependent schemes, this scheme is technically relatively complicated and defines $\tan\beta$ in a non-universal way. Therefore we assess the $\overline{DR}$-scheme as the best renormalization scheme for $\tan\beta$. It is technically the easiest, numerically the most well-behaved, and it is still gauge independent in the practically most important case.

References

[1] A. Freitas, D. Stöckinger, **hep-ph**/0205281, to be published in *Phys. Rev.* **D**.

[2] A. Dabelstein, *Z. Phys.* **C67** (1995) 495.

[3] P. Chankowski, S. Pokorski, J. Rosiek, *Nucl. Phys.* **B423** (1994) 437.

[4] H. Kluberg-Stern, J.B. Zuber, *Phys. Rev.* **D12** (1975) 467,482,3159.

[5] O. Piguet, K. Sibold, *Nucl. Phys.* **B253** (1985) 517.

[6] Y. Yamada, *Phys. Lett.* **B530** (2002) 174; Y. Yamada, these proceedings, **hep-ph**/0210324.

[7] M. Carena, S. Heinemeyer, C. Wagner, G. Weiglein, **hep-ph**/9912223.

[8] J.A. Coarasa, D. Garcia, J. Guasch, R.A. Jiménez, J. Solà, *Eur. Phys. J.* **C2** (1998) 373.

[9] M. Frank, S. Heinemeyer, W. Hollik, G. Weiglein, **hep-ph**/0202166.