CP violation in supersymmetry with universal strength of Yukawa couplings[1]

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Abstract

We analyse the CP problem in the context of a supersymmetric extension of the standard model with universal strength of Yukawa couplings. In these models we find a small amount of CP violation from the usual CKM mechanism and therefore a significant contribution from supersymmetry is required. The electric dipole moments impose severe constraints on the parameter space, forcing the trilinear couplings to be factorizable in matrix form. We find that the LL mass insertions give the dominant gluino contribution to saturate ε_K, while chargino contributions to ε'/ε are compatible with the experimental results. Due to significant supersymmetric contributions to $B_d - \bar{B}_d$ mixing, the recent large value of $a_{J/\psi K_S}$ can be accommodated.

1 Introduction

The understanding of the origin of fermion families and the observed pattern of fermion masses and mixings, together with the origin of CP violation, are among the major outstanding problems in particle physics.

Most extensions of the standard model (SM) naturally include new sources of CP violation. In supersymmetric (SUSY) extensions of the SM we find additional sources of CP violation, due to the presence of new CP violating phases. However, these new phases are severely constrained to be small by experimental bounds on the electric dipole moments (EDM's) [1, 2].

Since the question of CP-violation is closely related to the general flavour problem, one may wonder whether it is possible, within a supersymmetric extension of the SM, to establish a connection between the need for small CP violating phases and the observed pattern of quark masses and mixings. Such a connection might be possible if one imposes universality of strength for Yukawa couplings (USY) on a supersymmetric extension of the SM. In Ref. [3], we introduce the USY ansatz in the framework of SUSY, analysing the compatibility of this scenario with the EDM bounds, and discussing the contributions to K and B system CP violation observables.

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[1]Talk given by A. Teixeira.

2   Universal strength of Yukawa couplings

One of the most simple, and yet very attractive suggestions for the structure of the Yukawa couplings consists in assuming that they have a universal strength g_y, so that the mass matrices can be written as $m_{ij} = g_y v/\sqrt{2} \exp(i\phi_{ij})$, where i, j denote family indices. In the limit of small phases, the USY matrices can be viewed as a perturbation of the democratic type matrices [4]. Within the framework of universal strength of Yukawa couplings, the quark Yukawa matrices can be parametrized as $U_{ij} = \lambda_u/3 \; \exp[i\Phi_{ij}^u]$ and $D_{ij} = \lambda_d/3 \; \exp[i\Phi_{ij}^d]$, where $\lambda_{u,d}$ are overall real constants, and $\Phi^{u,d}$ are pure phase matrices. By performing appropriate weak-basis transformations the matrices U and D can, without loss of generality, be put in the form

$$U = \frac{\lambda_u}{3} \begin{pmatrix} e^{ip_u} & e^{ir_u} & 1 \\ e^{iq_u} & 1 & e^{it_u} \\ 1 & 1 & 1 \end{pmatrix} \; ; \qquad D = \frac{\lambda_d}{3} K^\dagger \begin{pmatrix} e^{ip_d} & e^{ir_d} & 1 \\ e^{iq_d} & 1 & e^{it_d} \\ 1 & 1 & 1 \end{pmatrix} K \; , \qquad (1)$$

where $K = \mathrm{diag}(1, e^{i\kappa_1}, e^{i\kappa_2})$. The phases (p, q, r, t) affect both the spectrum and the V_{CKM}, while κ_i only influence the V_{CKM}. The quark mass hierarchy forces the modulus of each (p, q, r, t) phase to be small, at most of order m_2/m_3. Also, the fact that $(|V_{cb}|^2 + |V_{ub}|^2)$ is small, constrains each one of the phases κ_1, κ_2 to be at most of order $|V_{cb}|$ [5, 6].

In Ref. [5], it was shown that USY can easily account for the quark mass spectrum at low energy scales. This class of Yukawa couplings can be generated at some high energy scale through the breaking of a flavour symmetry, as discussed in [3], and it has been also recently argued that USY textures may naturally arise within the framework of a theory with two "large extra dimensions" [7]. Furthermore, RG evolution preserves the USY texture that is responsible for the non-degenerate quark spectrum [3].

By scanning the parameter space, one can find values of $(p, q, r, t, \kappa_1, \kappa_2)$, such that the quark spectrum and the V_{CKM} can be correctly reproduced. To illustrate some of the features of this ansatz, let us consider, as an example, the following choice of USY phases:

	p	r	q	t	κ_1	κ_2
up	5.17×10^{-4}	-7.4×10^{-6}	-1.43×10^{-2}	1.58×10^{-3}	$-$	$-$
down	2.29×10^{-3}	2.83×10^{-2}	-9.27×10^{-2}	-0.145	0	1.38×10^{-2}

Table 1: Choice of USY phases for the structure considered in Eq. (1).

The overall factors have been defined as $\lambda_u/3 = m_t/v \sin\beta$ and $\lambda_d/3 = m_b/v \cos\beta$. The Yukawa couplings can be diagonalized as $S_R^{d(u)} \; Y^{d(u)T} \; S_L^{d(u)\dagger} = Y_{\mathrm{diag}}^{d(u)}$, where S_R and S_L are unitary matrices. It is worth stressing that within the context of the SM, the matrices $S_{L,R}^{u,d}$ do not have, by themselves, any physical meaning, only the combination $S_L^u . S_L^{d\dagger} \equiv V_{\mathrm{CKM}}$ is physically meaningful. However the matrices $S_{L,R}^{u,d}$ do play a significant rôle in some extensions of the SM, as for example the MSSM with non-universal soft-

breaking terms. In the present USY model the V_{CKM} is given by

$$V_{\text{CKM}} = \begin{pmatrix} 0.975 - 6.88 \times 10^{-4}\,i & 0.221 + 3.96 \times 10^{-3}\,i & 0.0027 - 4 \times 10^{-5}\,i \\ -0.220 - 9.71 \times 10^{-3}\,i & 0.973 + 0.061\,i & 0.041 + 1.92 \times 10^{-3}\,i \\ 0.0063 + 1.10 \times 10^{-4}\,i & -0.041 - 1.75 \times 10^{-3}\,i & 0.999 + 0.027\,i \end{pmatrix} . \tag{2}$$

Regarding CP violation, the above V_{CKM} leads to a strength of CP breaking (measured by $J \equiv \text{Im}\,(V_{ud}V_{cs}V_{us}^*V_{cd}^*)$) that is too small to account for the observed value of ε_K. This is a generic feature of the USY ansatz, and it provides further motivation to embed USY in a larger framework, where new sources of CP violation naturally arise, like the MSSM.

3   Supersymmetric USY models and CP violation

In Ref. [3], we considered the minimal supersymmetric standard model (MSSM), where a minimal number of superfields is introduced and R parity is conserved, with the following soft SUSY breaking terms

$$\begin{aligned} V_{SB} &= m_{0\alpha}^2 \phi_\alpha^* \phi_\alpha + \epsilon_{ab}(A_{ij}^u Y_{ij}^u H_2^b \tilde{q}_{L_i}^a \tilde{u}_{R_j}^* + A_{ij}^d Y_{ij}^d H_1^a \tilde{q}_{L_i}^b \tilde{d}_{R_j}^* + A_{ij}^l Y_{ij}^l H_1^a \tilde{l}_{L_i}^b \tilde{e}_{R_j}^* \\ &\quad - B\mu H_1^a H_2^b + \text{H.c.}) - \frac{1}{2}(m_3 \bar{\tilde{g}}\tilde{g} + m_2 \overline{\widetilde{W^a}}\widetilde{W^a} + m_1 \bar{\tilde{B}}\tilde{B}) , \end{aligned} \tag{3}$$

where the Yukawa couplings are of the USY form, and ϕ_α denotes all the scalar fields of the theory. The μ term and the gaugino soft terms are assumed to be real, so that the structure and phases of the trilinear terms will play a key rôle, regarding both the stringent bounds coming from the EDM's, and the enhancement of the CP observables. In this work we will also take as a guideline the assumption that all supersymmetric phases should be no greater than the largest of the USY phases in Table 1.

We begin by investigating how the EDM bounds constrain the parameter space for SUSY models with universal strength of Yukawa couplings. The current experimental bound on the EDM of the neutron and mercury atom are $d_n < 6.3 \times 10^{-26}\ e$ cm and $d_{Hg} < 2.1 \times 10^{-28}\ e$ cm, respectively [8]. These bounds can be translated into constraints for the imaginary parts of the flavour conserving LR mass insertions. Requiring that d_n does not exceed the experimental limit compels $\text{Im}\,(\delta_{11}^{d(u)})_{LR}$ to be less than 10^{-6}, while compatibility with the mercury EDM corresponds to having $\text{Im}(\delta_{11}^{d(u)})_{LR} \lesssim 10^{-7} - 10^{-8}$ and $\text{Im}(\delta_{22}^d)_{LR} \lesssim 10^{-5} - 10^{-6}$ [1].

We showed that, even in the limit of small (or vanishing) supersymmetric phases, the large mixing inherent to USY couplings (displayed in S_L and S_R), together with the EDM experimental limits, severely constrain any non-universality of the trilinear terms. In Fig. 1 we present the constraints from the EDM's on the off-diagonal entries of the A-terms, in particular A_{12} and A_{13}. In our analysis we assumed $\tan\beta = 5$, $m_0 = m_{1/2} = 250$ GeV and $A_{ij} = m_0$ for all elements except $A_{12,13}$, which we set in the range $[-3m_0, 3m_0]$.

From these figures, it is clear that any significant non-universality among the A terms leads to unacceptably large contributions to the EDM's. Similar constraints hold for the
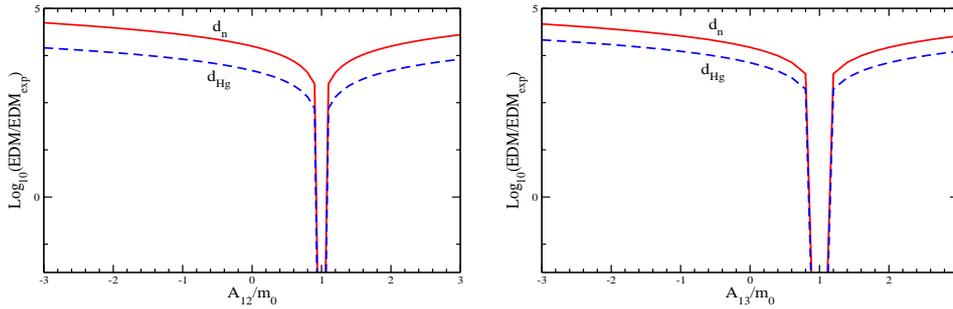
Figure 1: Neutron and mercury EDM's as function of the off-diagonal entries A_{12} and A_{13} for $\tan\beta = 5$ and $m_0 = m_{1/2} = 250$ GeV. $A_{ij} = 1 \; \forall ij \neq 12(13)$ on the left (right) figure.

other off-diagonal elements A_{21}, A_{31}, A_{23} and A_{32}, and this situation is far more severe than the case of hierarchical Yukawa couplings. Since Hermitian USY couplings lead to an unrealistic sum rule for the quark masses, the associated EDM problem cannot be overcome by taking simultaneously Hermitian Yukawa and trilinear couplings [9]. An interesting possibility is to have trilinear terms that can be factorized as $\hat{A} = A_{ij}Y_{ij} = A.Y$ or $Y.A$ [10]. This factorization implies that the mass insertion $(\delta^d_{11})_{LR}$ is suppressed by the ratio $m_d/m_{\tilde{q}}$. To be specific, let us consider the following structure

$$A = m_0 \begin{pmatrix} a & a & a \\ b & b & b \\ c & c & c \end{pmatrix} . \tag{4}$$

In this case the trilinear couplings $\hat{A}$ can be written as $\hat{A} = \mathrm{diag}(a, b, c).Y$, with complex (a, b, c). For $|A_{ij}| \simeq 3m_0$, the EDM's constraint on these phases is quite stringent:

$$\varphi_a \lesssim 0.02 \text{ rad} \quad ; \quad \varphi_b \lesssim 0.35 \text{ rad} \quad ; \quad \varphi_c \lesssim 0.02 \text{ rad} . \tag{5}$$

Thus, we can conclude that in USY models the maximal allowed non-universality for the A-terms is the structure presented in Eq. (4), with the associated SUSY phases (φ_{ij}) within the limits given in Eq. (5). In view of this, we analyse the contributions to the kaon system CP violating observables, ε_K and ε'/ε.

Let us start our analysis by considering the indirect CP violating parameter of the K sector, ε_K. In the presence of supersymmetric ($\tilde{g}$ and $\tilde{\chi}^{\pm}$) contributions, the off-diagonal entry in the kaon mass matrix can be decomposed as $M_{12} = M_{12}^{\mathrm{SM}} + M_{12}^{\tilde{g}} + M_{12}^{\tilde{\chi}^{\pm}}$. Within the present scenario, with USY phases as in Table 1, we find that the SM contribution to ε_K is $\mathcal{O}(10^{-5})$. Regarding the supersymmetric contributions, and taking the A-terms as in Eq. (4), we have studied the behaviour of the $(\delta^d_{12})_{LL}$, $(\delta^d_{12})_{LR}$ and $(\delta^u_{12})_{LL}$, mass insertions, focusing on the correlation between the $(\delta^d_{12})_{LL}$ and $(\delta^d_{12})_{LR}$. We have verified that gluino contributions clearly dominate over those of the charginos, and that the LL mass insertions provide the dominant gluino contribution. We notice that in our model, the large mixing displayed by the rotation matrices enhances any non-diagonal

contribution to $M_{\tilde{Q}_L}^2$ induced by the trilinear terms from RG evolution, hence LL mass insertions can account for the experimental value of ε_K.

We then consider supersymmetric contributions to the direct CP violation parameter, ε'/ε, which can be symbolically decomposed as a sum of the SM, gluino, and chargino terms, the latter receiving contributions from diagrams with one and two mass insertions in the squark internal line.

In USY scenarios, the SM prediction for ε'/ε is found to be $\mathcal{O}(10^{-6})$. The supersymmetric contributions were analysed with detail in [3]. In order to discuss the rôle of each contribution within our model, in Fig. 2(a) we plot the gluino and chargino contributions to ε'/ε as function of δA, which is defined as

$$\delta A = (A_{3i} - A_{1i,2i})/m_0 = c - a \quad \text{(assuming } a = b) \ . \tag{6}$$

The SUSY CP violating phases have been fixed as: $\varphi_a = \varphi_c = 0$ and $\varphi_b = 0.1$. As in the previous figures, we have assumed $\tan \beta = 5$ and $m_0 = m_{1/2} = 250$ GeV.
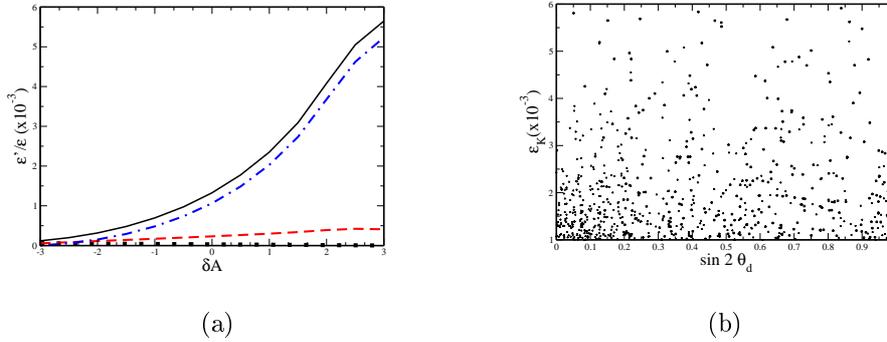

(a)   (b)

Figure 2: (a) Contributions to ε'/ε versus the non-universality parameter δA. The dotted line refers to gluino contributions, the dot-dashed and dashed lines to those of the charginos (with one and two mass insertions, respectively). The solid line stands for the total supersymmetric contribution to ε'/ε. (b) Correlation between ε_K and $\sin 2\theta_d$.

As can be seen from this figure, the chargino contributions with one mass insertion give the dominant contribution to ε'/ε. The chargino contributions with two mass insertions that arise from the SUSY effective $\bar{s}dZ$ vertex are also relevant and considerably larger than that of the gluino.

Finally, we address the issue of CP violation in the B_d meson system. The present world average, $a_{J/\psi K_S} = 0.79 \pm 0.12$, is dominated by the BaBar and Belle results [11]. In the case of supersymmetric contributions to the $\Delta B = 2$ transition, the ratio of the total and partial SM contributions can be parametrized as $r_d^2 e^{2i\theta_d} = M_{12}(B_d)/M_{12}^{\text{SM}}(B_d)$, hence the measurement of $a_{J/\psi K_S}$ does not determine $\sin 2\beta$ but rather $\sin(2\beta + 2\theta_d)$. The SM contributions to $\sin 2\beta$ associated with the phases of Table 1 are negligible, $\mathcal{O}(10^{-3})$.

Regarding SUSY contributions, we took into account charged-Higgs, chargino and gluino contributions to $M_{12}^{\text{SUSY}}(B_d)$. Since in this class of models flavour mixing is not suppressed, one finds that the $(\delta_{13}^{d,u})_{LL}$ and $(\delta_{13}^{d,u})_{LR}$ mass insertions can be significantly

large and saturate the experimental result. We have found that within our scenario, the leading supersymmetric contribution to $\sin(2\beta + 2\theta_d)$ was associated with the $\Delta B = 2$ chargino mediated box diagrams. In Fig. 2(b) we present the correlation between the values of ε_K and $\sin 2\theta_d$ for $\tan\beta = 5$ and $m_0 = m_{1/2} = 250$ GeV. The values of the parameters a, b, and c are randomly selected in the range $[-3, 3]$ and the phases fixed as $\varphi_a = \varphi_c = 0$ and $\varphi_b = 0.1$. From Fig. 2(b), it is clear that one can have $\sin 2\theta_d$ within the experimental range while having a prediction for ε_K and ε'/ε compatible with the measured value. This result appears as characteristic of the kind of models under consideration. Despite the smallness of the phases introduced in these models via the Yukawa and trilinear couplings, they are sufficient to account for CP violation in the K-system, as well as the observed value of $a_{J/\psi K_S}$.

4   Discussion and Conclusions

In this work we have studied the implications of having universal strength of Yukawa couplings within the unconstrained MSSM.

We have argued that the trilinear soft terms play a key rôle in embedding USY into SUSY. In fact, due to the large mixing and associated phases, the constraints from the EDM's on the SUSY parameter space are far more stringent than in the case of a standard Yukawa parametrization. We found that in order to satisfy the bound of the mercury EDM, the A-terms should be matrix factorizable, with phases constrained to be in the range $10^{-2} - 10^{-1}$.

We have investigated the new contributions to both K and B system CP observables, finding that gluino mediated boxes with LL mass insertions provide the leading contributions to ε_K, while ε'/ε is dominated by chargino loops, through LL flavour mixing. Regarding the B system, we argued that within this model supersymmetric chargino exchanges, provide the leading contributions, which are in agreement with the recent measurements at BaBar and Belle.

In conclusion, we have presented an alternative scenario for CP violation, where the strength of CP violation stemming from the SM is naturally small, so that new SUSY contributions are essential to generate the correct value of ε_K and ε'/ε, as well as the recently observed large value of $a_{J/\psi K_S}$.

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