The MSSM in the Light of Precision Data

S. Heinemeyer[1] and G. Weiglein[2]

[1]*Institut für theoretische Elementarteilchenphysik, LMU München, Theresienstr. 37, D–80333 München, Germany*

[2]*Institute for Particle Physics Phenomenology, University of Durham, Durham DH1 3LE, UK*

Abstract

The potential of present and anticipated future electroweak precision data, including the Higgs boson and top quark masses, for testing quantum effects of the electroweak theory is investigated in the context of the Minimal Supersymmetric Standard Model (MSSM). The present status of the theoretical predictions is analyzed. The impact of the parametric uncertainties from the experimental errors of the input parameters is studied, and an estimate for the remaining uncertainties from unknown higher-order corrections is given both in the Standard Model (SM) and the MSSM. Examples of electroweak precision tests in the mSUGRA scenario and the unconstrained MSSM are analyzed, and the status of the global fit to all data is discussed.

1 Introduction

Theories based on Supersymmetry (SUSY) [1] are widely considered as the theoretically most appealing extension of the Standard Model (SM). They are consistent with the approximate unification of the gauge coupling constants at the GUT scale and provide a way to cancel the quadratic divergences in the Higgs sector hence stabilizing the huge hierarchy between the GUT and the Fermi scales. Furthermore, in SUSY theories the breaking of the electroweak symmetry is naturally induced at the Fermi scale, and the lightest supersymmetric particle can be neutral, weakly interacting and absolutely stable, providing therefore a natural solution for the dark matter problem. SUSY predicts the existence of scalar partners $\tilde{f}_L, \tilde{f}_R$ to each SM chiral fermion, and spin–1/2 partners to the gauge bosons and to the scalar Higgs bosons. So far, the direct search for SUSY particles has not been successful. One can only set lower bounds of $\mathcal{O}(100)$ GeV on their masses [2].

An alternative way to probe SUSY is via the virtual effects of the additional particles to precision observables. This requires a very high precision of the experimental results as well as of the theoretical predictions. The most relevant electroweak precision observables

(EWPO) in this context are the W boson mass, M_W, the effective leptonic weak mixing angle, $\sin^2\theta_{\rm eff}$, and the mass of the lightest $\mathcal{CP}$-even MSSM Higgs boson, m_h. Contrary to the SM case, where the mass of the Higgs boson is a free parameter, within the MSSM the quartic couplings of the Higgs potential are fixed in terms of the gauge couplings as a consequence of SUSY [3]. Thus, at the tree-level, the Higgs sector is determined by just two independent parameters besides the SM electroweak gauge couplings g and g', conventionally chosen as $\tan\beta = v_2/v_1$, the ratio of the vacuum expectation values of the two Higgs doublets, and M_A, the mass of the $\mathcal{CP}$-odd A boson. As a consequence, the mass of the lightest $\mathcal{CP}$-even MSSM Higgs boson can be predicted in terms of the other model parameters.

An upper bound of $m_h \lesssim 135$ GeV [4, 5] can be established, taking into account all existing higher-order corrections (for $m_t = 175$ GeV and a common soft SUSY-breaking scale of $M_{\rm SUSY} = 1$ TeV). The prospective accuracy of the measurement of the Higgs-boson mass at the LHC of about 200 MeV [6] or at an e^+e^- linear collider (LC) of even 50 MeV [7–9] will promote m_h to a precision observable. Owing to the sensitive dependence of m_h on especially the scalar top sector, the measured value of m_h will allow to set stringent constraints on the parameters in this sector.

In the unconstrained MSSM no specific assumptions are made about the underlying SUSY-breaking mechanism, and a parameterization of all possible SUSY-breaking terms is used. This gives rise to the huge number of more than 100 new parameters in addition to the SM ones, which in principle can be chosen independently of each other. A phenomenological analysis of this model in full generality would clearly be very involved, and one usually restricts to certain benchmark scenarios, see e.g. Refs. [10–12]. On the other hand, models in which all the low-energy parameters are determined in terms of a few parameters at the Grand Unification scale (or another high-energy scale), employing a specific soft SUSY-breaking scenario, are much more predictive. The most prominent scenarios in the literature are minimal Supergravity (mSUGRA) [1], minimal Gauge Mediated SUSY Breaking (mGMSB) [13] and minimal Anomaly Mediated SUSY Breaking (mAMSB) [14–16]. Analyses comparing the Higgs sector in these scenarios and discussing implications for searches at present and future colliders can be found in Refs. [17, 18].

Examples for the current experimental status of EWPO are given in Tab. 1, including their relative experimental precision. The quantities in the first three lines, M_Z, G_F, and m_t, are usually employed as input parameters for the theoretical predictions. The observables M_W, $\sin^2\theta_{\rm eff}$, Γ_Z, on the other hand, are the three most prominent observables for testing the electroweak theory by comparing the experimental results with the theory predictions. Comparing the typical size of electroweak quantum effects, which is at the per cent level, with the relative accuracies in Tab. 1, which are at the per mille level, clearly shows the sensitivity of the electroweak precision data to loop effects.

The prospective accuracy that can be achieved for electroweak precision observables at the next generation of colliders, including m_t and m_h, has been analyzed in detail in Ref. [19] and is reviewed in Tab. 2.

	central value	absolute error	relative error
M_Z [GeV]	91.1875	± 0.0021	$\pm 0.002\%$
G_F [GeV^{-2}]	1.16637×10^{-5}	$\pm 0.00001 \times 10^{-5}$	$\pm 0.0009\%$
m_t [GeV]	174.3	± 5.1	$\pm 2.9\%$
M_W [GeV]	80.426	± 0.034	$\pm 0.04\%$
$\sin^2\theta_{\rm eff}$	0.23148	± 0.00017	$\pm 0.07\%$
Γ_Z [GeV]	2.4952	± 0.0023	$\pm 0.09\%$

Table 1: Examples of EWPO with their current absolute and relative experimental errors (see text).

	now	Tev. Run IIA	Run IIB	LHC	LC	GigaZ
$\delta \sin^2\theta_{\rm eff} (\times 10^5)$	17	78	29	14–20	(6)	1.3
δM_W [MeV]	34	27	16	15	10	7
δm_t [GeV]	5.1	2.7	1.4	1.0	0.2	0.13
δm_h [MeV]	—	—	$\mathcal{O}(2000)$	100	50	50

Table 2: Current and anticipated future experimental uncertainties for $\sin^2\theta_{\rm eff}$, M_W, m_t, and m_h. See Ref. [19] for a detailed discussion and further references.

2 Theory status of precision observables in the MSSM

In this section we discuss the theory status of the various EWPO in the MSSM and for sake of comparison also in the SM. In order to analyze virtual effects of SUSY, it is in general not sufficient to restrict to certain parameterizations, like the S, T, U parameters [20] (which are only applicable for specific types of new physics contributions and are intrinsically one-loop quantities; for a discussion of this issue, see Ref. [21]). Instead, the MSSM predictions for the actual observables need to be worked out in detail.

Concerning the situation in the SM, as will be described in detail below, the level of accuracy for EWPO is quite advanced. Obtaining predictions for observables in the MSSM at a certain order requires in general a higher effort than for the SM case. This is related to the fact that in the MSSM many additional parameters enter, in particular new mass scales. The level of accuracy achieved so far in the MSSM is therefore somewhat lower than in the SM.

Furthermore, besides the known sources of sizable corrections in the SM, e.g. contributions enhanced by powers of m_t or logarithms of light fermions, there are additional sources of possibly large corrections within the MSSM:

- Large corrections can arise not only from loops containing the top quark, but also its scalar superpartners. Corrections from the top and scalar top quark sector of the MSSM can be especially large in the MSSM Higgs sector, where one-loop corrections can reach the level of 100%. The leading one-loop term from the top and scalar top sector entering the predictions in the Higgs sector is given by [22]

$$\sim G_F\, m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) . \tag{2.1}$$

- Effects from the $b/\tilde{b}$ sector of the MSSM can also be very important for large $\tan\beta$.

- The b Yukawa coupling can receive large SUSY corrections, yielding a shift in the relation between the b quark mass and the corresponding Yukawa coupling [23],

$$y_b = \frac{\sqrt{2}}{v\,\cos\beta}\,\frac{m_b}{1+\Delta m_b} . \tag{2.2}$$

  The quantity Δm_b contains in particular a contribution involving a gluino in the loop, which gives rise to a correction proportional to $(\alpha_s\,\mu\,m_{\tilde{g}}\,\tan\beta)$, which can be large. For $\Delta m_b \to -1$ the b Yukawa coupling even becomes non-perturbative.

- In general, SUSY loop contributions can become large if some of the SUSY particles are relatively light.

2.1 Electroweak precision observables

Within the SM, very accurate results are in particular available for M_W, where meanwhile all ingredients of the complete two-loop result are known [24, 25] (as well as leading QCD and electroweak three-loop corrections). Taking into account the latest result obtained in Ref. [26], the remaining theoretical uncertainties from unknown higher-order corrections within the SM can be estimated to be (using the methods described in Refs. [19, 25, 27])

$$\text{SM}: \quad \delta M_W^{\text{th}} \approx \pm 4 \text{ MeV}, \quad \delta \sin^2\theta_{\text{eff}}^{\text{th}} \approx \pm 6 \times 10^{-5}. \tag{2.3}$$

They are considerably smaller at present than the parametric uncertainties from the experimental errors of the input parameters m_t and $\Delta\alpha_{\text{had}}$. The experimental errors of $\delta m_t = \pm 5.1$ GeV and $\delta(\Delta\alpha_{\text{had}}) = 36 \times 10^{-5}$ [28] induce parametric theoretical uncertainties of

$$\begin{aligned} \delta m_t: \quad & \delta M_W^{\text{para}} \approx \pm 31 \text{ MeV}, \quad && \delta \sin^2\theta_{\text{eff}}^{\text{para}} \approx \pm 16 \times 10^{-5}, \\ \delta(\Delta\alpha_{\text{had}}): \quad & \delta M_W^{\text{para}} \approx \pm 6.5 \text{ MeV}, \quad && \delta \sin^2\theta_{\text{eff}}^{\text{para}} \approx \pm 13 \times 10^{-5}. \end{aligned} \tag{2.4}$$

This has to be compared with the current experimental errors given in Tab. 1.

At one-loop order, complete results for the electroweak precision observables M_W and $\sin^2\theta_{\text{eff}}$ are also known within the MSSM. At the two-loop level, the leading corrections in $\mathcal{O}(\alpha\alpha_s)$ have been obtained [29], which enter via the quantity $\Delta\rho$,

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}. \tag{2.5}$$

It parameterises the leading universal corrections to the electroweak precision observables induced by the mass splitting between fields in an isospin doublet [30]. $\Sigma_{Z,W}(0)$ denote the transverse parts of the unrenormalized Z- and W-boson self-energies at zero momentum transfer, respectively. The induced shifts in M_W and $\sin^2\theta_{\rm eff}$ are in leading order given by (with $1 - s_W^2 \equiv c_W^2 = M_W^2/M_Z^2$)

$$\delta M_W \approx \frac{M_W}{2}\frac{c_W^2}{c_W^2 - s_W^2}\Delta\rho, \quad \delta \sin^2\theta_{\rm eff} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2}\Delta\rho. \tag{2.6}$$

For the gluonic corrections, results in $\mathcal{O}(\alpha\alpha_s)$ have also been obtained for the prediction of M_W [31]. The comparison with the contributions entering via $\Delta\rho$ showed that in this case indeed the full result is well approximated by the $\Delta\rho$ contribution. Contrary to the SM case, the two-loop $\mathcal{O}(\alpha\alpha_s)$ corrections turned out to increase the one-loop contributions, leading to an enhancement of up to 35% [29].

Recently the leading two-loop corrections to $\Delta\rho$ at $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ ($\alpha_{t,b} \equiv y_{t,b}^2/(4\pi)$, $y_{t,b}$ being the top and bottom Yukawa couplings, respectively) have been obtained for the case of a large SUSY scale, $M_{\rm SUSY} \gg M_Z$ [32, 33]. These contributions involve the top and bottom Yukawa couplings and contain in particular corrections proportional to m_t^4 and bottom loop corrections enhanced by $\tan\beta$. As an example, the effect of the $\mathcal{O}(\alpha_t^2)$ MSSM contributions on δM_W amounts up to -12 MeV, see Fig. 1. The 'effective' change in M_W in comparison with the corresponding SM result with the same value of the Higgs-boson mass is significantly smaller. It amounts up to -3 MeV and goes to zero for large M_A as expected from the decoupling behavior.

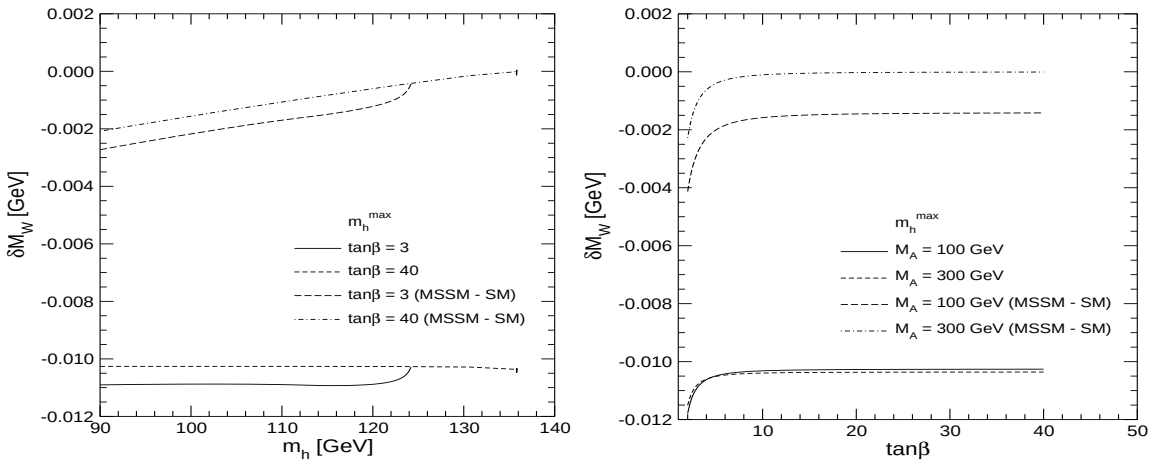

Figure 1: Contribution of the $\mathcal{O}(\alpha_t^2)$ MSSM corrections to M_W as a function of m_h (left) and $\tan\beta$ (right) in the $m_h^{\rm max}$ scenario [11].

Comparing the presently available results for the electroweak precision observables M_W and $\sin^2\theta_{\rm eff}$ in the MSSM with those in the SM (as given in Eq. (2.3)), a crude estimate yields

$$\text{MSSM}: \quad \delta M_W^{\rm th} \approx \pm 10 \text{ MeV}, \quad \delta \sin^2\theta_{\rm eff}^{\rm th} \approx \pm 12 \times 10^{-5}. \tag{2.7}$$

Thus, the uncertainties from unknown higher-order corrections within the MSSM are about twice as large as in the SM in the case of $\sin^2\theta_{\rm eff}$ and even larger for M_W.

2.2 The lightest $\mathcal{CP}$-even Higgs boson mass

The mass of the lightest $\mathcal{CP}$-even MSSM Higgs boson can be predicted from the other model parameters. At the tree-level, the two $\mathcal{CP}$-even Higgs boson masses are obtained by rotating the neutral $\mathcal{CP}$-even Higgs boson mass matrix with an angle α,

$$M_{\rm Higgs}^{2,\rm tree} = \begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}, \tag{2.8}$$

with α satisfying

$$\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \quad -\frac{\pi}{2} < \alpha < 0. \tag{2.9}$$

In the Feynman-diagrammatic approach the higher-order corrected Higgs boson masses are derived by finding the poles of the h, H-propagator matrix whose inverse is given by

$$(\Delta_{\rm Higgs})^{-1} = -i \begin{pmatrix} p^2 - m_{H,\rm tree}^2 + \hat{\Sigma}_{HH}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_{h,\rm tree}^2 + \hat{\Sigma}_{hh}(p^2) \end{pmatrix}, \tag{2.10}$$

where the $\hat{\Sigma}(p^2)$ denote the renormalized Higgs-boson self-energies, p being the momentum going through the external legs. Determining the poles of the matrix $\Delta_{\rm Higgs}$ in Eq. (2.10) is equivalent to solving the equation

$$\left[p^2 - m_{h,\rm tree}^2 + \hat{\Sigma}_{hh}(p^2)\right]\left[p^2 - m_{H,\rm tree}^2 + \hat{\Sigma}_{HH}(p^2)\right] - \left[\hat{\Sigma}_{hH}(p^2)\right]^2 = 0\,. \tag{2.11}$$

The status of the available results for the self-energy contributions to Eq. (2.10) can be summarized as follows. For the one-loop part, the complete result within the MSSM is known [22,34,35]. The by far dominant one-loop contribution is the $\mathcal{O}(\alpha_t)$ term due to top and stop loops. Concerning the two-loop effects, their computation is quite advanced and it has now reached a stage such that all the presumably dominant contributions are known, see Ref. [5,36] and references therein. They include the strong corrections, usually indicated as $\mathcal{O}(\alpha_t\alpha_s)$, and Yukawa corrections, $\mathcal{O}(\alpha_t^2)$, to the dominant one-loop $\mathcal{O}(\alpha_t)$ term, as well as the strong corrections to the bottom/sbottom one-loop $\mathcal{O}(\alpha_b)$ term, i.e. the $\mathcal{O}(\alpha_b\alpha_s)$ contribution. For the $b/\tilde{b}$ sector corrections also an all-order resummation of the $\tan\beta$-enhanced terms, $\mathcal{O}(\alpha_b(\alpha_s\tan\beta)^n)$, is known. Most recently the $\mathcal{O}(\alpha_t\alpha_b)$ and $\mathcal{O}(\alpha_b^2)$ corrections have been derived [37]. All two-loop corrections have been obtained by neglecting the external momentum.

An upper bound of $m_h \lesssim 135$ GeV [4,5] can be established [38] taking into account all existing higher-order corrections (in the $m_h^{\rm max}$ scenario with $m_t = 174.3$ GeV and $M_{\rm SUSY} = 1$ TeV, see Refs. [10,11]).

The remaining theoretical higher-order uncertainties in m_h have been analyzed in detail in Ref. [5]. This has been done by

- extrapolating from the size of the existing one-loop corrections to the missing two-loop contributions,
- changing the renormalization scale in the one-loop result [39] in order to estimate missing two-loop corrections

- changing the renormalization of the top quark mass at the two-loop level in order to estimate remaining three-loop contributions,
- from the result for the leading three-loop contribution.

As a result, the remaining theoretical uncertainty from unknown higher orders has been estimated to be

$$\delta m_h^{\rm th} \approx \pm 3 \text{ GeV} . \tag{2.12}$$

Concerning the parametric uncertainties from the experimental errors of the input parameters, in particular the current experimental error of the top-quark mass of $\delta m_t = \pm 5.1$ GeV has a very large effect [40],

$$\delta m_t : \quad \delta m_h^{\rm para} \approx \pm 5 \text{ GeV} . \tag{2.13}$$

In order to enable sensitive electroweak precision tests in the MSSM Higgs sector a drastic reduction of the parametric uncertainty induced by δm_t will be crucial [41]. This can be achieved with the measurement of m_t at the LC [7–9],

$$\delta m_t^{\rm exp,LC} \lesssim 100 \text{ MeV} . \tag{2.14}$$

Besides a drastically improved experimental precision on m_t, obviously also a big reduction of the theoretical uncertainties from unknown higher-order corrections will be necessary. In order to match the future precision on m_h with the accuracy of the theoretical prediction, the theoretical error has to be reduced by at least a factor of 10. This will require a complete two-loop calculation, including the external momentum, dominant three-loop and possibly even leading four-loop corrections.

2.3 $B \to X_s\gamma$ and the anomalous magnetic moment of the muon

Examples of further observables where virtual effects of SUSY particles can be very important are the branching ratio for $B \to X_s\gamma$ and the anomalous magnetic moment of the muon, $g_\mu - 2$.

The branching ratio BR($b \to s\gamma$) can receive large SUSY corrections for light charged Higgs bosons and large μ or $\tan\beta$. The flavour-changing neutral current processes impose very important constraints on the parameter space both of general two-Higgs-doublet models and of the MSSM. The currently available SUSY contributions to BR($b \to s\gamma$) include the one-loop result and leading higher-order corrections [42].

SUSY contributions to $g_\mu - 2$ are particularly important for large $\tan\beta$ and light gaugino and slepton masses. The one-loop result in the MSSM has been supplemented by leading logarithmic two-loop contributions [43]. The comparison of the theoretical predictions with the experimental result [44] is affected by sizable QCD uncertainties, see Ref. [45] for a discussion.

3 Precision tests of the SM and the MSSM

Before investigating the case of the unconstrained MSSM, we first focus on the example of mSUGRA as a particular SUSY-breaking scenario. It is interesting to note that the

rather restricted scenario of mSUGRA is still compatible with all available constraints from EWPO, the Higgs boson sector, cold dark matter (CDM) [46], BR($b \to s\gamma$) [47] and from the anomalous magnetic moment of the muon, $g_\mu - 2$ [44], see e.g. Refs. [48, 49]. The constraints from BR($b \to s\gamma$) and in particular $g_\mu - 2$ favor the positive sign of the parameter μ, see e.g. Ref. [50]. The prediction for $g_\mu - 2$, however, is still affected by sizable QCD uncertainties, as discussed above.

We now turn to the unconstrained MSSM and compare it with the EWPO data. In Fig. 2 we compare the SM and the MSSM prediction for M_W as a function of m_t [51]. The predictions within the two models give rise to two bands in the m_t–M_W plane with only a relatively small overlap region (indicated by a blue area in Fig. 2). The allowed parameter region in the SM (the red and blue bands) arises from varying the only free parameter of the model, the mass of the SM Higgs boson, from $M_H = 113$ GeV (upper edge of the blue area) to 400 GeV (lower edge of the red area). The green and the blue areas indicate the allowed region for the unconstrained MSSM. SUSY masses close to their experimental lower limit are assumed for the upper edge of the green area, while the decoupling limit with SUSY masses of $\mathcal{O}(2$ TeV) yields the lower edge of the blue area. Thus, the overlap region between the predictions of the two models corresponds in the SM to the region where the Higgs boson is light, i.e. in the MSSM allowed region ($m_h \lesssim 135$ GeV). In the MSSM it corresponds to the case where all superpartners are heavy, i.e. the decoupling region of the MSSM. The current 68% C.L. experimental results for m_t and M_W slightly favor the MSSM over the SM. The prospective accuracies for the LHC and the LC with GigaZ option, see Tab. 2, are also shown in the plot (using the current central values), indicating the potential for a significant improvement of the sensitivity of the electroweak precision tests [52].

In Fig. 3 the comparison between the SM and the MSSM is shown in the M_W–$\sin^2\theta_{\rm eff}$ plane. As above, the predictions in the SM (red and blue bands) and the MSSM (green and blue bands) are shown together with the current 68% C.L. experimental results and the prospective accuracies for the LHC and the LC with GigaZ option. Again the MSSM is slightly favored over the SM. It should be noted that the prospective improvements in the experimental accuracies, in particular at a LC with GigaZ option, will provide a high sensitivity to deviations both from the SM and the MSSM.

The central value for the experimental value of $\sin^2\theta_{\rm eff}$ in Fig. 3 is based on both leptonic and hadronic data. The fact that the two most precise measurements, $A_{\rm LR}$ from SLD [53] and $A_{\rm FB}^{\rm b}$ from LEP [54], differ from each other by about 3σ, giving rise to a relatively low fit probability of the SM global fit, has caused considerable attention in the literature. In particular, several analyses have been performed where the hadronic data on $A_{\rm FB}$ have been excluded from the global fit (see e.g. Refs. [55, 56]). It has been noted that in this case the SM global fit, possessing a much higher fit probability, yields an upper bound on M_H which is rather low in view of the experimental lower bound on M_H of $M_H > 114.4$ GeV [57]. The value of $\sin^2\theta_{\rm eff}$ corresponding to the measurement of $A_{\rm LR}$(SLD) alone is $\sin^2\theta_{\rm eff} = 0.23098 \pm 0.00026$ [53]. Fig. 3 shows that adopting the latter value of $\sin^2\theta_{\rm eff}$ makes the agreement between the data and the SM prediction much worse, while the MSSM provides a very good description of the data. In accordance with this result, in Ref. [56] it has been found that the contribution of light gauginos and scalar leptons in the MSSM (in a scenario with vanishing SUSY contribution to $\Delta\rho$) gives

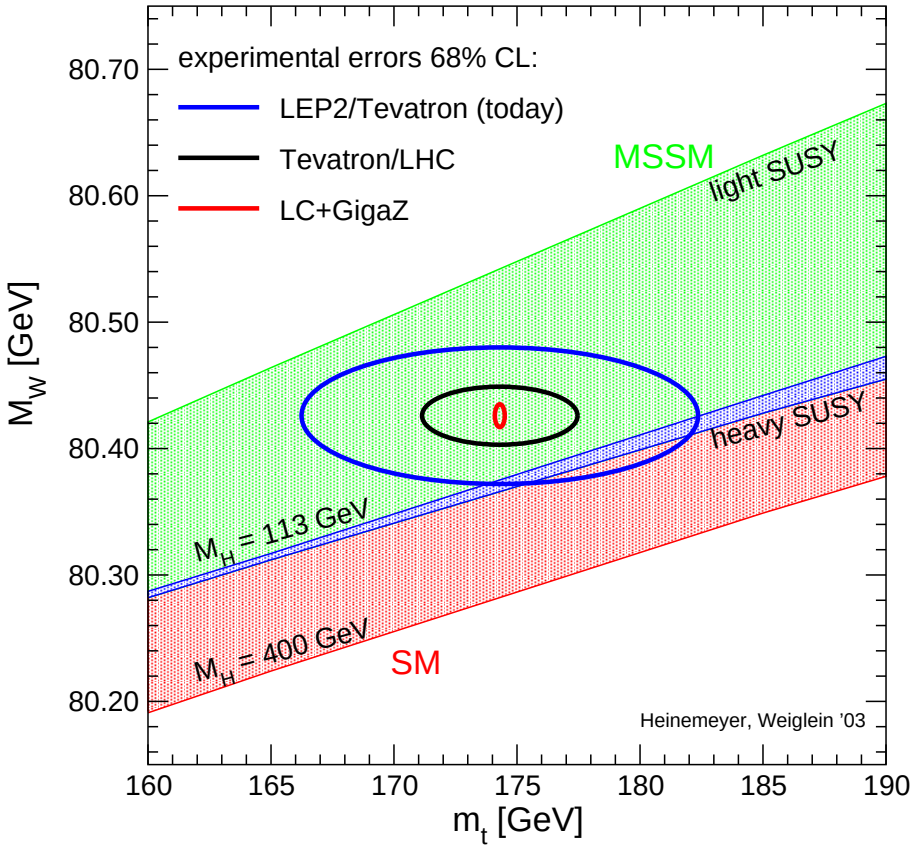

Figure 2: The current experimental results for M_W and m_t and the prospective accuracies at the next generation of colliders are shown in comparison with the SM prediction (red and blue bands) and the MSSM prediction (green and blue bands).

rise to a shift in M_W and $\sin^2\theta_{\rm eff}$ as compared to the SM case which brings the MSSM prediction in agreement with the experimental values of M_W and $A_{\rm LR}$(SLD).

On the other hand, it has also been investigated whether the discrepancy between $A_{\rm LR}$ and $A_{\rm FB}^{\rm b}$ could be explained in terms of contributions of some kind of new physics. The (loop-induced) contributions from SUSY particles in the MSSM are however too small to account for the 3σ difference between the two observables (see e.g. Ref. [56]). Thus, the quality of the fit to $A_{\rm LR}$ and $A_{\rm FB}^{\rm b}$ in the MSSM is similar to the one in the SM.

Another observable for which the SM prediction shows a large deviation by about 3σ from the experimental value is the neutrino–nucleon cross section measured at NuTeV [58]. Also in this case loop effects of SUSY particles in the MSSM are too small to account for a sizable fraction of the discrepancy (see e.g. Ref. [59]).

A global fit to all data has been performed within the MSSM in Ref. [60]. The results are shown in Fig. 4, where the predictions in the SM, the MSSM and the constrained MSSM (i.e. the mSUGRA scenario) are compared with the experimental data (the SUSY predictions are for $\tan\beta = 35$). Fig. 4 shows the features discussed above: the MSSM predictions for M_W and (for large $\tan\beta$) $g_\mu - 2$ are in better agreement with the data than in the SM (slight improvements also occur for the total width of the Z boson, Γ_Z, and for

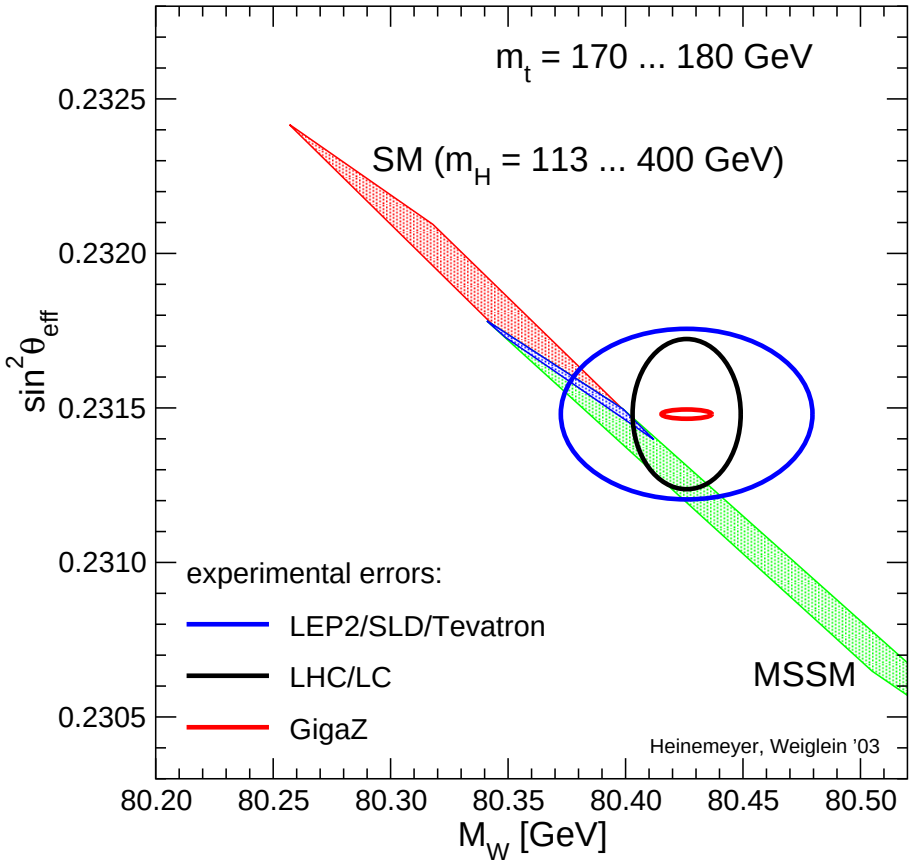

Figure 3: The current experimental results for M_W and $\sin^2\theta_{\rm eff}$ and the prospective accuracies at the next generation of colliders are shown in comparison with the SM prediction (red and blue bands) and the MSSM prediction (green and blue bands).

$B \to X_s\gamma$). On the other hand, for the observables with the largest deviations between theory and experiment, namely $A_{\rm FB}^{\rm b}$ and the neutrino–nucleon cross section measured at NuTeV (the latter is not shown in Fig. 4), the MSSM does not yield a significant improvement compared to the SM. The global fit in the MSSM has a lower χ^2 value than in the SM. Since the MSSM fit has less degrees of freedom than the SM one, the overall fit probability in the MSSM is only slightly better than in the SM.

4 Conclusions

We have investigated electroweak precision tests in the framework of the MSSM. Compared to the SM, the MSSM contains several new sources for potentially large radiative corrections. Of particular importance is the Higgs sector of the MSSM. While within the SM the Higgs-boson mass is a free parameter, the relation between the mass of the lightest $\mathcal{CP}$-even Higgs boson of the MSSM and the other model parameters is one of the most striking predictions of SUSY models.

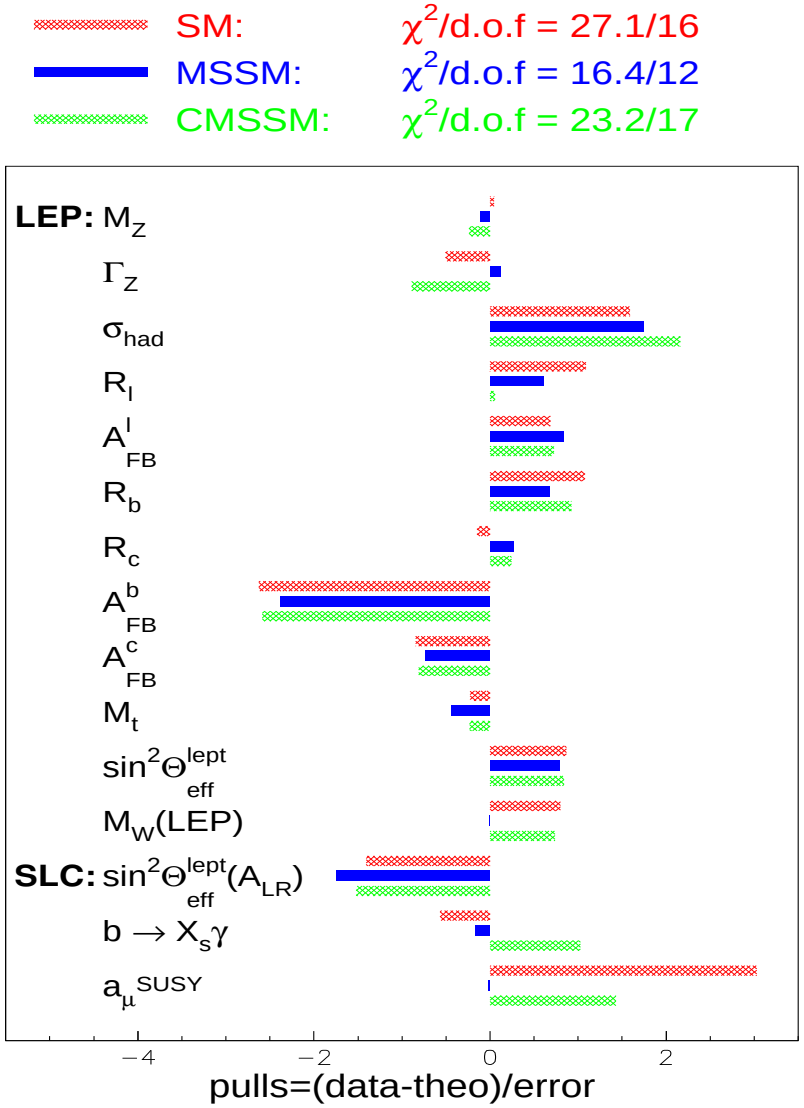

Figure 4: The predictions in the SM, the MSSM and the mSUGRA scenario (CMSSM) are compared with the data. Deviations between theory and experiment are indicated in units of one standard deviation of the experimental results (from Ref. [60]).

We have summarized the theory status of the precision observables in the MSSM and have given an estimate of the remaining theoretical uncertainties from unknown higher-order corrections. We find that the present theoretical uncertainties for M_W and the Z-boson observables in the MSSM are still significantly higher than in the SM.

We have discussed examples of electroweak precision tests in the context of the mSUGRA scenario and the unconstrained MSSM. The mSUGRA scenario, despite its small number of free parameters, can accommodate all experimental constraints, i.e. the ones from the electroweak precision data, the anomalous magnetic moment of the muon, $B \to X_s\gamma$, the cold dark matter constraints, and the lower bounds from the Higgs and SUSY particle

searches. The global fit to all data in the MSSM yields a better agreement for M_W and $g_\mu - 2$ than in the SM, while no significant improvements occur for $A_{\rm FB}^{\rm b}$ and the neutrino–nucleon cross section measured at NuTeV. The overall fit probability in the MSSM is only slightly better than the one in the SM.

We have furthermore analyzed the potential of anticipated future electroweak precision data, including the Higgs boson and the top-quark mass, for testing quantum effects of the MSSM. In order to match the prospective accuracy of m_h achievable at the LHC with the theoretical prediction, an improvement of the theoretical uncertainties from unknown higher-order corrections by more than a factor ten will be necessary. In order to reduce the parametric theoretical uncertainties to the same level, the precision measurement of the top-quark mass at the LC will be crucial. Significant improvements of both the parametric uncertainties and the ones from unknown higher-order corrections will also be necessary in view of the prospective experimental accuracies of M_W and $\sin^2\theta_{\rm eff}$ at the next generation of colliders. Thus, substantial progress in the theoretical predictions will be necessary in order to exploit electroweak precision physics in the MSSM, which might become possible at the next generation of colliders.

References

[1] H. P. Nilles, Phys. Rept. **110** (1984) 1;
H. E. Haber and G. L. Kane, Phys. Rept. **117** (1985) 75;
R. Barbieri, Riv. Nuovo Cim. **11N4** (1988) 1.

[2] K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66** (2002) 010001.

[3] J. Gunion, H. Haber, G. Kane and S. Dawson, *The Higgs Hunter's Guide*, Addison-Wesley, 1990.

[4] S. Heinemeyer, W. Hollik and G. Weiglein, Eur. Phys. J. C **9** (1999) 343 [arXiv:hep-ph/9812472].

[5] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, Eur. Phys. J. C **28** (2003) 133 [arXiv:hep-ph/0212020].

[6] ATLAS Collaboration, Technical Design Report 1999, Vol. II, CERN/LHC/99-15, ATLAS TDR 15.

[7] J. A. Aguilar-Saavedra *et al.* [ECFA/DESY LC Physics Working Group Collaboration], arXiv:hep-ph/0106315.

[8] T. Abe *et al.* [American Linear Collider Working Group Collaboration], in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, arXiv:hep-ex/0106056.

[9] K. Abe *et al.* [ACFA Linear Collider Working Group Collaboration], arXiv:hep-ph/0109166.

[10] M. Carena, S. Heinemeyer, C. E. Wagner and G. Weiglein, arXiv:hep-ph/9912223.

[11] M. Carena, S. Heinemeyer, C. E. Wagner and G. Weiglein, Eur. Phys. J. C **26** (2003) 601 [arXiv:hep-ph/0202167].

[12] B. C. Allanach *et al.*, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, Eur. Phys. J. C **25** (2002) 113 [eConf **C010630** (2001) P125] [arXiv:hep-ph/0202233].

[13] G. F. Giudice and R. Rattazzi, Phys. Rept. **322** (1999) 419 [arXiv:hep-ph/9801271].

[14] L. Randall and R. Sundrum, Nucl. Phys. B **557** (1999) 79 [arXiv:hep-th/9810155].

[15] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812** (1998) 027 [arXiv:hep-ph/9810442].

[16] T. Gherghetta, G. F. Giudice and J. D. Wells, Nucl. Phys. B **559** (1999) 27 [arXiv:hep-ph/9904378].

[17] S. Ambrosanio, A. Dedes, S. Heinemeyer, S. Su and G. Weiglein, Nucl. Phys. B **624** (2002) 3 [arXiv:hep-ph/0106255];
J. R. Ellis, S. Heinemeyer, K. A. Olive and G. Weiglein, Phys. Lett. B **515** (2001) 348 [arXiv:hep-ph/0105061].

[18] A. Dedes, S. Heinemeyer, S. Su and G. Weiglein, arXiv:hep-ph/0302174.

[19] U. Baur, R. Clare, J. Erler, S. Heinemeyer, D. Wackeroth, G. Weiglein and D. R. Wood, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, eConf **C010630** (2001) P122 [arXiv:hep-ph/0111314].

[20] M. E. Peskin and T. Takeuchi, Phys. Rev. D **46** (1992) 381.

[21] T. Abe *et al.* [American Linear Collider Working Group Collaboration], in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, arXiv:hep-ex/0106057.

[22] J. R. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B **257** (1991) 83;
Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. **85** (1991) 1;
H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66** (1991) 1815.

[23] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D **50** (1994) 7048 [arXiv:hep-ph/9306309].

[24] A. Freitas, W. Hollik, W. Walter and G. Weiglein, Phys. Lett. B **495** (2000) 338 [arXiv:hep-ph/0007091];
M. Awramik and M. Czakon, Phys. Rev. Lett. **89** (2002) 241801 [arXiv:hep-ph/0208113];
A. Onishchenko and O. Veretin, Phys. Lett. B **551** (2003) 111 [arXiv:hep-ph/0209010];
M. Awramik and M. Czakon, arXiv:hep-ph/0305248.

[25] A. Freitas, W. Hollik, W. Walter and G. Weiglein, Nucl. Phys. B **632** (2002) 189 [arXiv:hep-ph/0202131].

[26] M. Faisst, J. H. Kuhn, T. Seidensticker and O. Veretin, arXiv:hep-ph/0302275.

[27] A. Freitas, S. Heinemeyer and G. Weiglein, hep-ph/0212068.

[28] M. W. Grunewald, Nucl. Phys. Proc. Suppl. **117** (2003) 280 [arXiv:hep-ex/0210003].

[29] A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Junger and G. Weiglein, Phys. Rev. Lett. **78** (1997) 3626 [arXiv:hep-ph/9612363]; Phys. Rev. D **57** (1998) 4179 [arXiv:hep-ph/9710438].

[30] M. J. Veltman, Nucl. Phys. B **123** (1977) 89.

[31] S. Heinemeyer, PhD thesis, Universität Karlsruhe, Shaker Verlag, Aachen 1998, see `www-itp.physik.uni-karlsruhe.de/prep/phd/`;
G. Weiglein, arXiv:hep-ph/9901317;
S. Heinemeyer, W. Hollik and G. Weiglein, *in preparation*.

[32] S. Heinemeyer and G. Weiglein, in *Proc. of the 5th International Symposium on Radiative Corrections (RADCOR 2000)* ed. Howard E. Haber, arXiv:hep-ph/0102317.

[33] S. Heinemeyer and G. Weiglein, JHEP **0210** (2002) 072 [arXiv:hep-ph/0209305].

[34] P. H. Chankowski, S. Pokorski and J. Rosiek, Phys. Lett. B **286** (1992) 307; Nucl. Phys. B **423** (1994) 437 [arXiv:hep-ph/9303309].

[35] A. Dabelstein, Nucl. Phys. B **456** (1995) 25 [arXiv:hep-ph/9503443]; Z. Phys. C **67** (1995) 495 [arXiv:hep-ph/9409375].

[36] S. P. Martin, Phys. Rev. D **66** (2002) 096001 [arXiv:hep-ph/0206136].

[37] A. Dedes, G. Degrassi and P. Slavich, arXiv:hep-ph/0305127.

[38] S. Heinemeyer, W. Hollik and G. Weiglein, Comput. Phys. Commun. **124** (2000) 76 [arXiv:hep-ph/9812320];
The *FeynHiggs* code can be obtained from `www.feynhiggs.de` .

[39] M. Frank, S. Heinemeyer, W. Hollik and G. Weiglein, arXiv:hep-ph/0202166.

[40] S. Heinemeyer, W. Hollik and G. Weiglein, JHEP **0006** (2000) 009 [arXiv:hep-ph/9909540].

[41] S. Heinemeyer, S. Kraml, W. Porod and G. Weiglein, arXiv:hep-ph/0306181.

[42] G. Degrassi, P. Gambino and G. F. Giudice, JHEP **0012** (2000) 009 [arXiv:hep-ph/0009337];
M. Carena, D. Garcia, U. Nierste and C. E. Wagner, Phys. Lett. B **499** (2001) 141 [arXiv:hep-ph/0010003];
D. A. Demir and K. A. Olive, Phys. Rev. D **65** (2002) 034007 [arXiv:hep-ph/0107329].

[43] G. Degrassi and G. F. Giudice, Phys. Rev. D **58** (1998) 053007 [arXiv:hep-ph/9803384].

[44] H. N. Brown *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **86** (2001) 2227 [arXiv:hep-ex/0102017];
G. W. Bennett *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **89** (2002) 101804 [Erratum-ibid. **89** (2002) 129903] [arXiv:hep-ex/0208001].

[45] F. Jegerlehner, these proceedings.

[46] A. Melchiorri and J. Silk, Phys. Rev. D **66** (2002) 041301 [arXiv:astro-ph/0203200];
A. Benoit *et al.* [the Arch gsave initmatrix /Helvetica findfont 12 scalefont setfont (161) dup string [arXiv:astro-ph/0210306];
J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive and M. Srednicki, Nucl. Phys. B **238** (1984) 453;
H. Goldberg, Phys. Rev. Lett. **50** (1983) 1419.

[47] S. Ahmed et al., CLEO CONF 99-10;
[Belle Collaboration], BELLE-CONF-0003, contribution to the 30th International conference on High-Energy Physics, Osaka, 2000. See also
K. Abe *et al.* [Belle Collaboration], Phys. Lett. B **511** (2001) 151 [arXiv:hep-ex/0103042];
K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0107065;
L. Lista [BABAR Collaboration], arXiv:hep-ex/0110010.

[48] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, arXiv:hep-ph/0303043.

[49] J. R. Ellis, S. Heinemeyer, K. A. Olive and G. Weiglein, JHEP **0301** (2003) 006 [arXiv:hep-ph/0211206].

[50] W. de Boer, M. Huber, C. Sander and D. I. Kazakov, Phys. Lett. B **515** (2001) 283.

[51] S. Heinemeyer and G. Weiglein, arXiv:hep-ph/0012364.

[52] J. Erler, S. Heinemeyer, W. Hollik, G. Weiglein and P. M. Zerwas, Phys. Lett. B **486** (2000) 125 [arXiv:hep-ph/0005024].

[53] M. Swartz, these proceedings.

[54] P. Wells, these proceedings.

[55] M. S. Chanowitz, arXiv:hep-ph/0304199, these proceedings.

[56] G. Altarelli, F. Caravaglios, G. F. Giudice, P. Gambino and G. Ridolfi, JHEP **0106** (2001) 018 [arXiv:hep-ph/0106029].

[57] G. Abbiendi et al. [ALEPH, DELPHI, L3, OPAL Collaborations], arXiv:hep-ex/0306033, to appear in *Phys. Lett.* **B**.

[58] K. McFarland, these proceedings.

[59] S. Davidson, S. Forte, P. Gambino, N. Rius and A. Strumia, JHEP **0202** (2002) 037 [arXiv:hep-ph/0112302].

[60] W. de Boer and C. Sander, appeared in the proceedings of "SUSY 02", DESY Hamburg, Germany, June 17-23, 2002, p. 1121-1126.