Unified Description of the Non – singlet Spin–dependent Structure Function g_1 Incorporating Altarelli-Parisi Evolution and the Double Logarithmic $\ln^2(1/x)$ Effects at Low x

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Abstract

A unified equation for the non-singlet spin dependent structure function $g_1^{ns}(x,Q^2) = g_1^p(x,Q^2) - g_1^n(x,Q^2)$ which incorporates the complete leading order Altarelli-Parisi evolution at finite x and double logarithmic $\ln^2(1/x)$ effects at $x \to 0$ is formulated and solved assuming a simple form of the non-perturbative part of g_1^{ns}. Reasonable agreement is obtained with the data of the SMC. Predictions for the small x behaviour of g_1^{ns} in the range relevant for the HERA measurements are given. Extrapolation of g_1^{ns} to the region of small x and small Q^2 is also discussed.

1 Introduction

Measurements of polarised lepton scattering on polarised proton, deuteron and neutron targets allow verification of sum rules like e.g. the Bjorken sum rule, [1], which is a fundamental relation of QCD and which refers to the first moment of the non-singlet spin dependent structure function, $g_1^{ns}(x,Q^2)$. Evaluation of the sum rules thus requires knowledge of spin dependent structure functions over the entire region of x. Since the experimentally accessible x range is limited (presently $x > 0.003$, [2]), extrapolations to $x = 0$ and $x = 1$ are unavoidable. The latter is not critical, contrary to the former; the small x behaviour of $g_1(x)$ is theoretically not well established and results on the moments depend drastically on the assumptions made for this extrapolation, cf. [2,3]. Theoretical studies are just awaited for, especially in view of a possible future polarising the proton beam at HERA.

Small x behaviour of both singlet and non-singlet spin dependent structure function $g_1(x,Q^2)$ is controlled by the double logarithmic terms i.e. by those terms which correspond to powers of $\alpha_s \ln^2(1/x)$ [4]. Let us recall that the Regge pole model gives the following small x behaviour of $g_1^i(x,Q^2)$: $g_1^i \sim x^{-\alpha_i(0)}$, where $i = s, ns$ denote either singlet or non-singlet combination and $\alpha_{s,ns}(0)$ denote the intercept of the Regge pole trajectory corresponding to the axial vector mesons, [5], with $I = 0$ or $I = 1$ respectively. We expect $\alpha_{s,ns}(0) \leq 0$. The Regge behaviour of the $g_1^i(x,Q^2)$ is unstable against the resummation of the $\ln^2(1/x)$ terms which generate more singular x dependence than that implied by $\sim x^{-\alpha_i(0)}$ for $\alpha_{s,ns}(0) \leq 0$. Thus the double logarithmic terms can generate the leading small x behaviour of the g_1.

The double logarithmic $\ln^2(1/x)$ effects go beyond the standard QCD evolution of spin dependent parton densities [6]. They can be accomodated for in the QCD evolution formalism

based upon the renormalisation group equations, [7]. Here we show an alternative approach based on unintegrated distributions (cf. the unpolarised case [8]), for simplicity limited to the non-singlet structure function $g_1^{ns}(x,Q^2)$. We summarize only the main results; details can be found in [9]. For simplicity we shall drop the superscript 'ns', i.e. we will denote g_1^{ns} as g_1 etc.

2 Double logarithmic $\ln^2(1/x)$ resummation

The double logarithmic terms in the non-singlet part of the $g_1(x,Q^2)$ are generated by ladder diagrams with quark (antiquark) exchange, [10]. To be precise for the $g_1(x,Q^2)$ one has also to include certain class of non-ladder diagrams [4]. In the non-singlet case however their contribution is non-leading in the large N_c limit (N_c is a number of colours). It turns out to be numerically small for $N_c = 3$ and will be neglected.

When discussing the leading $\ln^2(1/x)$ resummation effects in the $g_1(x,Q^2)$ it is convenient to work with the (combination) of the unintegrated spin dependent quark distributions, $f(x',k^2) = (f_u(x',k^2) - f_d(x',k^2))/6$ where x' is the longitudinal momentum fraction of the parent proton carried by a quark and k^2 denotes the quark transverse momentum squared. The sum of $\ln^2(1/x)$ terms corresponding to ladder diagrams is generated by the following integral equation:

$$f(x',k^2) = f^{(0)}(x',k^2) + \bar{\alpha}_s(k^2)\int_{x'}^1 \frac{dz}{z}\int_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2} f\left(\frac{x'}{z},k'^2\right) \tag{1}$$

where $\bar{\alpha}_s(k^2) = 2\alpha_s(k^2)/(3\pi)$. In this equation only the leading part of the splitting function $P_{qq}(z)$ in the $z=0$ limit is retained. The variable k_0^2 is the infrared cut-off and the inhomogeneous term $f^{(0)}(x',k^2)$ will be specified later. The integration limit k^2/z follows from the requirement that the quark virtuality is controlled by the tranverse momentum squared. The structure function $g_1(x,Q^2)$ is related in the following way to $f(x',k^2)$:

$$g_1(x,Q^2) = g_1^{(0)}(x) + \int_{k_0^2}^{W^2}\frac{dk^2}{k^2} f(x' = x(1+\frac{k^2}{Q^2}),k^2) \tag{2}$$

where $g_1^{(0)}(x)$ is a non-perturbative part of the g_1 and $W^2 = Q^2(1-x)/x$. The kinematic limit for the integration over dk^2 should in fact be set equal to $W^2/4$ but we have checked that the results are relativly insensitive to this change. The relation $x' = x(1+k^2/Q^2)$ follows from the on-shell condition imposed upon the struck quark, i.e. $(q+k)^2 = 0$. The non-perturbative part $g_1^{(0)}(x)$ originates from the non-perturbative region $k^2 < k_0^2$, i.e.

$$g_1^{(0)}(x) = \int_0^{k_0^2}\frac{dk^2}{k^2} f(x,k^2) \tag{3}$$

Origin of the $\ln^2(1/x)$ terms in g_1 can be traced to the fact that the conventional single logarithmic terms coming from the logarithmic integration over the longitudinal momentum fraction z are enhanced by the logarithmic integration over the transverse momentum up to the z dependent limit k^2/z in eq.(1) and up to the x dependent limit $W^2 = Q^2(1/x-1)$ in eq. (2). Those $\ln^2(1/x)$ terms will generate the leading small x behaviour of $g_1(x,Q^2)$ provided that the input structure functions $g_1^{(0)}(x,Q^2)$ and $f^{(0)}(x,k^2)$ are non-singular for $x \to 0$. For fixed (i.e. non-running) coupling $\bar{\alpha}_s(k^2) \to \tilde{\alpha}_s$ the small x behaviour is then $g_1(x,Q^2) \sim x^{-\lambda}$ where $\lambda = 2\sqrt{\tilde{\alpha}_s}$ and where we have neglected the slowly varying logarithmic factors.

3 Unified equation incorporating Altarelli-Parisi evolution and the double $\ln^2(1/x)$ resummation.

Equation (1) generates correctly the leading small x behaviour but it is inaccurate in describing the evolution in Q^2 both for very small and, of course, for large values of x. In order to have the correct Q^2 evolution one should thus include in the formalism the complete splitting function $P_{qq}(z)$ and not only its leading component at $z \to 0$. The corresponding equation then reads [9]:

$$f(x',k^2) = f^{(0)}(x',k^2) + \bar{\alpha}_s(k^2)\int_{x'}^1 \frac{dz}{z}\int_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2} f\left(\frac{x'}{z},k'^2\right) +$$

$$\frac{\alpha_s(k^2)}{2\pi}\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\left[\frac{4}{3}\int_{x'}^1\frac{dz}{z}\frac{(z+z^2)f(\frac{x'}{z},k^2)-2zf(x',k^2)}{1-z}+\left(\frac{1}{2}+\frac{8}{3}ln(1-x')\right)f(x',k^2)\right] \quad (4)$$

where

$$f^{(0)}(x',k^2) = \frac{\alpha_s(k^2)}{2\pi}\left[\frac{4}{3}\int_{x'}^1\frac{dz}{z}\frac{(1+z^2)g_1^{(0)}(\frac{x'}{z})-2zg_1^{(0)}(x')}{1-z}+\left(\frac{1}{2}+\frac{8}{3}ln(1-x')\right)g_1^{(0)}(x')\right] \quad (5)$$

and the function $\bar{\alpha}_s(k^2)$ is defined in section 2. The structure function $g_1(x,Q^2)$ is related to $f(x',k^2)$ by eq.(2). Equations (4),(5) and (2) which treat both potentially 'large logarithms' $\ln(1/x)$ and $\ln(Q^2)$ on equal footing will be the basis of our analysis. Similar treatment of the unpolarised (singlet) structure function and of gluon distributions has been proposed in refs [8].

4 Numerical results

We solved eq.(4) assuming k_0^2 =1 GeV^2 and

$$g_1^{(0)}(x) = \frac{2}{3}g_A(1-x)^3 \quad (6)$$

where g_A is the axial vector coupling constant, $g_A = 1.257$. In the small x limit we have $g_1^{(0)}(x) \to const$ that corresponds to the assumption that $\alpha_{A_1}(0) = 0$ where $\alpha_{A_1}(0)$ is the intercept of the A_1 Regge pole trajectory. The function $g_1^{(0)}(x)$ given by eq. (6) does also satisfy the Bjorken sum rule, i.e. $\int_0^1 dx g_1^{(0)}(x) = g_A/6$.

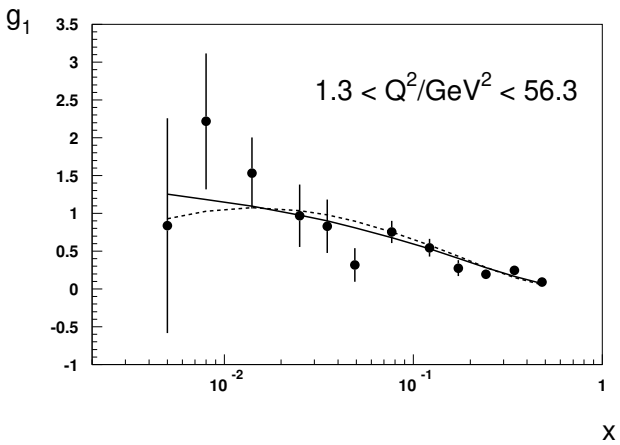

Fig.1. Non-singlet part of the proton spin structure function $g_1(x,Q^2)$. Points show the recent SMC results, [2, 3], where $g_1^{ns} \equiv g_1^p - g_1^n = 2(g_1^p - g_1^d/(1-3\omega_D/2))$ was obtained from the proton and deuteron data (ω_D denotes a probability of the D state of the deuteron; ω_D=0.05). Error bars are statistical; systematic errors are of the order of 20% of the statistical ones (in the lowest x bin). Values of Q^2 change with x as marked in the figure. Continuous line corresponds to the model calculations, eqs (2), (4) and (6), performed at the measured values of (x,Q^2); broken line is a pure LO Altarelli–Parisi prediction at the same points.

In Fig.1 we compare the calculated $g_1(x,Q^2)$ with recent measurements of the SMC which extend to presently lowest values of x. In view of the simplicity of the parametrisation (6) with no free parameters, agreement with the data can be regarded as quite satisfactory.

In Fig.2 we show $g_1(x, Q^2 = 10\ \mathrm{GeV}^2)$, obtained from eqs(2) and (4) and from the standard LO QCD evolution as well as the input $g_1^{(0)}(x)$ given by eq.(6). the $\ln^2(1/x)$ resummation gives g_1 steeper than that generated from the LO evolution and this effect is already visible for $x \lesssim 10^{-2}$.

Equations (2) and (4) generate dynamically the small x extrapolation of $g_1(x, Q^2)$ and so one may in particular determine the contribution to the Bjorken integral coming from the region of very small values of x which is not being experimentally probed. In principle g_1 obtained from eqs (2,4) can violate the Bjorken sum rule but we have found that this violation is less than 0.1 %. We have estimated a contribution $\Delta I(x_a, x_b, Q^2) = \int_{x_a}^{x_b} dx g_1(x, Q^2)$ to the Bjorken integral and found that $\Delta I(0, 0.003, 10\ \mathrm{GeV}^2) = 0.0057$ for the pure Altarelli-Parisi evolution calculations and 0.0080 for these with double logarithmic terms included. These numbers have to be compared with 0.004 obtained when a $g_1 = const$ behaviour, consistent with Regge prediction has been assumed and fitted to the lowest x data points for proton and deuteron targets (cf. [2, 3]). An analogous number in the present case ($g_1^{(0)}$ used instead of g_1 in the integrand of ΔI) is $\Delta I(0, 0.003)$=0.0026.

Let us finally comment upon the possible low Q^2 extrapolation of g_1. For $Q^2 \to 0$ (for fixed pq), g_1 should be a finite function of pq, free from any kinematical singularities or zeros at $Q^2 = 0$. The g_1 defined by eqs (2,4) and with $g_1^{(0)}(x)$ given by (6) fulfills those criteria. This may not always hold for arbitrary parametrisation of $g_1^{(0)}$ which may include kinematical singularities at $x = 0$. In such a case one may just use the same prescription as that which was adopted in ref. [11] i.e. to replace in eq.(2) the function $g_1^{(0)}(x)$ by $g_1^{(0)}(\bar{x})$ where $\bar{x} = x\,(1 + k_0^2/Q^2)$ and to leave remaining parts of the calculation unchanged. After that, g_1 can be extrapolated to the low Q^2 region (for fixed $2pq = Q^2/x$) including the point $Q^2 = 0$. It has however to be remembered that the (extrapolated) partonic contribution to the low Q^2 region may not be the only one there.

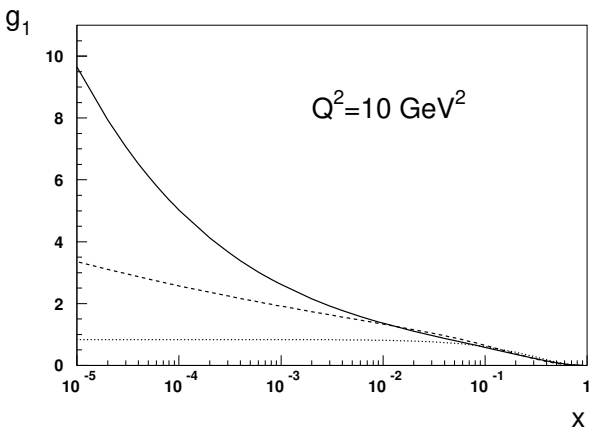

Fig.2. Non-singlet part of the proton spin structure function $g_1(x, Q^2 = 10\ \mathrm{GeV}^2)$. Upper curve – model calculations, eqs (2), (4) and (6), middle – a pure leading order Altarelli–Parisi prediction and a bottom one – $g_1^{(0)}$, eq.(6).

5 Summary and conclusions

We have formulated the unified equation for the spin dependent non-singlet structure function $g_1^{ns}(x, Q^2)$ which incorporated the LO Altarelli-Parisi evolution and the double logarithmic $\ln^2(1/x)$ effects at small x. The equation treats on equal footing two potentially 'large logarithms': $ln(Q^2)$ and $ln(1/x)$. It allows to extrapolate g_1^{ns} dynamically to the region of small x.

The non-singlet case has been chosen for its simplicity; dominant part of the double logarithmic $\ln^2(1/x)$ terms and the QCD evolution here are generated by ladder diagrams with quark and antiquark exchange. It has also been chosen in order to meet the experimental need to know the low x behaviour of g_1^{ns}, necessary when evaluating the Bjorken sum rule from the data.

We solved that equation starting from the simple parametrisation of the non-perturbative part of g_1^{ns}. We found that the $\ln^2(1/x)$ effects are very significant already for $x \lesssim 10^{-2}$ which may be probed in the possible future HERA measurements. We have also estimated the

contribution from the small x region ($x < 0.003$) to the Bjorken sum rule and found it to be around 4% of the value of the sum. We have pointed out that $g_1^{ns}(x, Q^2)$ obtained from our formalism can easily be extrapolated to the region of low Q^2 although it may not be the only contribution in this region. Elaboration of similar formalism for the singlet case is in progress.

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References

[1] J.D. Bjorken, *Phys. Rev.* **148** (1966) 1467; *Phys. Rev.* **D1** (1970) 1376.

[2] SMC; B. Adeva et al., `CERN-PPE/97-118` and to appear in *Phys. Lett.* **B**.

[3] SMC; D. Adams et al., *Phys. Lett.* **B396** (1997) 338;

[4] J. Bartels, B.I. Ermolaev and M.G. Ryskin, *Z. Phys.* **C70** (1996) 273. J. Bartels, B.I. Ermolaev and M.G. Ryskin, *Z. Phys.* **C72** (1996) 627; J. Kwieciński, *Acta. Phys. Polon.* **B27** (1996) 893.

[5] B.L. Ioffe, V.A. Khoze and L.N.Lipatov, Hard Processes, (North Holland, 1984); J. Ellis and M. Karliner, *Phys. Lett.* **B231** (1988) 72.

[6] M. Ahmed and G. Ross, *Phys. Lett.* **B56** (1976) 385; *Nucl. Phys.* **111** (1976) 298. G. Altarelli and G. Parisi, *Nucl. Phys.* **B126** (1977) 298; M. Stratmann, A. Weber and W. Vogelsang, *Phys. Rev.* **D53** (1996) 138; M. Glück, et al., *Phys. Rev.* **D53** (1996) 4775; M. Glück, E. Reya and W. Vogelsang, *Phys. Lett.* **B359** (1995) 201; T. Gehrmann and W. J. Stirling, *Phys. Rev.* **D53** (1996) 6100; *Phys. Lett.* **B365** (1996) 347; C. Bourrely and J. Soffer, *Nucl. Phys.* **B445** (1995) 341; *Phys. Rev.* **D53** (1996) 4067; R. D. Ball, S. Forte and G. Ridolfi, **Nucl. Phys. B444** (1995) 287; J. Bartelski and S. Tatur, *Acta Phys. Polon.* **B27** (1996) 911; *Z. Phys.* **C71** (1996) 595.

[7] J. Blümlein and A. Vogt, *Acta Phys. Polon.* **B27** (1996) 1309; *Phys.Lett.* **B386** (1996) 350; J. Blümlein, S. Riemersma and A. Vogt *Nucl. Phys.* B (Proc. Suppl) **51C** (1996) 30.

[8] L.N. Gribov, E.N. Levin and M.G. Ryskin, *Phys. Rep.* **100** (1983) 1; J. Kwieciński, A.D. Martin and A.M. Staśto, *Phys. Rev.* **D56** (1997) 3991.

[9] B. Badełek and J. Kwieciński, `hep-ph/9709363` and to appear in *Phys. Lett.* **B**.

[10] V.G. Gorshkov et al., *Yad. Fiz.* **6** (1967) 129 (*Sov. J. Nucl. Phys.* **6** (1968) 95); L.N. Lipatov, *Zh. Eksp. Teor. Fiz.* **54** (1968) 1520 (*Sov. Phys. JETP* **27** (1968) 814); R. Kirschner and L.N. Lipatov, *Nucl. Phys.* **B213** (1983) 122; R. Kirschner, *Z. Phys.* **C67** (1995) 459; J. Kwieciński, *Phys. Rev.* **D26** (1982) 3293.

[11] B. Badełek and J. Kwieciński, *Phys. Lett.* **B295** (1992) 263; *Rev. Mod. Phys.* **68** (1996) 445.