QCD Predictions for g_1^p at Small x Incorporating Double $ln^2(1/x)$ Resummation

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Abstract

The proton spin dependent structure function g_1^p is analysed using the unified scheme incorporating both Altarelli-Parisi evolution and the double $ln^2(1/x)$ effects at low x. The latter are found to be very important in the region of x which can possibly be probed at HERA. Predictions for the polarized gluon distribution $\Delta G(x)$ at low x are also given.

It has recently been pointed out that the spin dependent structure function g_1 at low x is dominated by the *double* logarithmic $ln^2(1/x)$ contributions i.e. by those terms of the perturbative expansion which correspond to the powers of $ln^2(1/x)$ at each order of the expansion [1, 2]. Those contributions go *beyond* the LO or NLO order QCD evolution of polarised parton densities [3] and in order to take them into account one has to include the resummed double $ln^2(1/x)$ terms in the coefficient and splitting functions [4]. In ref. [5] we have developed an alternative approach based on unitegrated distributions which for simplicity was formulated for the non-singlet distributions. In this note we wish to present the preliminary results for the singlet structure functions concentrating on the proton polarized structure function $g_1^p(x,Q^2)$. Our main aim is to study $g_1^p(x,Q^2)$ in the region of the small values of x which can possibly be probed at HERA [6] and confront obtained results with those obtained from the LO Altarelli-Parisi evolution. We will also show similar results for the polarised gluon distribution $\Delta G(x,Q^2)$.

The dominant contribution to the double $ln^2(1/x)$ resummation comes from the ladder diagrams with quark and gluon exchanges along the ladder. In what follows we shall neglect for simplicity possible non-ladder bremsstrahlung terms which are relatively unimportant [1, 2].

It is convenient to introduce the unintegrated (spin dependent) parton distributions $f_i(x',k^2)$ $(i = u_v, d_v, \bar{u}, \bar{d}, \bar{s}, g)$ where k^2 is the transverse momentum squared of the parton i and x' the longitudinal momentum fraction of the parent proton carried by a parton. The conventional (integrated) distributions $\Delta p_i(x,Q^2)$ are related in the following way to the unintegrated distributions $f_i(x',k^2)$:

$$\Delta p_i(x,Q^2) = \Delta p_i^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f_i(x' = x(1+\frac{k^2}{Q^2}), k^2) \tag{1}$$

where $\Delta p_i^{(0)}(x)$ is the nonperturbative part of the of the distributions and

$$W^2 = Q^2(\frac{1}{x} - 1) \tag{2}$$

where as usual x is the Bjorken parameter and $Q^2 = -q^2$ with q denoting the four momentum transfer between leptons. The parameter k_0^2 is the infrared cut-off which will be set equal to 1 GeV^2. The origin of the nonperturbative part $\Delta p_i^{(0)}(x)$ can be viewed upon as originating from the non-perturbative region $k^2 < k_0^2$, i.e.

$$\Delta p_i^{(0)}(x) = \int_0^{k_0^2} \frac{dk^2}{k^2} f_i(x, k^2) \tag{3}$$

The spin dependent structure function $g_1^p(x, Q^2)$ of the proton is related in a standard way to the (integrated) parton distributions:

$$g_1^p(x, Q^2) = \frac{1}{2}\left[\frac{4}{9}(\Delta u_v(x, Q^2) + 2\Delta\bar{u}(x, Q^2)) + \frac{1}{9}(\Delta d_v(x, Q^2) + 2\Delta\bar{u}(x, Q^2) + 2\Delta\bar{s}(x, Q^2))\right] \tag{4}$$

where $\Delta u_v(x, Q^2) = \Delta p_{u_v}(x, Q^2)$ etc. We assume $\Delta\bar{u} = \Delta\bar{d}$ and confine ourselves to the number of flavours N_F equal to three.

The valence quarks distributions and asymmetric part of the sea

$$f_{US}(x', k^2) = f_{\bar{u}}(x', k^2) - f_{\bar{s}}(x', k^2) \tag{5}$$

will correspond to ladder diagrams with quark exchange along the ladder. The singlet distributions

$$f_S(x', k^2) = f_{u_v}(x', k^2) + f_{d_v}(x', k^2) + 4f_{\bar{u}}(x', k^2) + 2f_{\bar{s}}(x', k^2) \tag{6}$$

and the gluon distributions $f_g(x', k^2)$ will correspond to ladder diagrams with both quark (anti-quark) and gluon exchanges along the ladder.

The sum of double logarithmic $ln^2(1/x)$ terms corresponding to ladder diagrams is generated by the following integral equations:

$$f_k(x', k^2) = f_k^{(0)}(x', k^2) + \frac{\alpha_s}{2\pi}\Delta P_{qq}(0)\int_{x'}^1 \frac{dz}{z}\int_{k_0^2}^{k^2/z} \frac{dk'^2}{k'^2} f_k\left(\frac{x'}{z}, k'^2\right) \tag{7}$$

$(k = u_v, d_v, US)$

$$f_S(x', k^2) = f_S^{(0)}(x', k^2) +$$
$$\frac{\alpha_s}{2\pi}\int_{x'}^1 \frac{dz}{z}\int_{k_0^2}^{k^2/z} \frac{dk'^2}{k'^2}\left[\Delta P_{qq}(0) f_S\left(\frac{x'}{z}, k'^2\right) + \Delta P_{qg}(0) f_g\left(\frac{x'}{z}, k'^2\right)\right]$$

$$f_g(x', k^2) = f_g^{(0)}(x', k^2) +$$
$$\frac{\alpha_s}{2\pi}\int_{x'}^1 \frac{dz}{z}\int_{k_0^2}^{k^2/z} \frac{dk'^2}{k'^2}\left[\Delta P_{gq}(0) f_S\left(\frac{x'}{z}, k'^2\right) + \Delta P_{gg}(0) f_g\left(\frac{x'}{z}, k'^2\right)\right] \tag{8}$$

with $\Delta P_{ij}(0) = \Delta P_{ij}(z = 0)$ where $\Delta P_{ij}(z)$ denote the LO splitting functions describing evolution of spin dependent parton densities and the inhomogeneous terms $f^{(0)}(x,', k^2)$ will be specified later. To be precise we have:

$$\Delta P_{qq}(0) = \frac{4}{3}$$

$$\Delta P_{qg}(0) = -N_F$$
$$\Delta P_{gq}(0) = \frac{8}{3}$$
$$\Delta P_{gg}(0) = 12 \tag{9}$$

Equation (7) is similar to the corresponding equation in QED describing the double logarithmic resummation generated by ladder diagrams with fermion exchange [7]. The problem of double logarithmic asymptotics in QCD in the non-singlet channels was also discussed in ref [8].

Equations (7, 8) generate singular power behaviour of the spin dependent parton distributions and of the spin dependent structure functions g_1 at small x i.e.

$$g_1^{NS}(x,Q^2) \sim x^{-\lambda_{NS}}$$
$$g_1^{S}(x,Q^2) \sim x^{-\lambda_{S}}$$
$$\Delta G(x,Q^2) \sim x^{-\lambda_{S}} \tag{10}$$

where $g_1^{NS} = g_1^p - g_1^n$ and $g_1^S = g_1^p + g_1^n$ respectively and ΔG is the spin dependent gluon distribution. This behaviour reflects similar small x' behaviour of the unintegrated distributions. Exponents $\lambda_{NS,S}$ are given by the following formulas:

$$\lambda_{NS} = 2\sqrt{\left[\frac{\alpha_s}{2\pi}\Delta P_{qq}(0)\right]}$$
$$\lambda_{S} = 2\sqrt{\left[\frac{\alpha_s}{2\pi}\gamma^+\right]} \tag{11}$$

where

$$\gamma^+ = \frac{\Delta P_{qq}(0) + \Delta P_{gg}(0) + \sqrt{(\Delta P_{qq}(0) - \Delta P_{gg}(0))^2 + 4\Delta P_{qg}(0)\Delta P_{gq}(0)}}{2} \tag{12}$$

Both equations (7,8) and the exponents $\lambda_{NS,S}$ acquire additional contributions from the non-ladder diagrams which are however relatively small [1, 2]. The power-like behaviour (11) remains the leading small x behaviour of the structure functions provided that their non-perturbative parts are less singular. This takes place if the latter are assumed to have the Regge pole like behaviour with the corresponding intercept(s) being near 0.

Following ref. [5] we extend equations (7,8) and add to their right hand side the contributions coming from the remaining parts of the splitting functions $\Delta P_{ij}(z)$. We also allow the coupling α_s to run setting k^2 as the relevant scale. In this way we obtain unified system of equations which contain both the complete LO Altarelli-Parisi evolution and the double logarithmic $ln^2(1/x)$ effects at low x. The corresponding system of equations reads:

$$f_k(x',k^2) = f_k^{(0)}(x',k^2) + \frac{\alpha_s(k^2)}{2\pi}\frac{4}{3}\int_{x'}^1 \frac{dz}{z}\int_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2} f_k\left(\frac{x'}{z},k'^2\right) +$$
$$\frac{\alpha_s(k^2)}{2\pi}\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\frac{4}{3}\int_{x'}^1\frac{dz}{z}\frac{(z+z^2)f_k(\frac{x'}{z},k'^2) - 2z f_k(x',k'^2)}{1-z} +$$
$$\frac{\alpha_s(k^2)}{2\pi}\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\left[2 + \frac{8}{3}ln(1-x')\right] f_k(x',k'^2) \tag{13}$$

$(k = u_v, d_v, US)$,

$$f_S(x',k^2) = f_S^{(0)}(x',k^2) + \frac{\alpha_s(k^2)}{2\pi}\int_{x'}^1 \frac{dz}{z}\int_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}\frac{4}{3}f_S\left(\frac{x'}{z},k'^2\right) +$$

$$\frac{\alpha_s(k^2)}{2\pi}\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\frac{4}{3}\int_{x'}^1\frac{dz}{z}\frac{(z+z^2)f_S(\frac{x'}{z},k'^2) - 2z f_S(x',k'^2)}{1-z} +$$

$$\frac{\alpha_s(k^2)}{2\pi}\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\left[2+\frac{8}{3}ln(1-x')\right]f_S(x',k'^2) +$$

$$\frac{\alpha_s(k^2)}{2\pi}N_F\left[-\int_{x'}^1\frac{dz}{z}\int_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}f_g\left(\frac{x'}{z},k'^2\right) + \int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\int_{x'}^1\frac{dz}{z}2z f_g(\frac{x'}{z},k'^2)\right]$$

$$f_g(x',k^2) = f_g^{(0)}(x',k^2) + \frac{\alpha_s(k^2)}{2\pi}\int_{x'}^1\frac{dz}{z}\int_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}\frac{8}{3}f_S\left(\frac{x'}{z},k'^2\right) +$$

$$\frac{\alpha_s(k^2)}{2\pi}\left[\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\int_{x'}^1\frac{dz}{z}(-\frac{4}{3})z f_S\left(\frac{x'}{z},k'^2\right) + 12\int_{x'}^1\frac{dz}{z}\int_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}f_g\left(\frac{x'}{z},k'^2\right)\right] +$$

$$\frac{\alpha_s(k^2)}{2\pi}\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\int_{x'}^1\frac{dz}{z}6z\left[\frac{f_g\left(\frac{x'}{z},k'^2\right) - f_g(x',k'^2)}{1-z} - 2f_g\left(\frac{x'}{z},k'^2\right)\right] +$$

$$\frac{\alpha_s(k^2)}{2\pi}\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\left[\frac{11}{2}-\frac{N_F}{3}+6ln(1-x')\right]f_g(x',k'^2) \qquad (14)$$

The inhomogeneous terms $f_i^{(0)}(x',k^2)$ are expressed in terms of the input (integrated) parton distributions and are the same as in the case of the LO Altarelli Parisi evolution [5]:

$$f_k^{(0)}(x',k^2) = \frac{\alpha_s(k^2)}{2\pi}\frac{4}{3}\int_{x'}^1\frac{dz}{z}\frac{(1+z^2)\Delta p_k^{(0)}(\frac{x'}{z}) - 2z)\Delta p_k^{(0)}(x')}{1-z} +$$

$$\frac{\alpha_s(k^2)}{2\pi}\left[2+\frac{8}{3}ln(1-x')\right]\Delta p_k^{(0)}(x') \qquad (15)$$

$(k = u_v, d_v, US)$,

$$f_S^{(0)}(x',k^2) = \frac{\alpha_s(k^2)}{2\pi}\frac{4}{3}\int_{x'}^1\frac{dz}{z}\frac{(1+z^2)\Delta p_S^{(0)}(\frac{x'}{z}) - 2z\Delta p_S^{(0)}(x')}{1-z} +$$

$$\frac{\alpha_s(k^2)}{2\pi}\left[(2+\frac{8}{3}ln(1-x'))\Delta p_S^{(0)}(x') + N_F\int_{x'}^1\frac{dz}{z}(1-2z)\Delta p_g^{(0)}(\frac{x'}{z})\right]$$

$$f_g^{(0)}(x',k^2) = \frac{\alpha_s(k^2)}{2\pi}\left[\frac{4}{3}\int_{x'}^1\frac{dz}{z}(2-z)\Delta p_S^{(0)}(\frac{x'}{z}) + (\frac{11}{2}-\frac{N_F}{3}+6ln(1-x'))\Delta p_g^{(0)}(x')\right] +$$

$$\frac{\alpha_s(k^2)}{2\pi}6\int_{x'}^1\frac{dz}{z}\left[\frac{\Delta p_g^{(0)}(\frac{x'}{z}) - z\Delta p_g^{(0)}(x')}{1-z} + (1-2z)\Delta p_g^{(0)}(\frac{x'}{z})\right] \qquad (16)$$

Equations (13,14) together with (15,16) and (1) reduce to the LO Altarelli-Parisi evolution equations with the starting (integrated) distributions $\Delta p_i^0(x)$ after we set the upper integration

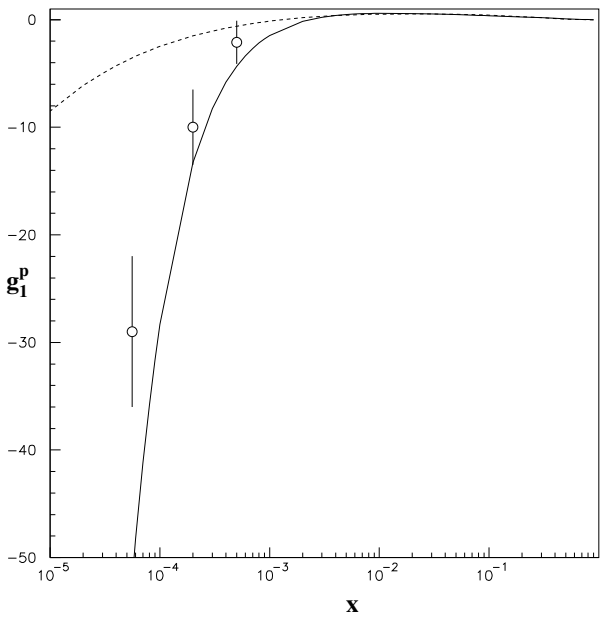

Figure 1: The structure function $g_1^p(x,Q^2)$ for $Q^2 = 10GeV^2$ plotted as the function of x. Solid line represents this structure function with the double $ln^2(1/x)$ terms included and the dashed line corresponds to g_1^p obtained from the LO Altarelli-Parisi equations The "experimental" points are based on the NLO QCD predictions with the statistical errors expected at HERA [6].

limit over dk'^2 equal to k^2 in all terms in equations (13,14) and if we set Q^2 in place of W^2 as the upper integration limit in the integral in eq. (1).

We solved equations (13,14) assuming the following simple parametrisation of the input distributions:

$$\Delta p_i^{(0)}(x) = N_i(1-x)^{\eta_i} \tag{17}$$

where we set $\eta_{u_v} = \eta_{d_v} = 3$, $\eta_{\bar{u}} = \eta_{\bar{s}} = 7$ and $\eta_g = 5$. The normalisation constants N_i were determined by imposing the Bjorken sum-rule for $\Delta u_v^{(0)} - \Delta d_v^{(0)}$ and requiring that the first moments of all other distributions are the same as those determined from the recent QCD analysis [9]. All distributions $\Delta p_i^{(0)}(x)$ behave as x^0 in the limit $x \to 0$ that corresponds to the implicit assumption that the Regge poles which correspond to axial vector mesons, which should control the small x behaviour of g_1 [10] have their intercept equal to 0. We checked that the parametrisation (17) combined with equations (1,4,13,14) gives reasonable description of the recent SMC data on $g_1^p(x,Q^2)$ [11].

In Fig.1 we show $g_1^p(x,Q^2)$ for $Q^2 = 10GeV^2$ in the small x region which can possibly be probed at HERA. We show predictions based on the equations (13,14,1) and confront them with the expectations which follow from solving the LO Altarelli-Parisi evolution equations with the input distributions at $Q_0^2 = 1GeV^2$ given by equation(s) (17). We also show in this Figure the "experimental" points which were obtained from the extrapolations based on the NLO QCD analysis together with estimated statistical errors [6]. We see that the structure function $g_1^p(x,Q^2)$ which contains effects of the double $ln^2(1/x)$ resummation begins to differ from that calculated within the LO Altarelli Parisi equations already for $x \sim 10^{-3}$. It is however compara-

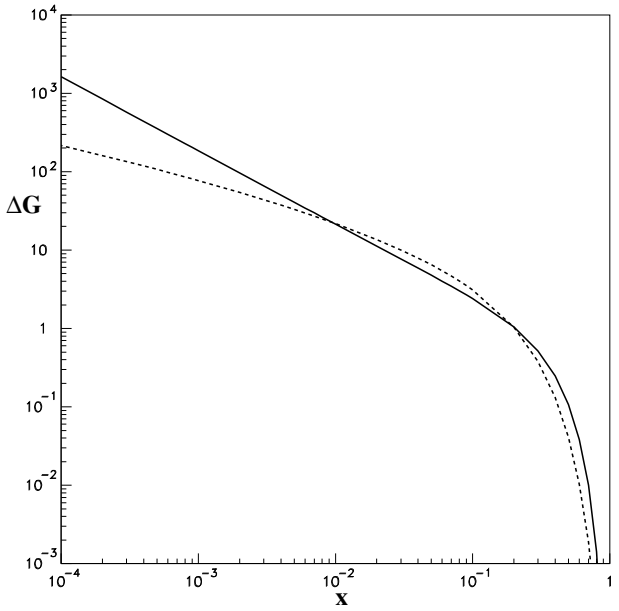

Figure 2: The spin dependent gluon distribution $\Delta G(x,Q^2)$ for $Q^2 = 10GeV^2$ plotted as the function of x. Solid line represents $\Delta G(x,Q^2)$ with the double $ln^2(1/x)$ terms included and the dashed line corresponds to the $\Delta G(x,Q^2)$ obtained from the LO Altarelli-Parisi equations.

ble to the structure function obtained from the NLO analysis for $x > 10^{-4}$ which is indicated by the "experimental" points. This is presumably partially an artifact of the difference in the input distributions but it also reflects the fact that the NLO approximation contains the first two terms of the double $ln^2(1/x)$ resummation in the corresponding splitting and coefficient functions. It can also be seen from Fig.1 that the (complete) double $ln^2(1/x)$ resummation generates the structure function which is significantly steeper than that obtained from the NLO QCD analysis and the difference between those two extrapolations becomes significant for $x < 10^{-4}$.

In Fig. 2 we show our predictions for the polarized gluon distribution $\Delta G(x,Q^2) = \Delta p_g(x,Q^2)$ for $Q^2 = 10GeV^2$ and confront it with the polarized gluon distribution obtained from the LO Altarelli-Parisi equations. The gluon distributions exhibits characteristic $x^{-\lambda_S}$ behaviour with $\lambda_S \sim 1$. Similar behaviour is exhibited by the structure function $g_1^p(x,Q^2)$ itself.

To sum up we have presented results of the analysis of the "unified" equations which contain the LO Altarelli Parisi evolution and the double $ln^2(1/x)$ effects at low x. As the first approximation we considered those double $ln^2(1/x)$ effects which are generated by ladder diagrams. The double logarithmic effects were found to be very important and they should in principle be visible in possible HERA measurements (cf. Fig. 1).

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