Two-loop renormalization of $\tan\beta$ and its gauge dependence [1]

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Abstract

Renormalization of two-loop divergent corrections to the vacuum expectation values (v_1, v_2) of the two Higgs doublets in the minimal supersymmetric standard model, and their ratio $\tan\beta = v_2/v_1$, is discussed for general R_ξ gauge fixings. When the renormalized (v_1, v_2) are defined to give the minimum of the loop-corrected effective potential, it is shown that, beyond the one-loop level, the dimensionful parameters in the R_ξ gauge fixing term generate gauge dependence of the renormalized $\tan\beta$. Additional shifts of the Higgs fields are necessary to realize the gauge-independent renormalization of $\tan\beta$.

The minimal supersymmetric (SUSY) standard model (MSSM) [2, 3] has two Higgs boson doublets

$$H_1 = (H_1^0, H_1^-), \quad H_2 = (H_2^+, H_2^0). \tag{1}$$

Both H_1^0 and H_2^0 acquire the vacuum expectation values (VEVs) v_i ($i = 1, 2$) which spontaneously break the SU(2) $\times$ U(1) gauge symmetry. H_i^0 are then expanded about the minimum of the Higgs potential as

$$H_i^0 = v_i/\sqrt{2} + \phi_i^0, \quad \langle\phi_i^0\rangle = 0. \tag{2}$$

Here I ignore the CP violation in the Higgs sector and take v_i as real and positive.

A lot of physical quantities of the theory depend on the Higgs VEVs. In calculating radiative corrections to these quantities, v_i have to be renormalized. In the MSSM, the renormalization is usually performed [4, 5] by specifying the weak boson masses, which are proportional to $v_1^2 + v_2^2$, and the ratio $\tan\beta \equiv v_2/v_1$. However, since $\tan\beta$ itself is not a physical observable, a lot of renormalization schemes for $\tan\beta$ have been proposed [6, 7] in the studies of the radiative corrections in the MSSM. One method

[1] Talk based on Ref. [1].

is to define the renormalized $\tan\beta$ as a tree-level function of the physical observables. This method is manifestly independent of the gauge fixing and renormalization scale. However, the form of the counterterm strongly depends on the chosen observables and is often very complicated. Here I concentrate on another method, the process-independent definition of $\tan\beta$, which is given by the ratio of the renormalized VEVs v_i. I discuss the renormalization of the ultraviolet (UV) divergent corrections to v_i and $\tan\beta$, working in the $\overline{\rm DR}$ scheme [8]. The results are presented as the renormalization group equations (RGEs) for v_i and $\tan\beta$. Since they are not physical observables, they may depend on the gauge fixing in general. I therefore investigate [1] their gauge dependence in the general R_ξ gauge fixing [9]. The results for the gauge dependence can be generalized for other models with two or more Higgs doublets.

In the $\overline{\rm DR}$ scheme [10, 4, 5], we absorb Δv_i by the shift of quadratic terms in the Higgs potential, as

$$(m_i^2+\delta m_i^2)|\phi_i^0|^2 \;\Rightarrow\; \delta L \ni -\sqrt{2}(v_i\delta m_i^2){\rm Re}\phi_i^0. \tag{3}$$

The renormalized v_i then give the minimum of the loop-corrected effective potential $V_{\rm eff}(H_1,H_2)$. This scheme is very convenient in practical calculation, because of very simple counterterm for $\tan\beta$, and that the explicit forms of the tadpole diagrams are necessary only for two-point functions of the Higgs bosons. However, the effective potential is generally dependent on the gauge fixing [11]. The gauge dependence of the renormalized v_i and their ratio $\tan\beta$ then might be a serious problem in calculating radiative corrections. I will therefore discuss the gauge dependence of the running $\tan\beta$ in this definition, in general R_ξ gauges and to the two-loop order.

The RGE for v_i can be obtained from the UV divergent corrections to the two quark masses m_b and m_t, ignoring the masses of all other quarks and leptons. These mass terms are generated from the $b\bar{b}H_1$ and $t\bar{t}H_2$ Yukawa couplings, respectively, as

$$L_{\rm int} = -h_b\bar{b}_Rb_L(v_1/\sqrt{2}+\phi_1^0) - h_t\bar{t}_Rt_L(v_2/\sqrt{2}+\phi_2^0) + {\rm h.c.} \tag{4}$$

One then obtain

$$\frac{dv_i}{dt} \;=\; \frac{1}{h_q}\left[\sqrt{2}\frac{d}{dt}(m_q) - \frac{dh_q}{dt}v_i\right], \tag{5}$$

where $q=(b,t)$ for $i=(1,2)$, respectively. $t\equiv \ln Q_{\overline{\rm DR}}$ is the $\overline{\rm DR}$ renormalization scale.

The R_ξ gauge fixing term takes the form

$$\begin{aligned} L_{GF} \;=\; & -\frac{1}{2\xi_Z}(\partial^\mu Z_\mu - \rho_Z G_Z)^2 - \frac{1}{\xi_W}|\partial^\mu W_\mu^+ - i\rho_W G_W^+|^2 \\ & -\frac{1}{2\xi_\gamma}(\partial^\mu\gamma_\mu)^2 - \frac{1}{2\xi_g}\sum_{a=1}^{8}(\partial^\mu g_\mu^a)^2. \end{aligned} \tag{6}$$

The would-be Nambu-Goldstone bosons G_V for $V=(Z,W)$ appear in Eq. (6). The parameters $\rho_V\equiv\xi_V m_V$, where $m_V^2 = g_V^2(v_1^2+v_2^2)/4$ ($g_W^2=g_2^2$, $g_Z^2=g_2^2+g_Y^2$) are masses of Z and $W^\pm$, are introduced in Eq. (6). This is to emphasize that the gauge symmetry breaking terms $\xi_V m_V$ in L_{GF}, and also in the accompanied Fadeev-Popov ghost term, has very different nature from v_i generated by the shifts (2), as shown later. The terms $\rho_V G_V$ in Eq. (6) are expressed in the gauge basis (1) of the Higgs bosons as

$$\rho_Z G_Z = \xi_Z m_Z G_Z \equiv -\sqrt{2}{\rm Im}(\rho_{1Z}\phi_1^0 - \rho_{2Z}\phi_2^0), \tag{7}$$

$$\rho_W G_W^\pm = \xi_W m_W G_W^\pm \equiv -(\rho_{1W} H_1^\pm - \rho_{2W} H_2^\pm), \tag{8}$$

with parameters ρ_{iV}. The usual form of the R_ξ gauge fixing in the MSSM is recovered by the substitution [5, 3]

$$(\rho_{1V}, \rho_{2V}) = \xi_V g_V (v_1, v_2)/2 = \xi_V m_V (\cos\beta, \sin\beta). \tag{9}$$

The UV divergent corrections to m_b contain one source for the SU(2)×U(1) gauge symmetry breaking. It is either v_1 originated from the shift (2) of H_1^0, or ρ_{1V} in the R_ξ gauge fixing term (6) and the Fadeev-Popov ghost term. The former contribution is obtained from that to the $\bar{b}_R b_L \phi_1^0$ Yukawa coupling h_b by replacing external ϕ_1^0 by $v_1/\sqrt{2}$, except for the wave function correction of H_1^0 to h_b. Similar argument holds for the UV divergent corrections to m_t and to the $\bar{t}_R t_L \phi_2^0$ Yukawa coupling h_t. As a result, if the ρ_{iV} contributions are absent, the runnings of v_i are the same as those of the wave functions of H_i^0, namely

$$\frac{dv_i}{dt} = -\gamma_i v_i. \tag{10}$$

The anomalous dimensions γ_i of H_i^0 generally depend on the gauge fixing parameters ξ. Their explicit forms are

$$(4\pi)^2 \gamma_i^{(1)} = N_c h_q^2 - \frac{3}{4} g_2^2 \left(1 - \frac{2}{3}\xi_W - \frac{1}{3}\xi_Z\right) - \frac{1}{4} g_Y^2 (1 - \xi_Z), \tag{11}$$

at the one-loop, and

$$\begin{aligned}(4\pi)^4 \gamma_1^{(2)} &= -N_c(3h_b^4 + h_b^2 h_t^2) + 2N_c h_b^2 \left(\frac{8}{3} g_3^2 - \frac{1}{9} g_Y^2\right) + L(g), \\ (4\pi)^4 \gamma_2^{(2)} &= -N_c(3h_t^4 + h_b^2 h_t^2) + 2N_c h_t^2 \left(\frac{8}{3} g_3^2 + \frac{2}{9} g_Y^2\right) + L(g).\end{aligned} \tag{12}$$

at the two-loop. Here $h_q^2 = (h_b^2, h_t^2)$ for $i = (1, 2)$, respectively, and $N_c = 3$. The results in Eq. (12) are obtained from the general formula [12] in the $\overline{\text{MS}}$ scheme, after conversion into the $\overline{\text{DR}}$ scheme [13]. Their last term $L(g)$ is a gauge-dependent $\mathcal{O}(g^4)$ polynomial and is common both for $\gamma_1^{(2)}$ and $\gamma_2^{(2)}$, while the $\mathcal{O}(h_q^2 g^2)$ terms are ξ independent [12]. It is therefore seen that the gauge dependence of γ_i cancels in the RGE of the ratio $\tan\beta$ to the two-loop order.

However, in general R_ξ gauges, ρ_{iV} in the gauge fixing terms (6) may give additional contributions to the quark mass running, as $\bar{b}b\rho_{1V}$ and $\bar{t}t\rho_{2V}$. Since they have no corresponding contributions to the $\bar{b}b\phi_1$ and $\bar{t}t\phi_2$ couplings, the RGEs for v_i deviate [14, 5] from Eq. (5), as shown in Fig. 1 at the one-loop level. The general forms of the RGEs are then

$$\frac{dv_i}{dt} = -\gamma_i v_i + Y_{iV} \rho_{iV}, \tag{13}$$

where Y_{iV} are polynomials of dimensionless couplings. Therefore, the RGE for $\tan\beta$ becomes, substituting Eq. (9),

$$\frac{d}{dt} \tan\beta = \tan\beta \left(-\gamma_2 + \gamma_1 + \frac{\xi_V g_V}{2}(Y_{2V} - Y_{1V})\right). \tag{14}$$

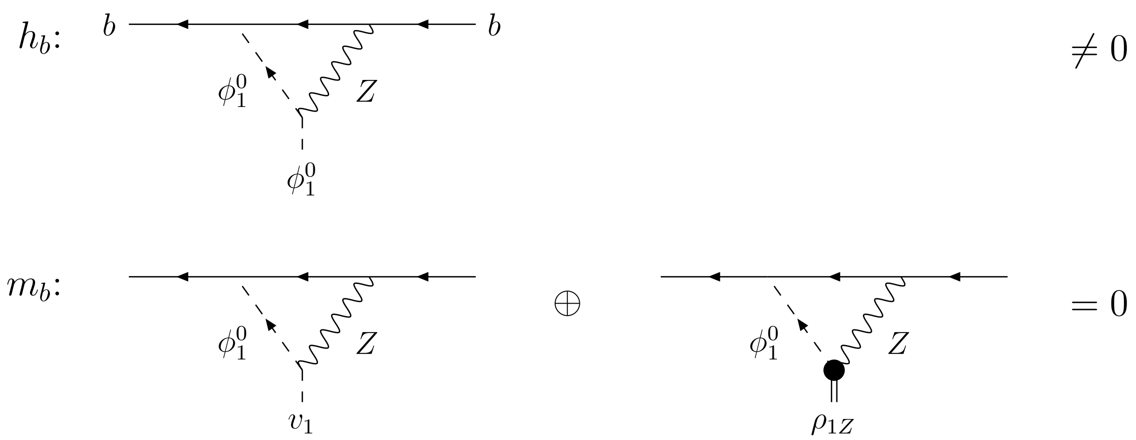

Figure 1: One-loop difference between the runnings of h_b and m_b by the ρ_{1Z} contribution.

I then give explicit form of the RGE for $\tan\beta$ to the two-loop order. First, one-loop RGEs for v_i ($i = 1, 2$) are

$$\left.\frac{dv_i}{dt}\right|_{1\text{loop}} = -\gamma_i^{(1)} v_i + \frac{1}{(4\pi)^2}(g_Z \rho_{iZ} + 2g_2\rho_{iW}) = v_i\left[-\gamma_i^{(1)} + \frac{1}{(4\pi)^2}\left(\frac{\xi_Z g_Z^2}{2} + \xi_W g_2^2\right)\right], \tag{15}$$

The ρ_{iV} contributions to m_q are obtained from the diagrams similar to that in Fig. 1. Eq. (15) is consistent with the result in Refs. [5] for $\xi = 1$. Since the gauge dependence of γ_i, as well as the contribution from (ρ_{iZ}, ρ_{iW}) satisfying Eq. (9), cancels in the ratio (14), the one-loop running $\tan\beta$ is gauge parameter independent in the R_ξ gauge. Note that this one-loop gauge independence of the running $\tan\beta$ does not hold in more general gauge fixings where Eq. (9) is not satisfied [7].

The two-loop ρ_{iV} contributions to dv_i/dt have $\mathcal{O}(h_q^2 g\rho_{iV})$ and $\mathcal{O}(g^3\rho_{iV})$ terms. The latter is common for both $i = 1$ and 2, and cancels out in the ratio $\tan\beta$ if Eq. (9) is satisfied. Therefore, only the former $\mathcal{O}(h_q^2 g\rho_{iV})$ contributions are explicitly calculated. For example, the $\mathcal{O}(h_b^2 g_Z \rho_{1Z})$ contribution to v_1 comes from the diagram in Fig. 2. The two-loop RGEs for v_i are finally

$$\left.\frac{dv_i}{dt}\right|_{2\text{loop}} = -\gamma_i^{(2)} v_i - \frac{N_c h_q^2}{(4\pi)^4}(g_Z\rho_{iZ} + 2g_W\rho_{iW}) + P_V(g)\rho_{iV}, \tag{16}$$

where again $h_q^2 = (h_b^2, h_t^2)$ for $i = (1, 2)$, respectively. $P_V(g)$ are possibly gauge-dependent $\mathcal{O}(g^3)$ functions which are common for both ρ_{1V} and ρ_{2V}. It is therefore seen that, due to the ρ_{iV} contributions in Eq. (16), the running $\tan\beta$ has the $\mathcal{O}(h_q^2 g_2^2, h_q^2 g_Y^2)$ gauge parameter dependence. Although existing higher-order calculations of the corrections to the MSSM Higgs sector [15, 16, 17, 18] have not included the contributions of these orders yet, the gauge dependence of $\tan\beta$ may cause theoretical problem in future studies of the higher-order corrections in the MSSM.

A possible way to restore the gauge independence of renormalized running $\tan\beta$ is to introduce gauge-dependent shifts of ϕ_i^0 such as to cancel the ρ_{iV} contributions to the

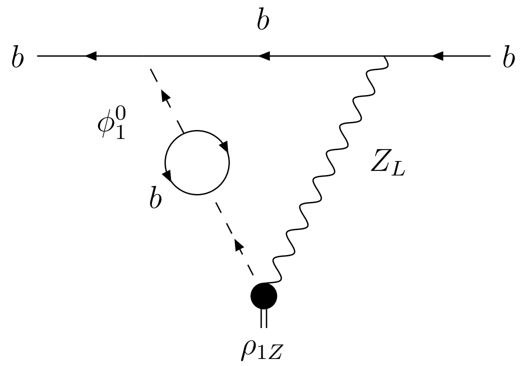

Figure 2: Two-loop divergent $\mathcal{O}(h_b^2 g\rho_{1Z})$ contribution to m_b.

effective action. This modification corresponds to the addition of extra shifts of v_i to all diagrams. The running v_i in this new definition then obey the same RGEs as those for H_i, namely Eq. (10). The modified renormalized $\tan\beta$ becomes gauge independent to the two-loop order. However, an extra two-loop shift $\delta(v_2/v_1)$ has to be added to any quantities which depend on $\tan\beta$. The concrete procedure for this modification is now under investigation.

In conclusion, I discussed the two-loop UV renormalization of the ratio $\tan\beta = v_2/v_1$ of the Higgs VEVs in the MSSM in general R_ξ gauges. When renormalized v_i are given by the minimum of the loop-corrected effective potential, the contributions of ρ_{iV} in the R_ξ gauge fixing term cause two-loop gauge dependence of the RGE for $\tan\beta$. To avoid this gauge dependence, the contributions of ρ_{iV} have to be cancelled by extra shifts of the Higgs boson fields ϕ_i^0.

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