Lepton Flavor Violating Process in Bi-large Texture of Neutrino Mixing

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- A.Kageyama, S.K., N.Shimoyama, M.Tanimoto
  Phys.Lett **B527**:206-214,2002
- A.Kageyama, S.K., N.Shimoyama, M.Tanimoto
  Phys.Rev. **D65**:096010,2002

I. Motivations

• Neutrino Oscillations [Super-K, SNO]

⟹ **Bi-large neutrino mixing**

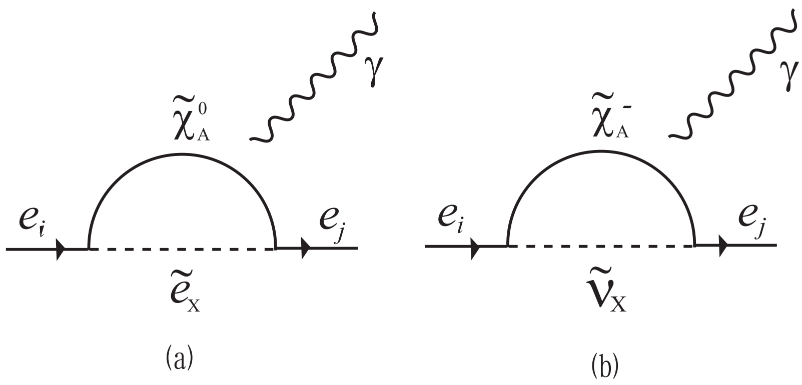
- $\sin^2 2\theta_{\rm sol} = 0.6 - 0.85$ (best fit : $\theta_{\rm sol} = 30°$)
- $\sin^2 2\theta_{\rm atm} \geq 0.92$ (best fit : $\theta_{\rm atm} = 45°$)
- $\triangle m^2_{\rm sol} = 5 \times 10^{-5}{\rm eV}^2$ (best fit)
- $\triangle m^2_{\rm atm} = 2.5 \times 10^{-3}{\rm eV}^2$ (best fit)

⟹ **Small neutrino mass**

- **See-saw mechanism** (introducing $\nu_{\rm R}$)
- Small Lepton Flavor Violation in SM + $\nu_{\rm R}$
  BR($\mu \to e\gamma$) $\sim 10^{-50}$

• Supersymmetry

- Solving Gauge Hierarchy Problem
- Gauge Coupling Unification
- **Large Lepton Flavor Violation ($\to$ Sec.II)**
  [F.Borzumati and A.Masiero]



MSSM + $\nu_{\rm R}$ (see-saw)

In this framework, **LFV is related to neutrino mass spectrum and neutrino mixing matrix elements.**

Spectrums:

1. Hierarchical
   - J.Hisano,T.Moroi,K.Tobe and M.Yamaguchi, Phys.Rev.D53(1996)2442
   - J.Sato,K.Tobe and T.Yanagida, Phys.Lett.B498(2001)498

     ...
2. Degenerate
3. Inverse-hierarchical → **Sec.III**

Neutrino mixing matrix:

$$U_{\rm MNS} \sim \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & U_{e3} < 0.2 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (1)$$

We can compare between the theorecical predictions and experimental bound for $\mu \to e\gamma$ and $\tau \to \mu\gamma$:

$${\rm BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}$$ **(MEGA;1999)**

$${\rm BR}(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$$ **(CLEO;2000)**

for three types of neutrino mass spectrum.

II. LFV in MSSM + ν_R

[F.Borzumati and A.Masiero(1986)]
[J.Hisano,T.Moroi,K.Tobe and M.Yamaguchi(1996)]

Superpotential

$$W_{\rm lepton} = \mathbf{Y}_e L H_d e_R^c + \mathbf{Y}_\nu L H_u \nu_R^c + \frac{1}{2}\nu_R^{cT}\mathbf{M}_R \nu_R^c$$

$$(\mathbf{Y}_e : \text{diagonal}, \mathbf{M}_R : \text{diagonal}\ ,\ \mathbf{Y}_\nu : \text{non–diagonal}) \qquad (2)$$

Soft SUSY breaking terms

$$\begin{aligned} -\mathcal{L}_{\rm soft} &= (\mathbf{m}_{\tilde{L}}{}^2)_{ij}\tilde{L}_i^\dagger \tilde{L}_j + (\mathbf{m}_{\tilde{e}}{}^2)_{ij}\tilde{e}_{Ri}^* \tilde{e}_{Rj} + (\mathbf{m}_{\tilde{\nu}}{}^2)_{ij}\tilde{\nu}_{Ri}^* \tilde{\nu}_{Rj} \\ &\quad + \mathbf{A}_{ij}^e H_d \tilde{e}_{Ri}^* \tilde{L}_j + \mathbf{A}_{ij}^\nu H_u \tilde{\nu}_{Ri}^* \tilde{L}_j + \cdots \end{aligned} \qquad (3)$$

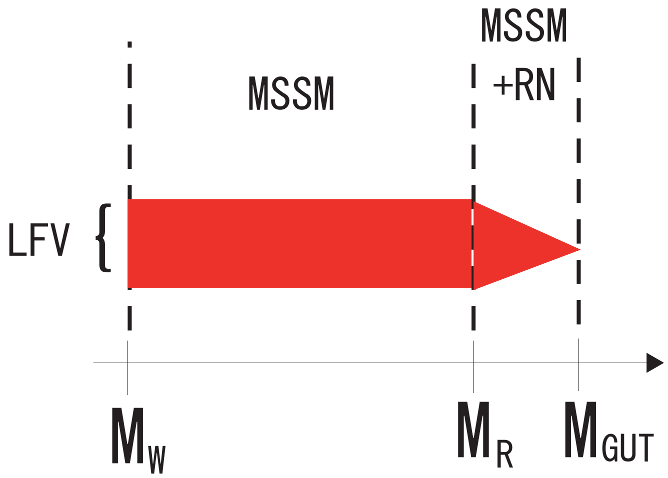
We take universality condition:

$$\begin{aligned} (\mathbf{m}_{\tilde{L}}{}^2)_{ij} &= (\mathbf{m}_{\tilde{e}}{}^2)_{ij} = (\mathbf{m}_{\tilde{\nu}}{}^2)_{ij} = \cdots = \delta_{ij} {m_0}^2\ , \\ \mathbf{A}^\nu &= \mathbf{Y}_\nu a_0 m_0, \\ \mathbf{A}^e &= \mathbf{Y}_e a_0 m_0, \quad (a_0 = 0) \end{aligned} \qquad (4)$$

at GUT scale ($\sim 10^{16}$GeV).

There are no flavor violations at GUT scale.
This assumptions (4) are the most conservative one for lepton flavor violations.

However, lepton flavor violation are induced through Renormalization Group Equations effect. This is due to generation off-diagonal elements of $(\mathbf{m}^2_{\tilde{L}})_{ij}$.



$$\mu\frac{d}{d\mu}(\mathbf{m}^2_{\tilde{L}})_{ij} = \mu\frac{d}{d\mu}(\mathbf{m}^2_{\tilde{L}})_{ij}\Big|_{\mathrm{MSSM}} + \frac{1}{16\pi^2}\left[(\mathbf{m}^2_{\tilde{L}}\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu + \mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{m}^2_{\tilde{L}})_{ij} + 2(\mathbf{Y}^\dagger_\nu\mathbf{m}_{\tilde{\nu}}\mathbf{Y}_\nu + \tilde{m}^2_{H_u}\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu + \mathbf{A}^\dagger_\nu\mathbf{A}_\nu)_{ij}\right] \quad (5)$$

Source of LFV comes from second and third lines including neutrino Yukawa coupling matrix $\mathbf{Y}_\nu$.

$$(\triangle\mathbf{m}^2_{\tilde{L}})_{ij} \simeq -\frac{(6+2a_0^2)m_0^2}{16\pi^2}(\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu)_{ij}\ln\frac{M_X}{M_R}\,, \quad (6)$$

$$BR(\mu\to e\gamma) \propto |(\triangle\mathbf{m}^2_{\tilde{L}})_{21}|^2\tan^2\beta\,, \quad (7)$$

$$BR(\tau\to \mu\gamma) \propto |(\triangle\mathbf{m}^2_{\tilde{L}})_{32}|^2\tan^2\beta\,, \quad (8)$$

Y_ν depends on $m_1, m_2, m_3, M_R, U_{MNS}$ (next section). Therefore, discussing neutrino masses, mixings and M_R is crucial for estimating LFV.

III. LFV in three types of Neutrino Spectrums

Neutrino Mass Spectrum

1. $m_1 \ll m_2 \ll m_3$ **Hierarchical**

$$m_1 \sim 0, \quad m_2 = \sqrt{\Delta m^2_{\rm sol}}, \quad m_3 = \sqrt{\Delta m^2_{\rm atm}}$$

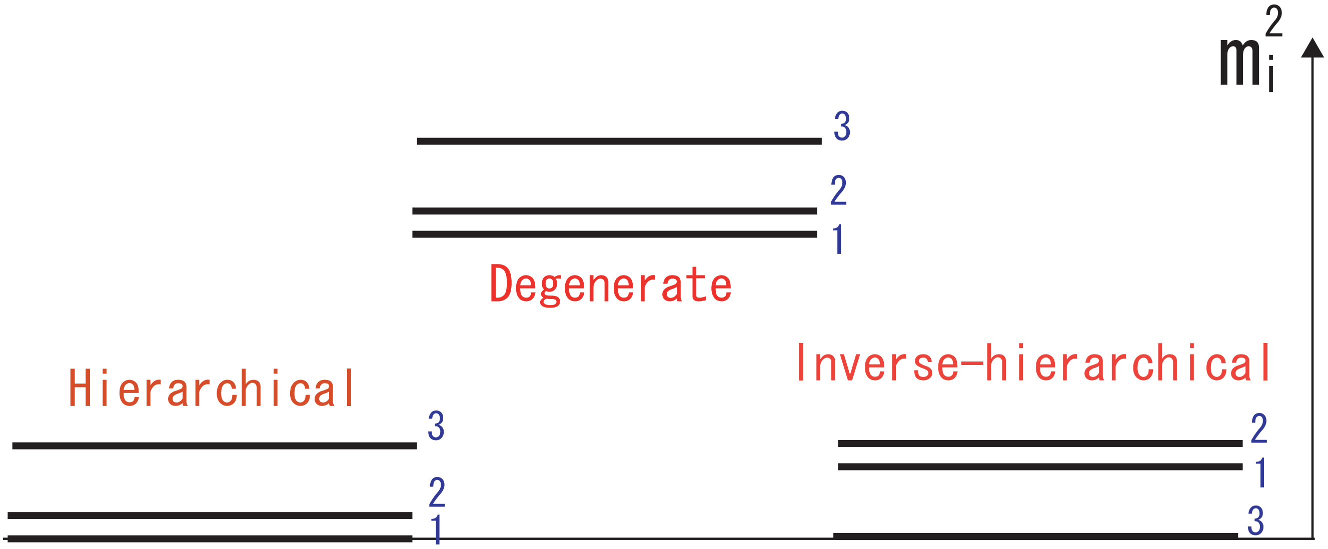
2. $m_1 \sim m_2 \sim m_3$ **Degenerate**

$$m_1 \equiv m_\nu,\ m_2 = m_\nu + \frac{1}{2m_\nu}\Delta m^2_{\rm sol},\ m_3 = m_\nu + \frac{1}{2m_\nu}\Delta m^2_{\rm atm}$$

3. $m_1 \sim m_2 \gg m_3$ **Inverse-hierarchical**

$$m_1 = m_2 - \frac{1}{2m_2}\Delta m^2_{\rm sol}, \quad m_2 \equiv \sqrt{\Delta m^2_{\rm atm}}, \quad m_3 \simeq 0$$

Taking **Large Mixing Angle MSW** solution :
$\triangle m^2_\odot = 7 \times 10^{-5}{\rm eV}^2$, $\triangle m^2_{\rm atm} = 3 \times 10^{-3}{\rm eV}^2$



Neutrino Yukawa coupling

$$\mathbf{Y}_\nu = \frac{1}{v_u}\sqrt{\mathbf{M}_R^{\rm diag}}\,\mathbf{R}\begin{pmatrix}\sqrt{m_1} & 0 & 0\\ 0 & \sqrt{m_2} & 0\\ 0 & 0 & \sqrt{m_3}\end{pmatrix}\mathbf{U}_{MNS}^{\rm T} \quad (9)$$

R is an orthogonal matrix ($\mathrm{RR}^{\mathrm{T}} = 1$), which is model dependent.

[1] $M_{R1} = M_{R2} = M_{R3} = M_R \rightarrow$ **Simple formula**
[2] $M_{R1} \neq M_{R2} \neq M_{R3} \rightarrow$ **Sec. V** (Discussion)

♣ **No CP violating phase**
♣ $\mathrm{U}_{e3} \leq 0.2$ **(CHOOZ)**

In the case of [1],

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} = \frac{M_R}{v_u^2}\left[U_{\mu 2}U_{e2}^*(m_2 - m_1) + U_{\mu 3}U_{e3}^*(m_3 - m_1)\right]$$

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32} = \frac{M_R}{v_u^2}\left[U_{\tau 2}U_{\mu 2}^*(m_2 - m_1) + U_{\tau 3}U_{\mu 3}^*(m_3 - m_1)\right]$$

↑ **These are R independent !**

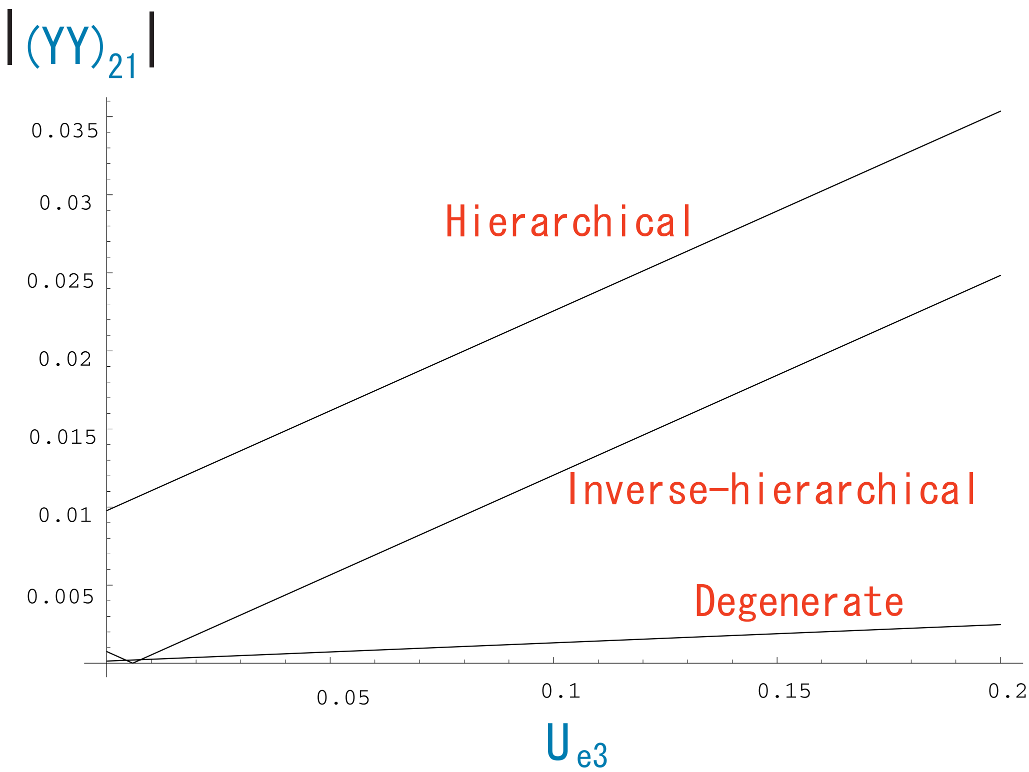
$\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is expressed in terms of neutrino oscillation parameters for three types of neutrino spectrums:

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} \simeq \frac{M_R}{\sqrt{2} v_u^2}\sqrt{\Delta m^2_{\rm atm}}\left[\frac{1}{\sqrt{2}} U^*_{e2}\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}}} + U^*_{e3}\right] \quad \textbf{Hier.}$$

$$\simeq \frac{M_R}{\sqrt{2} v_u^2}\frac{\Delta m^2_{\rm atm}}{2 m_\nu}\left[\frac{1}{\sqrt{2}} U^*_{e2}\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} + U^*_{e3}\right] \quad \textbf{Deg.}$$

$$(m_\nu = 0.3\,\mathrm{eV})$$

$$\simeq \frac{M_R}{\sqrt{2} v_u^2}\sqrt{\Delta m^2_{\rm atm}}\left[\frac{1}{2\sqrt{2}} U^*_{e2}\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} - U^*_{e3}\right] \quad \textbf{Inv.}$$



$\uparrow \; H \geq I \sim D$ $\qquad\qquad$ $\uparrow \; H \sim I \geq D$

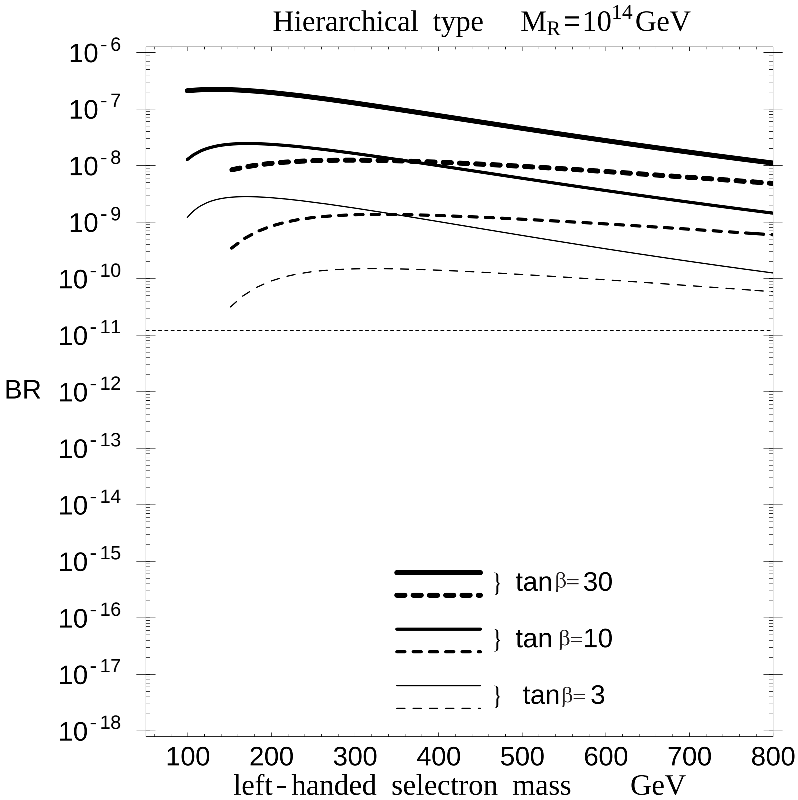


Figure 1: **Hierarchical** BR($\mu \to e\gamma$); $M_R = 10^{14}$GeV, $U_{e3} = \mathbf{0.2}$

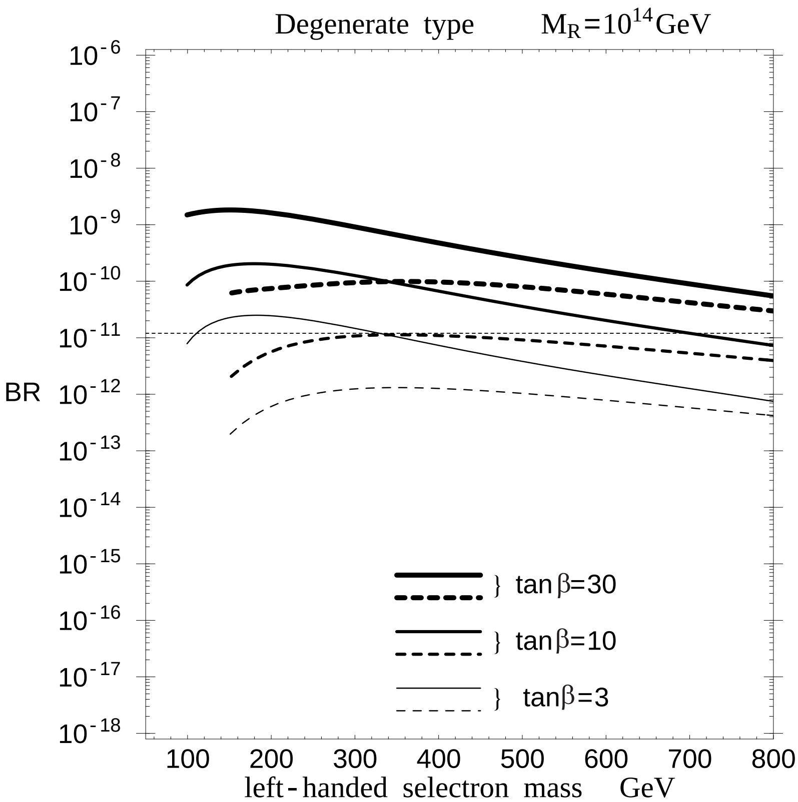


Figure 2: **Degenerate** BR($\mu \to e\gamma$); $M_R = 10^{14}$GeV,$U_{e3} = \mathbf{0.2}$

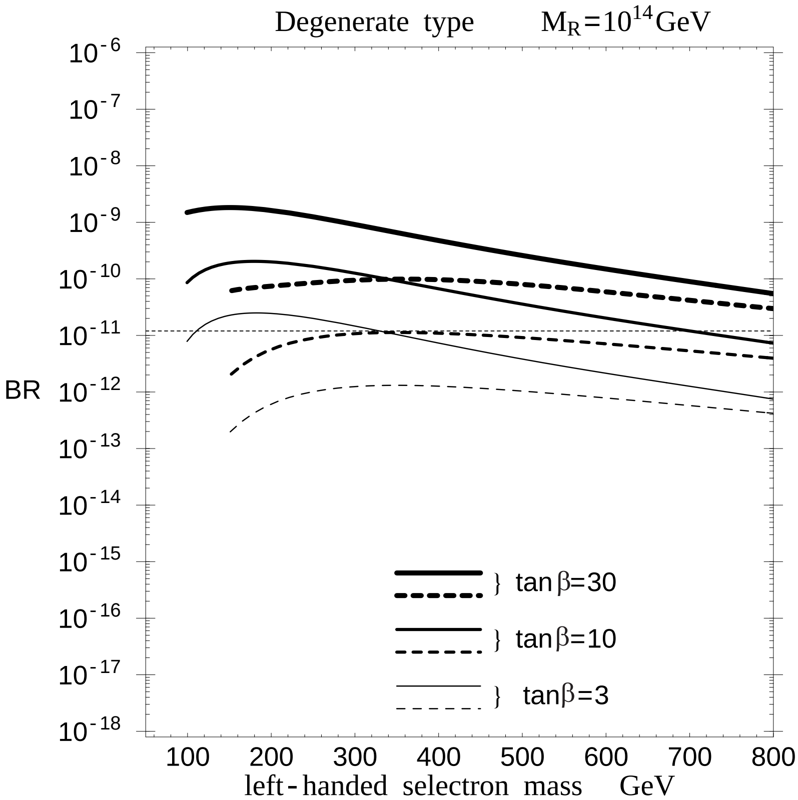


Figure 2: **Degenerate** BR($\mu \to e\gamma$); $M_R = 10^{14}$GeV,$U_{e3} = \mathbf{0.2}$

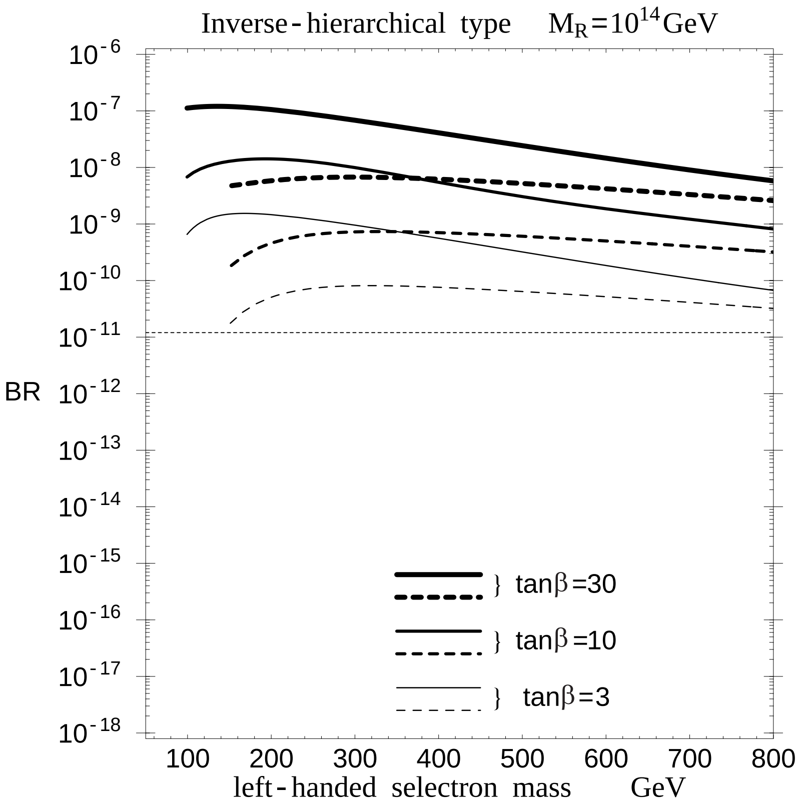


Figure 3: **Inverse-hier.**BR($\mu \to e\gamma$); $M_R = 10^{14}$GeV, $U_{e3} = \mathbf{0.2}$

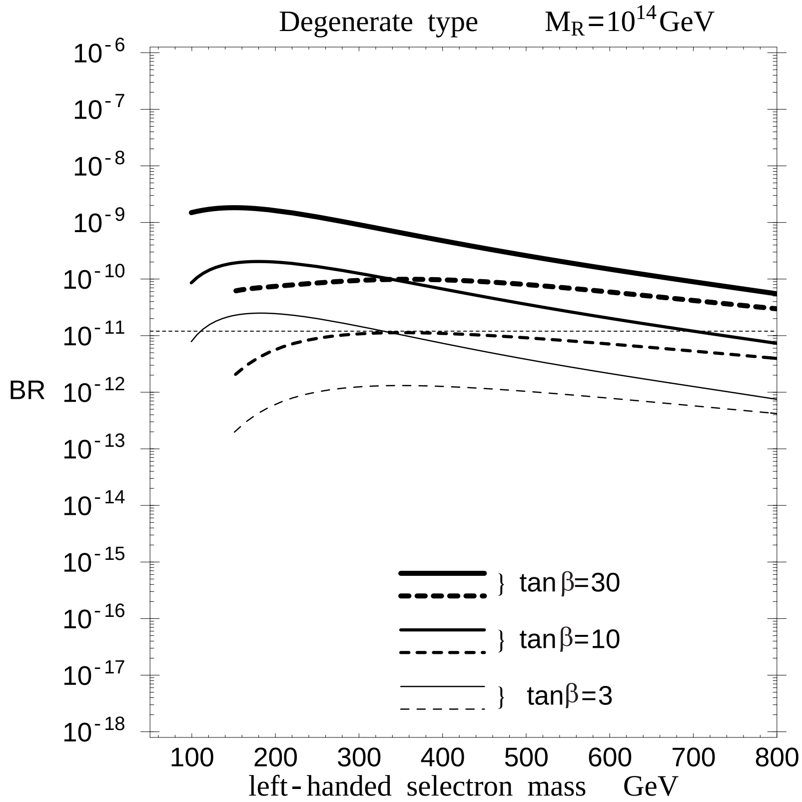


Figure 2: **Degenerate** BR($\mu \to e\gamma$); $M_R = 10^{14}$GeV, $U_{e3} = \mathbf{0.2}$

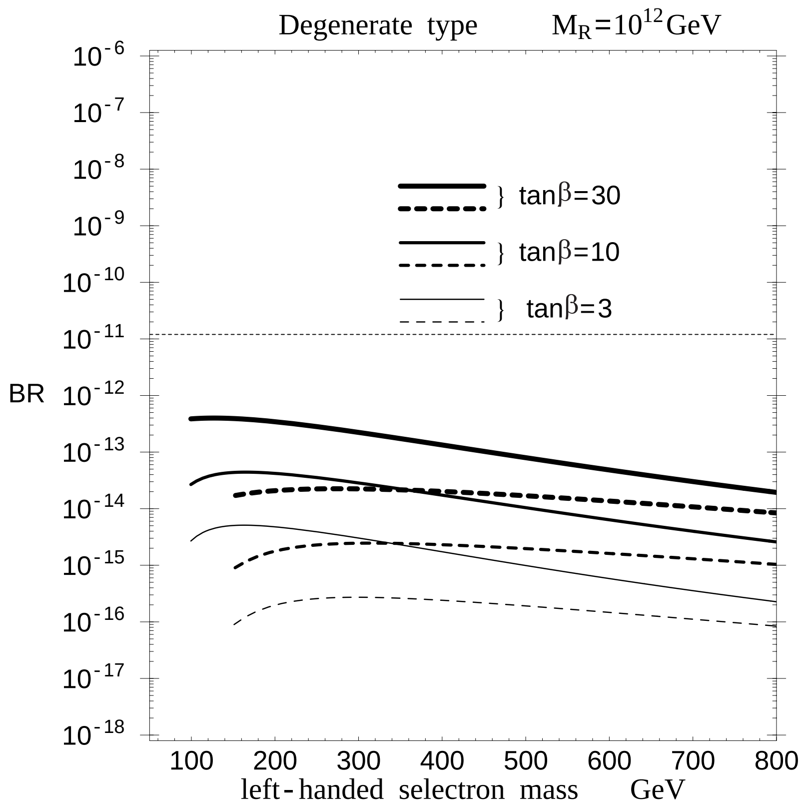


Figure 4: **Degenerate** BR($\mu \to e\gamma$); $M_R = 10^{12}$GeV, $U_{e3} = \mathbf{0.2}$

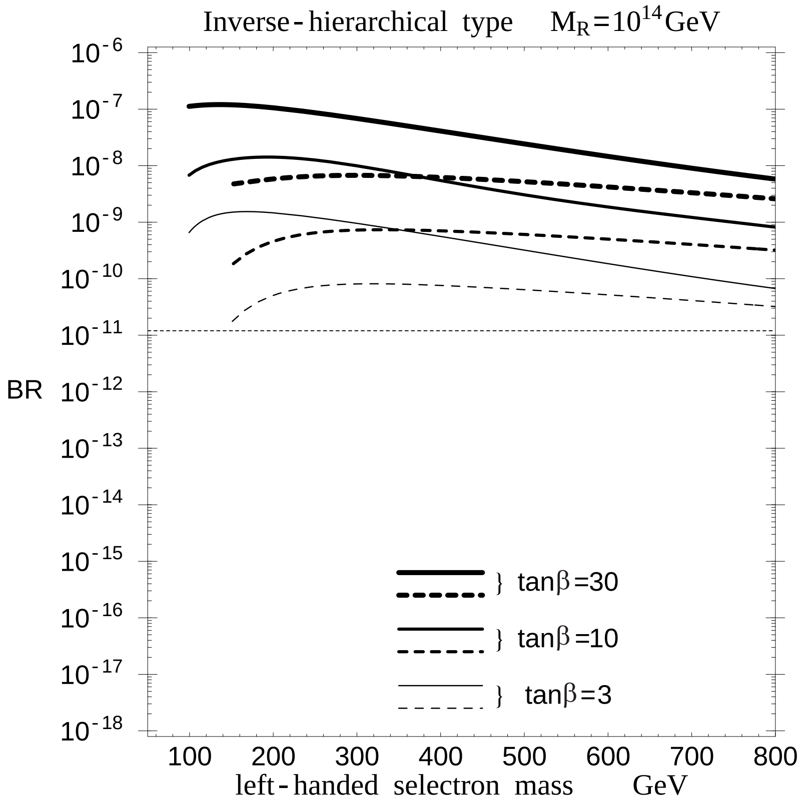


Figure 3: **Inverse-hier.**BR$(\mu \to e\gamma)$; $M_R = 10^{14}$GeV, $\underline{U_{e3} = \mathbf{0.2}}$

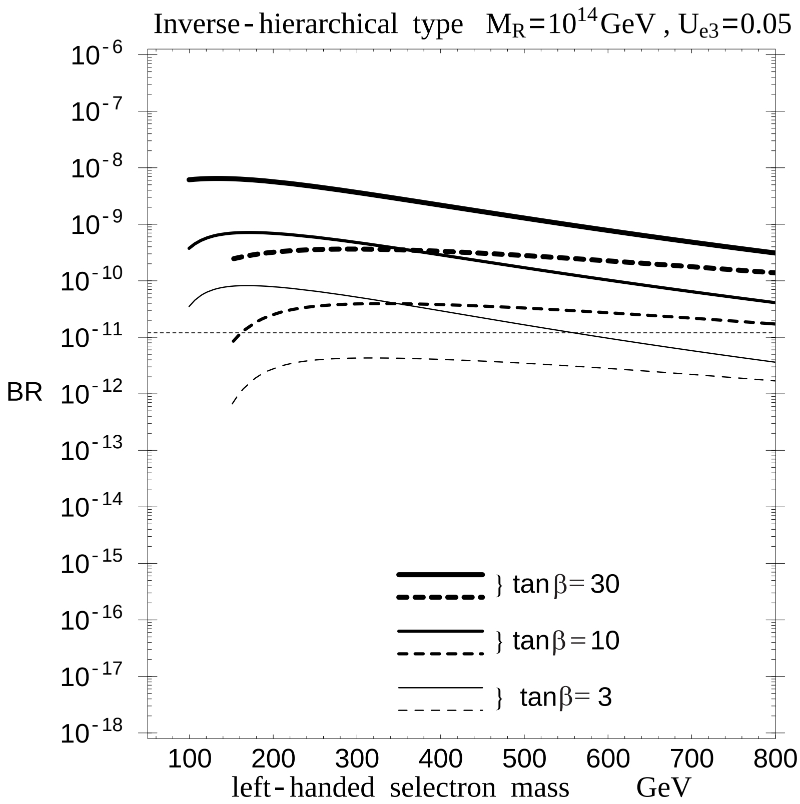


Figure 5: **Inverse-hier.**BR$(\mu \to e\gamma)$;$M_R = 10^{14}$GeV, $\underline{U_{e3} = \mathbf{0.05}}$

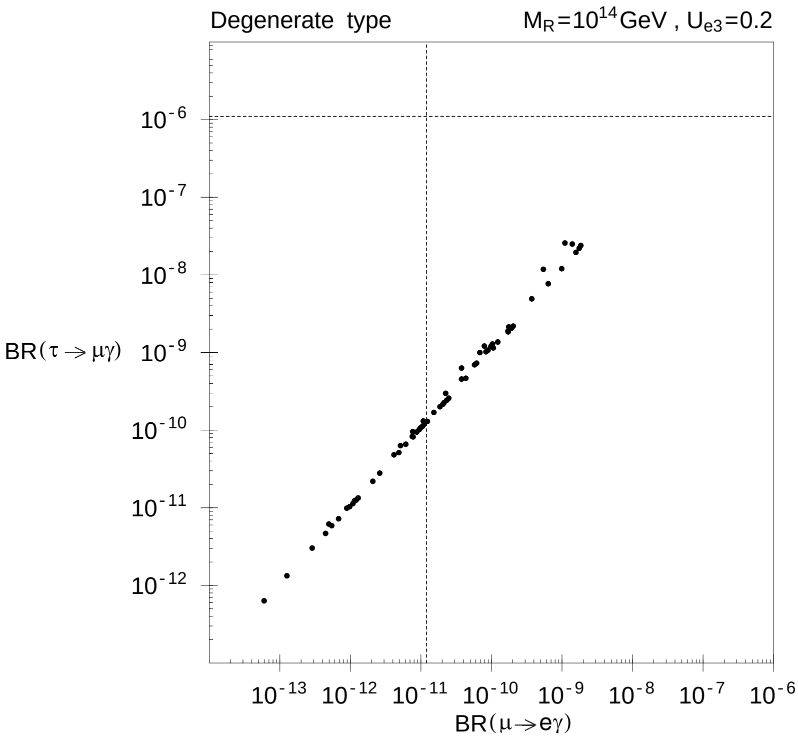


Figure 6: **Degenerate** BR($\tau \to \mu\gamma$) vs BR($\mu \to e\gamma$)
$M_R = 10^{14}$GeV, $U_{e3} = \mathbf{0.2}$

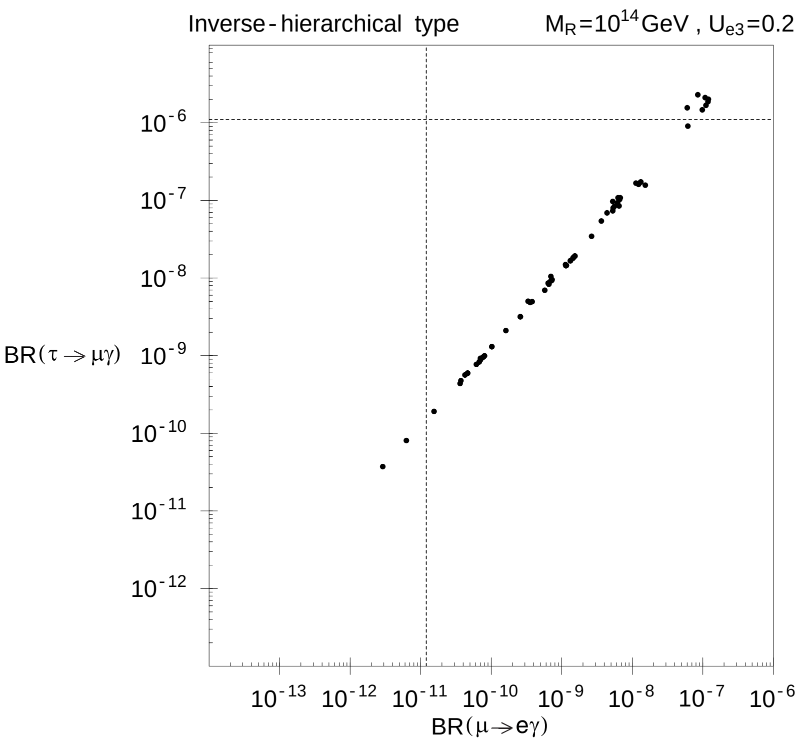


Figure 7: **Inverse-hier.** BR($\tau \to \mu\gamma$) vs BR($\mu \to e\gamma$)
$M_R = 10^{14}$GeV, $U_{e3} = \mathbf{0.2}$

V. Summary and Discussion

We investigate lepton flavor violations ($\mu \to e\gamma$, $\tau \to \mu\gamma$) in MSSM+ν_R in bi-large texture of neutrino mixing.

$[\mathbf{1}]\ \mathbf{M_{R1} = M_{R2} = M_{R3} = M_R}$

$$\text{BR(deg.)} \underset{\times 100}{<} \text{BR(Inverse-hier.)} \sim \text{BR(Hier.)} \quad @\ U_{e3} > 0.05$$

$[\mathbf{2}]\ \mathbf{M_{R1} \neq M_{R2} \neq M_{R3}}$

$$\text{Degenerate}: \quad \frac{\mathbf{BR}_{M_{R1} \neq M_{R2} \neq M_{R3}}}{\mathbf{BR}_{M_{R1}=M_{R2}=M_{R3}=M_R}} \leq 5.5$$

$$\text{Inverse}-\text{hierarchical}: \quad \frac{\mathbf{BR}_{M_{R1} \neq M_{R2} \neq M_{R3}}}{\mathbf{BR}_{M_{R1}=M_{R2}=M_{R3}=M_R}} \leq 1.04$$

M_{R}, U_{e3} and m_ν are crucial quantities for estimating lepton flavor violations.

Future experiment

- ♣ **BR($\mu \to e\gamma$) $\to$ 10^{-14} @ PSI**
- ♣ **BR($\tau \to \mu\gamma$) $\to$ 10^{-8} @KEK, SLAC**
- ♣ **Double beta decay : m_ν**
- ♣ **LBL experiment ($\nu_\mu \to \nu_e$) : U_{e3}**

More precise test of LFV are possible in future.