Infrared Contribution to the Double-Logarithmic Small-x Asymptotics of Structure Functions

S.I. Manayenkov, M.G. Ryskin

Petersburg Nuclear Physics Institute, Russian Academy of Science,
Gatchina, St.Petersburg district, 188350, Russia

Abstract

The role of the soft region ($k_T < 1$ GeV/c) in the small-x behaviour of the structure functions is studied with the help of the effective QCD Lagrangian taking into account the lightest degrees of freedom (constituent quarks and pions). It is shown that the pion exchange contribution increases few times a value of the non-singlet structure function if the pion coupling constant g is positive (for the isosinglet component of f_1 and for the isovector one of g_1) and decreases it for $g < 0$ (for the isosinglet part of g_1 and for the isovector component of f_1) . The pion exchange contribution changes the flavour singlet component of g_1 less than the non-singlet one.

1 Introduction

The small-x behaviour of the deep inelastic structure functions (SF) plays an important role in the precise description of the parton distributions. As it is known (see [1] and references therein), one cannot use the conventional DGLAP evolution equations for the region $x \ll 1$. Indeed, the double-logarithmic (DL) contributions of the ladder graphs at $x \ll 1$ come both from the region $|k^2_{i+1,T}| \ll |k^2_{i,T}|$ (as in the DGLAP evolution) and from $|k^2_{i+1,T}| \gg |k^2_{i,T}|$. Hence there is no k_T-ordering and $k_{i,T}$ can become close to the infrared cut off parameter μ in the middle of the evolution chain as at the beginning of the evolution.

In the DL approximation with fixed QCD coupling α_s a convenient way to calculate SF is to use the infrared evolution equation [2]. Using this method the small-x asymptotics of the non-singlet (NS) unpolarized ($f_{1,NS}$) and polarized ($g_{1,NS}$) structure functions have been obtained in [3], [4] $f_1^{NS}(x,Q^2) \sim x^{-\omega^{(+)}}$, $g_1^{NS}(x,Q^2) \sim x^{-\omega^{(-)}}$ with $\omega^{(+)} = \sqrt{2C_F\alpha_s/\pi}$, $\omega^{(-)} = 1.04\omega^{(+)}$, $C_F = (N^2-1)/(2N)$ for the $SU_c(N)$ colour group. The powerlike behaviour ($\sim x^{-\lambda}$) of SF is valid for running QCD coupling $\alpha_s(q^2) = 4\pi/[b\ln(q^2/\Lambda^2)]$ too. The value of λ depends on μ and in the extreme case $\mu = \lambda$ the power $\lambda = 2C_F/b$ [5]. But the region $k_T \leq 1$ GeV/c is the nonperturbative one. To obtain more realistic estimates we consider the perturbative QCD evolution at large transverse momenta $|k_T^2| > t_0 \approx 1$ (GeV/c)^2 and for the low k_T we use the effective Lagrangian QCD [6] which takes into account the lightest degrees of freedom only (light constituent quarks with mass $M = 350$ MeV and π^+, π^-, π^0 (the Goldstone bosons)). Recently

[7] a reasonable explanation of the "spin crisis" and the Gottfried sum rule violation has been proposed with the aid of the Lagrangian [6].

This paper is a short version of the work [8] where one can find more details and references.

2 Double-logarithmic evolution

We shall use the DL evolution equation in the integral form as it has been done in [3]. First such an equation has been considered in [9] for the case of QED. The amplitude A satisfies the equation [3]

$$A(t,\beta) = \int_\beta^1 \frac{d\beta'}{\beta'} \int_{\mu^2}^{t_m} \frac{dt'}{t'} G(t_m) A(t',\beta') + A_0(t,\beta) \tag{1}$$

where $t_m = t\beta'/\beta$. We have applied the Sudakov parameterization for the quark four-momentum $k = -\alpha\tilde{q} + \beta p + k_T$ (and the analogous relation for k') with $\tilde{q} = q - xp$, $x = -q^2/s$, $s = 2(pq)$ where q and p are the momenta of the heavy photon and the initial quark, respectively. We denote as k_T the transversal part of k ($qk_T = pk_T = 0$), $t = -k_T^2$ and α, β are the Sudakov parameters.

The non-singlet structure function looks like

$$f_{1,NS}(x,Q^2) = e_q^2 \Big[\delta(x-1) + \int_{\mu^2}^{S_m} A(t,\tilde{\beta}) \frac{dt}{t}\Big] \tag{2}$$

with $\tilde{\beta} = x + t/s$, $Q^2 = -q^2$ and $S_m = Q^2/(4x)$. In (2) e_q is the fractional electrical charge of a quark. The quantity $G(t_m)$ is given by

$$G(t_m) = \frac{H(t_m)}{2\pi}[1 - \theta(t_0 - t)\theta(t_0 - t')] + g\theta(t_0 - t)\theta(t_0 - t') \ , \tag{3}$$

where $H(t_m) = C_F\alpha_s(t_m)$. The first term in (3) describes the perturbative QCD evolution and the second one is related with the pion exchange contributing at $t < t_0$, $t' < t_0$. A value of g depends on the type of SF and on the isospin I in the t-channel. For the structure function f_1

$$g^{I=0} = 3M^2/[16\pi^2 f_\pi^2] \ , \quad g^{I=1} = -M^2/[16\pi^2 f_\pi^2] \tag{4}$$

and for the spin-dependent structure function g_1 we have

$$g^{I=0} = -3M^2/[16\pi^2 f_\pi^2] \ , \quad g^{I=1} = M^2/[16\pi^2 f_\pi^2] \ , \tag{5}$$

where f_π denotes the pion coupling constant $f_\pi = 93$ MeV and $M = 350$ MeV is the mass of the constituent quark. The inhomogeneous term contains also two terms

$$A_0(t,\beta) = \frac{H(t/\beta)}{2\pi}\theta(t - t_0) + g\theta(t_0 - t) \tag{6}$$

with $H(t/\beta) = C_F\alpha_s(t/\beta)$. The former and the latter terms correspond to the perturbative QCD contribution and to the pion exchange one, respectively.

For the flavour singlet case we have to consider both the gluon and quark components. Now A becomes a matrix A_d^c with c, d = gluon or quark. The lower index denotes a sort of a parent parton and the upper one denotes a current parton type. DL equation (1) remains valid but instead of H and g in (3), (6) we have to use the matrices $\hat{H}$ and $\hat{g}$, respectively

$$\hat{H} = \begin{pmatrix} 4N_c\alpha_s(\tilde{t}) & 2C_F\alpha_s(t) \\ -n_f\alpha_s(t') & C_F\alpha_s(t_m) \end{pmatrix}, \quad \hat{g} = \begin{pmatrix} 0 & 0 \\ 0 & g \end{pmatrix} \ , \tag{7}$$

where n_f is a number of active quark flavours and $\tilde{t} = t'\beta/\beta'$. For A_0 in (6) we are to put $\beta' = 1$ and $t' = M^2$ in $\hat{H}$ defined by (7). When the argument of α_s in some matrix element of $\hat{H}$ is smaller than t_0 we put it equal to t_0. The formula for the singlet part of SF $g_1(x, Q^2)$ reads

$$g^d_{1,S}(x, Q^2) = e_d^2\delta(1-x) + \frac{\sum_{q=1}^{n_f} e_q^2}{n_f}\int_{\mu^2}^{S_m} A^q_d(t, \tilde{\beta})\frac{dt}{t} \tag{8}$$

where $d = q$ ($d = g$) corresponds to the initial quark (gluon).

3 Discussion of numerical results

Our calculations have been performed with the aid of formulae (1) - (8) and presented in Figs. 1 - 6. The x-dependence of the NS isovector structure function $f^{I=1}_{1,NS}(x, Q^2)$ is shown in Fig. 1a and a behaviour of $\lambda = -\partial \ln f^{I=1}_{1,NS}/\partial \ln x$ is presented in Fig. 1b. We see that at $x \leq 10^{-3}$ the logarithmic derivative λ becomes a constant which does not depend on Q^2 and SF $f^{I=1}_{1,NS}(x, Q^2)$ has a powerlike behaviour ($\sim x^{-\lambda}$) at small x. These numerical predictions are in agreement with the asymptotical formula

$$f_{NS}(x, Q^2) = B_1 x^{-\lambda_1}\{\ln^{\kappa_1}(Q^2/\Lambda^2) - \ln^{\kappa_1}(\mu^2/\Lambda^2) + \kappa_1 \ln^{\kappa_1 - 1}(Q^2/\Lambda^2)[\psi(1) - \psi(\lambda_1)]\} \tag{9}$$

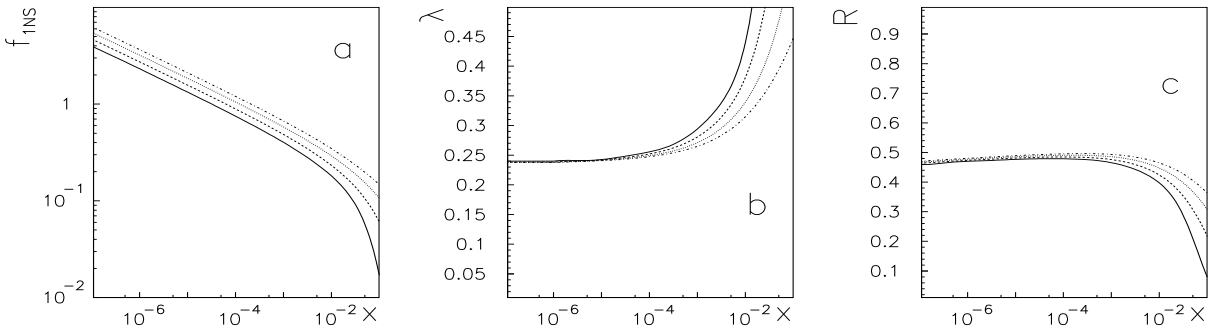

Fig. 1: Non-singlet isovector structure function $f^{I=1}_{1,NS}(x, Q^2)$. Dependence on x and Q^2 of a) $f^{I=1}_{1NS}(x, Q^2)$ b) logarithmic derivative λ c) ratio R. Solid, dashed, dotted and dash-dotted lines correspond to $Q^2 = 10$, 30, 100 and 300 (GeV/c)^2, respectively.

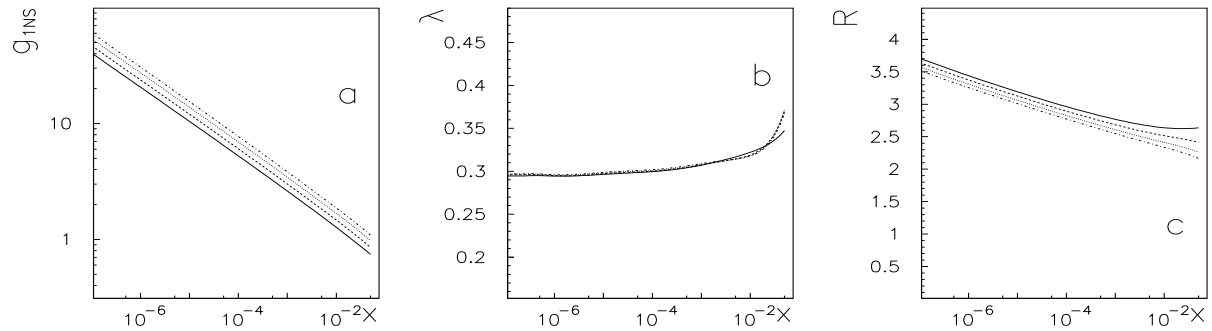

Fig. 2: Non-singlet isovector structure function $g^{I=1}_{1,NS}(x, Q^2)$. Dependence on x and Q^2 of a) $f^{I=1}_{1NS}(x, Q^2)$ b) logarithmic derivative λ c) ratio R. Solid, dashed, dotted and dash-dotted lines correspond to $Q^2 = 10$, 30, 100 and 300 (GeV/c)^2, respectively.

which has been obtained in [8] from the analytical solution of DL integral equation (1) for the NS case. In (9) B_1 and λ_1 are some constants, $\kappa_1 = 8/[(33 - 2n_f)\lambda_1]$ and $\psi(\zeta)$ denotes the digamma

function. Fig. 1c shows the ratio R of $f_{1,NS}^{I=1}$ calculated with the pion exchange contribution (PEC) and without it ($g = 0$ in (3), (6)). The value $R \approx 0.45$ means that PEC decreases the quark density at small x. In principal, this effect can be responsible for the Gottfried sum rule violation recently observed by NMC. The analogous calculations for the spin-dependent structure function $g_{1,NS}^{I=1}$ predict the powerlike asymptotics at small x with $\lambda \approx 0.3$ (see Fig. 2a, b). Strictly speaking equation (1) sums up the ladder graph contributions only. To take effectively into account the non-ladder DL contributions we multiply α_s in (3) by 1.08 in accordance with the result of [4]. Fig. 2c shows that PEC increases the quark density at small x ($R > 2.5$ for $x < 10^{-2}$). This effect can be related with a large difference between the proton and neutron structure

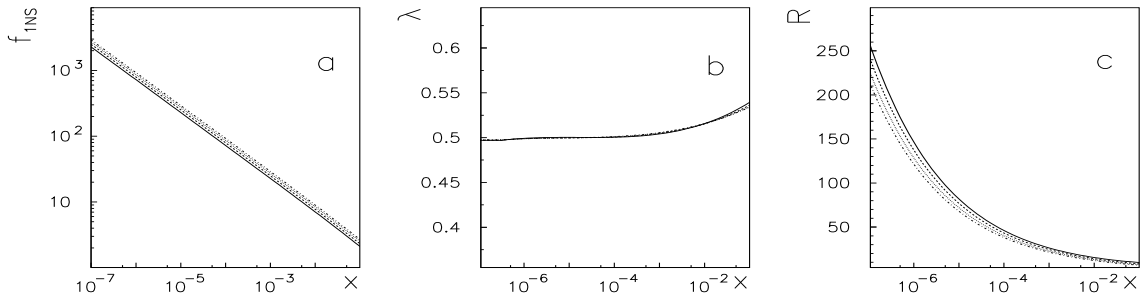

Fig. 3: Non-singlet isoscalar structure function $f_{1,NS}^{I=0}(x, Q^2)$. Curves are the same as in Fig. 1 but for $I = 0$.

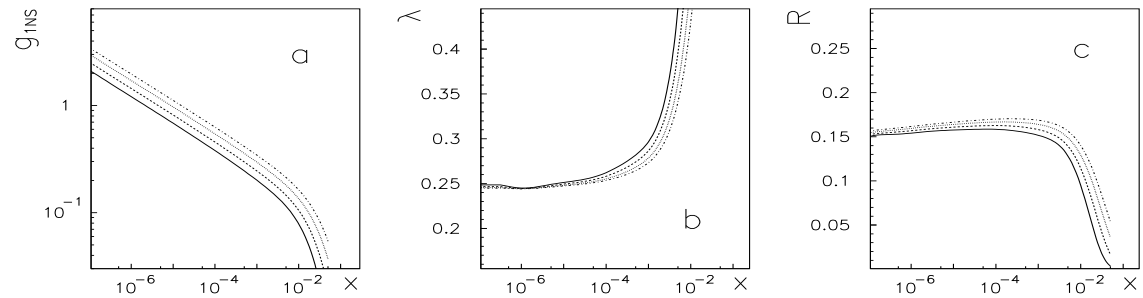

Fig. 4: Non-singlet isoscalar structure function $g_{1,NS}^{I=0}(x, Q^2)$. Curves are the same as in Fig. 2 but for $I = 0$.

functions $g_1^p - g_1^n$ observed experimentally by SMC. It is interesting to note that for $I = 0$ the asymptotical power $\lambda \approx 0.5$ which is in agreement with the Regge phenomenology prediction (see Fig. 3b). For $f_{1,NS}^{I=0}$ PEC is crucially important as one can see from Fig. 3c ($R > 20$ at $x < 10^{-3}$). Analyzing formulae (4), (5) and Figs. 1 - 4 we conclude that PEC increases SF ($R > 1$) and λ for $g > 0$ and decreases them for $g < 0$, a value of λ being less sensitive to PEC than R.

The results of our calculations of the flavour singlet contributions to $g_1(x, Q^2)$ are presented in Fig. 5 for the initial gluon and in Fig. 6 for the parent quark. To take approximately into account the non-ladder DL contributions one can use $\hat{H}_{eff} = \hat{H} + \hat{\delta}$ instead of $\hat{H}$ [10] where

$$\hat{\delta} = \begin{pmatrix} -2.016\alpha_s(\tilde{t}) & -0.179\alpha_s(t) \\ 0.168\alpha_s(t') & 0.025\alpha_s(t_m) \end{pmatrix} . \tag{10}$$

We see from Figs. 5, 6 that $g_1(x, Q^2)$ has a powerlike behaviour but the effective power λ is larger than 1 both for the initial gluon and quark. This means presumably that the next-to-leading order corrections are crucially important. Hence our calculations are not true theoretical

predictions but estimates only. The ratios shown in Fig. 5c and Fig. 6c are the ones of the structure functions calculated with the DL contributions of the ladder and non-ladder Feynman graphs to SF obtained without the non-ladder DL contribution ($\hat{\delta} = 0$). We see that the smaller x is the more important is the non-ladder DL contribution. We have considered PEC to the singlet structure functions and found them less important than for the NS case.

4 Conclusions

The model describing both the perturbative QCD and pion exchange DL contributions to SF at low x is proposed. The powerlike asymptotics at $x \ll 1$ both for non-singlet and singlet SF is

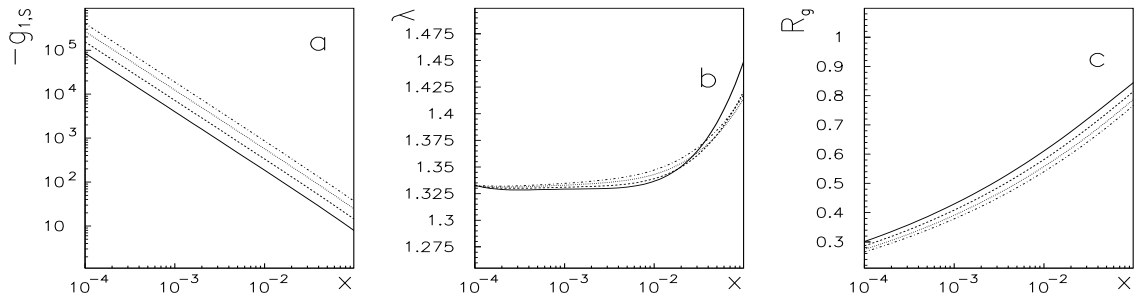

Fig. 5: Singlet structure function $g_{1,S}(x, Q^2)$ for parent gluon. Curves are the same as in Fig. 2 but R denotes ratio of total calculation to DL contribution of ladder graphs only.

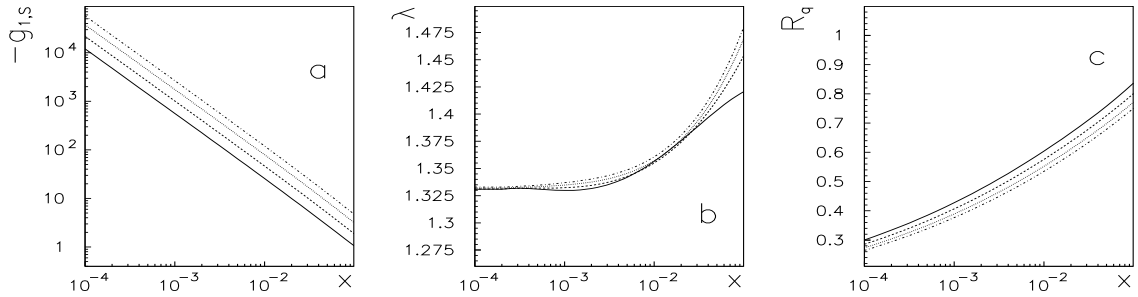

Fig. 6: Singlet spin-dependent structure function $g_{1,S}(x, Q^2)$ for parent quark. Curves are the same as in Fig. 5 but for parent quark.

predicted and values of the powers are calculated. For SF $f_{1,NS}$ in the isosinglet t-channel the predicted value of λ agrees with the Regge phenomenology. PEC diminishes appreciably $f_{1,NS}^{I=1}$ and increases $g_{1,NS}^{I=1}$. These predictions do not contradict the NMC and SMC data. Our calculations for the singlet case show an importance of the non-ladder DL contributions and point presumably to an importance of the next-to-leading order corrections to the DL calculations.

References

[1] V. G. Gorshkov, *Sov. Phys. Uspekhi* **16** (1973) 322.

[2] R. Kirschner and L. N. Lipatov, *Nucl. Phys.* **B213** (1983) 122.

[3] B. I. Ermolaev, S. I. Manayenkov, M. G. Ryskin, *Z. Phys.* **C69** (1996) 259.

[4] J. Bartels, B. I. Ermolaev, M. G. Ryskin, *Z. Phys.* **C70** (1996) 273.

[5] S. I. Manayenkov, *Z. Phys.* **C75** (1997) 685.

[6] A. Manohar, H. Georgi, *Nucl. Phys.* **B234** (1984) 189.

[7] A. A. Anselm and M. G. Ryskin, *Z. Phys.* **C68** (1995) 297.

[8] S. I. Manayenkov, M. G. Ryskin, to be published in *Z. Phys.* **C**.

[9] V. G. Gorshkov,V. N. Gribov, L. N. Lipatov and G. V. Frolov, *Sov. Journ. Nucl. Phys.* **6** (1967) 95.

[10] J. Bartels, B. I. Ermolaev, M. G. Ryskin, *Z. Phys.* **C72** (1996) 627.