Searching for tetraquarks on the lattice

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We address the question whether the lightest scalar mesons σ and κ are tetraquarks. We present a search for possible light tetraquark states with $J^{PC} = 0^{++}$ and $I = 0,\ 1/2,\ 3/2,\ 2$ in the dynamical and the quenched lattice simulations using tetraquark interpolators. In all the channels, we unavoidably find lowest scattering states $\pi(k)\pi(-k)$ or $K(k)\pi(-k)$ with back-to-back momentum $k = 0,\ 2\pi/L, \cdots$. However, we find an additional light state in the $I = 0$ and $I = 1/2$ channels, which may be related to the observed resonances σ and κ with a strong tetraquark component. In the exotic repulsive channels $I = 2$ and $I = 3/2$, where no resonance is observed, we find no light state in addition to the scattering states.

It is still not known whether the lightest observed nonet of scalar mesons σ, κ, $a_0(980)$ and $f_0(980)$ [2] are conventional $\bar{q}q$ states or exotic tetraquark $\bar{q}\bar{q}qq$ states. Tetraquark interpretation was proposed by Jaffe back in 1977 [1] and it is supported by many phenomenological studies, for example [2, 3]. The tetraquarks, composed of a scalar diquark and anti-diquark, form a flavor nonet and are expected to be light. The observed ordering $m_\kappa < m_{a_0(980)}$ favors tetraquark interpretation since the $I = 1$ state $[\bar{d}\bar{s}][us]$ with additional valence pair $\bar{s}s$ is naturally heavier than the $I = 1/2$ state $[\bar{s}\bar{d}][du]$.

It is important to determine whether QCD predicts any scalar tetraquark states below 1 GeV from a first principle lattice QCD calculation. Previous lattice simulations [4, 5] have not given the final answer yet. The strongest claim for σ as tetraquark was obtained using the sequential Bayes method to extract the spectrum [4] and needs confirmation using a different method. Our new results, given in this proceeding, are presented with more details in [6, 7].

We calculate the energy spectrum of scalar tetraquark states with $I = 0,\ 2,\ 1/2,\ 3/2$ in dynamical and quenched lattice simulations. Our dynamical simulation ($a \simeq 0.15$ fm, $V = 16^3{\times}32$) uses dynamical Chirally Improved u/d quarks [8] and it is the first dynamical simulation intended to study tetraquarks. The quenched simulation ($a \simeq 0.20$ fm, $V = 16^3{\times}28$) uses overlap fermions, which have exact chiral symmetry even at finite a.

The energies of the lowest three physical states are extracted from the correlation functions $C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle_{\vec{p}=\vec{0}} = \sum_n Z_i^n Z_j^{n*} e^{-E_n\, t}$ with tetraquark interpolators $\mathcal{O} \sim \bar{q}\bar{q}qq$, where $Z_i^n \equiv \langle 0|\mathcal{O}_i|n\rangle$. In all the channels we use three different interpolators that are products of two color-singlet currents [6]. In addition, we use two types of diquark anti-diquark interpolators in $I = 0,\ 1/2$ channels [6].

When calculating the $I = 0,\ 1/2$ correlation matrix, we neglect the so-called single and double disconnected quark contractions [5], as in all previous tetraquark studies. The resulting states have only a $\bar{q}\bar{q}qq$ Fock component in this approximation, while they would contain also a $\bar{q}q$ component if single disconnected contractions were taken into account [5]. Since we are searching for "pure" tetraquark states in this pioneering study, our approximation is physically motivated.

All physical states n with given $J^{PC} = 0^{++}$ and I propagate between the source and the sink in the correlation functions. Besides possible tetraquark states, there are unavoidable contributions from scattering states $\pi(k)\pi(-k)$ for $I = 0,\ 2$ and scattering states $\pi(k)K(-k)$ for $I = 1/2,\ 3/2$. Scattering states have discrete momenta $\vec{k} = \frac{2\pi}{L}\vec{j}$ on the lattice of size L and energy $(m_\pi^2 + \vec{k}^2)^{1/2} + (m_{\pi,K}^2 + \vec{k}^2)^{1/2}$ in the non-interacting approximation. Our main question is whether we find some light state in addition the scattering states in $I = 0,\ 1/2$ channels. If such a state is found, it could be related to the resonances σ or κ with a strong tetraquark component.

The energies E_n are extracted from the correlation functions $C_{ij}(t)$ via the eigenvalues $\lambda^n(t) \propto e^{-E_n(t-t_0)}$ of the generalized eigenvalue problem $C(t)\vec{u}^n(t) = \lambda^n(t,t_0)C(t_0)\vec{u}^n$ at some reference time t_0 [9].

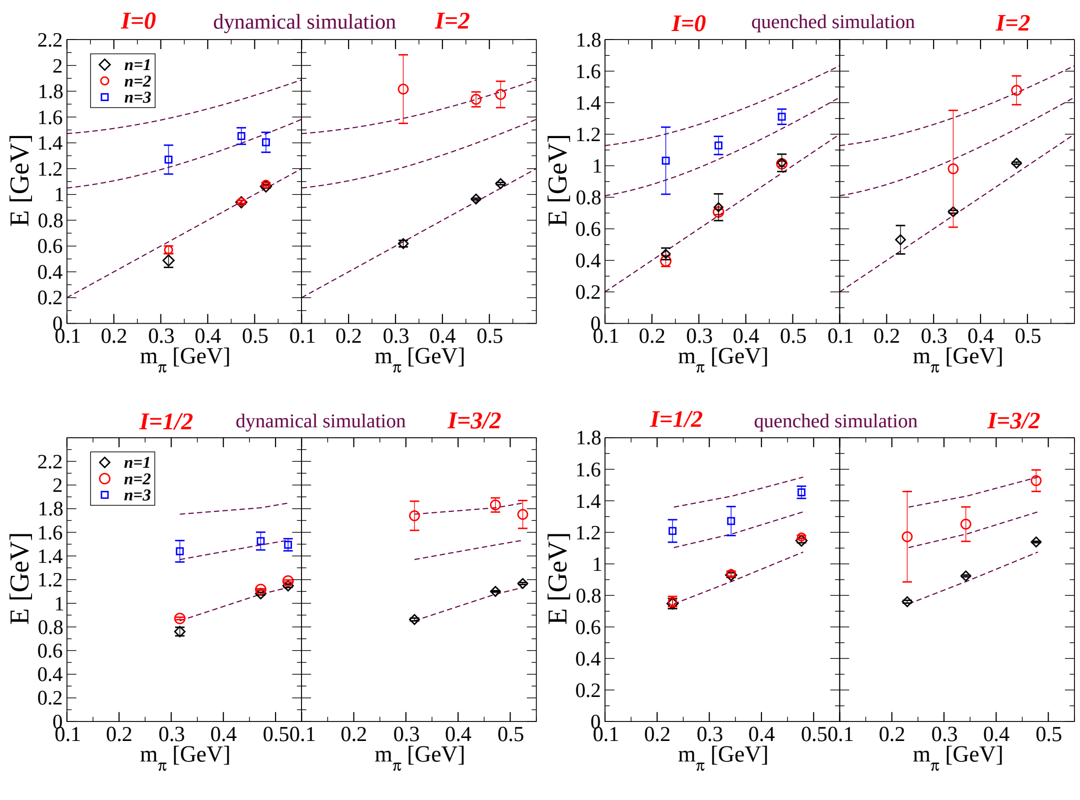

Figure 1: The resulting spectrum E_n for $I = 0,\ 2,\ 1/2,\ 3/2$ in the dynamical (left) and the quenched (right) simulations. Note that there are two states (black and red) close to each other in $I = 0$ and $I = 1/2$ cases. The lines at $I = 0,\ 2$ present the energies of non-interacting $\pi(k)\pi(-k)$ with $k = j\frac{2\pi}{L}$ and $j = 0, 1, \sqrt{2}$. Similarly, lines at $I = 1/2,\ 3/2$ present energies of $\pi(k)K(-k)$.

The resulting spectrum E_n for all four isospins is shown in Fig. 1. The lines present the energies of the scattering states in the non-interacting approximation. Our dynamical and quenched results are in qualitative agreement.

In the repulsive channel $I = 2$, where no resonance is expected, we indeed find only the candidates for the scattering states $\pi(0)\pi(0)$ and $\pi(\frac{2\pi}{L})\pi(-\frac{2\pi}{L})$ with no additional light state. The first excited state is higher than expected due to the smallness of 3×3 basis. Similar conclusion applies for the repulsive $I = 3/2$ channel with πK scattering states.

In the attractive channel $I = 0$ we find two (orthogonal) states close to the threshold $2m_\pi$ and another state consistent with $\pi(\frac{2\pi}{L})\pi(-\frac{2\pi}{L})$, so we do find an additional light state. This leads to a possible interpretation that one of the two light states is the scattering state $\pi(0)\pi(0)$ and the other one corresponds to σ resonance with strong tetraquark component (see more general discussion in [10]). In the attractive $I = 1/2$ channel we similarly find the candidates for the lowest two $\pi(k)K(-k)$ scattering states and a candidate for a κ resonance with a large tetraquark component. These results have to be confirmed by another independent lattice simulation before making firm conclusions.

We investigate two criterion for distinguishing the one-particle (tetraquark) and two-particle (scattering) states in [7]. The first criteria is related to the time dependence of $C_{ij}(t)$ and $\lambda^n(t)$ at finite temporal extent of the lattice. The second is related to the volume dependence of the couplings $\langle 0|\mathcal{O}_i|n\rangle$.

The ultimate method to study σ and κ on the lattice would involve the study of the spectrum and couplings in presence of $\bar{q}\bar{q}qq \leftrightarrow \bar{q}q \leftrightarrow vac \leftrightarrow glue$ mixing, using interpolators that cover these Fock components. Such a study has to be done as a function of lattice size L in order to extract the resonance mass and width using the Lüscher's finite volume method [10, 11].

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