Renormalization of Fermion-Flavour Mixing

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We report on an explicit on-shell framework to renormalize the fermion-flavour mixing matrices in the Standard Model and its extensions, at one-loop level. It is based on a novel procedure to separate the external-leg mixing corrections into gauge-independent self-mass and gauge-dependent wave-function renormalization contributions.

1 Introduction

Renormalizability endows the Standard Model (SM) with enhanced predictive power due to the fact that ultraviolet (UV) divergences from quantum effects can be eliminated by a redefinition of a finite number of independent parameters, such as masses and coupling constants. Furthermore, it has been known for a long time that, in the most frequently employed formulations in which the complete bare mass matrices of quarks are diagonalized, the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix must be also renormalized. In fact, this problem has been the object of several interesting studies over the last two decades. A matter of considerable interest is the generalization of these considerations to minimal renormalizable extensions of the SM.

2 On-shell renormalization prescription

The on-shell renormalization framework we propose is a generalization of Feynman's approach in QED [1]. Recall that in QED the self-energy contribution to an outgoing fermion is given by

$$\Delta\mathcal{M}^{\text{leg}} = \overline{u}(p)\Sigma(\not{p})\frac{1}{\not{p}-m}, \quad \Sigma(\not{p}) = A(p^2) + B(p^2)(\not{p}-m) + \Sigma^{\text{fin}}(\not{p})$$

where $\Sigma(\not{p})$ is the self-energy, A and B are divergent constants, and Σ^{fin} is a finite part which is proportional to $(\not{p}-m)^2$ in the vicinity of $\not{p}=m$ and therefore does not contribute to $\Delta\mathcal{M}^{\text{leg}}$. The contribution of A to $\Delta\mathcal{M}^{\text{leg}}$ is singular at $\not{p}=m$ and gauge independent and that of B is regular but gauge dependent. They are called self-mass (sm) and wave-function renormalization (wfr) contributions. A is cancelled by the mass counterterm δm while B is combined with proper vertex diagrams leading to a finite and gauge-independent physical amplitude.

In the case of fermion-flavour mixing we encounter not only diagonal terms as in QED but also off-diagonal contributions. The self-energy corrections to an external fermion leg are now

$$\Delta\mathcal{M}^{\text{leg}}_{ij} = \overline{u}_i(p)\Sigma_{ij}(\not{p})\frac{1}{\not{p}-m_j},$$

where i denotes the external fermion of momentum p and mass m_i, and j the virtual fermion of mass m_j. Using a simple algorithm that treats i and j on an equal footing, we write the self-energy as:

$$\Sigma_{ij}(\not p) = A_{ij}(p^2) + (\not p - m_i)B_{1,ij}(p^2) + B_{2,ij}(p^2)(\not p - m_j) + (\not p - m_i)\Sigma^{\rm fin}_{ij}(p^2)(\not p - m_j),$$

in analogy to QED. Similarly, we identify the contributions to $\Delta\mathcal{M}^{\rm leg}$ coming from A as sm and those coming from $B_{1,2}$ as wfr contributions. Again, $\Sigma^{\rm fin}$ gives zero contribution.

We consider next the cancellation of the sm contributions with the mass counterterms. We start from the bare mass term in the Lagrangian, $-\overline{\Psi}'m'_0\Psi'$, and decompose the bare mass into a so-called renormalized mass and a corresponding counterterm, $m'_0 = m' + \delta m'$. We then apply a bi-unitary transformation on the fermion fields $\overline{\Psi}', \Psi'$ that diagonalizes m' leading to the transformed mass term $-\overline{\Psi}(m + \delta m^{(-)}P_L + \delta m^{(+)}P_R)\Psi$. Here $P_{R,L} = (1 \pm \gamma_5)/2$ are the chiral projectors, m is real, diagonal and positive and $\delta m^{(\pm)}$ are arbitrary non-diagonal matrices subject to the Hermiticity constraint

$$\delta m^{(+)} = \delta m^{(-)\dagger}. \tag{1}$$

Further we adjust $\delta m^{(\pm)}$ to cancel, as much as possible, the sm contributions to $\Delta\mathcal{M}^{\rm leg}$.

The diagonalization of the complete mass matrix $M = m + \delta m^{(-)}P_L + \delta m^{(+)}P_R$ by means of a bi-unitary transformation of the form:

$$\psi_{L,R} = U_{L,R}\hat{\psi}_{L,R} \approx (1 + ih_{L,R})\hat{\psi}_{L,R}, \tag{2}$$

naturally induces a mixing counterterm matrix. Note that the second equality holds only at one-loop level. The matrices $h_{L,R}$ are chosen such that $\hat{M}$ is diagonal and are found to be:

$$i(h_{L,R})_{ij} = -\frac{m_i\delta m^{(\mp)}_{ij} + \delta m^{(\pm)}_{ij}m_j}{m_i^2 - m_j^2}, \qquad (h_{L,R})_{ii} = 0. \tag{3}$$

Due to the transformation in Eq. (2) the $Vf_i\overline{f}_j$ bare interaction term in the Lagrangian transforms as well

$$\mathcal{L}_{Vf_i\overline{f}_j} \propto \overline{\psi}^{f_i}_L K_0\gamma^\lambda\psi^{f_j}_L V_\lambda + H.c. \quad \xrightarrow{U_{L,R}} \quad \overline{\hat{\psi}}^{f_i}_L (K + \delta K)\gamma^\lambda\hat{\psi}^{f_j}_L V_\lambda + H.c.,$$

with $\delta K = i(Kh^{f_j}_L - h^{f_i}_L K)$. $K_0 = K + \delta K$ and K are explicitly gauge independent and preserve the basic properties of the theory. K is finite and identified with the renormalized mixing matrix. δK is identified with the mixing counterterm matrix.

3 Particular cases

Following the procedure outlined in Sec. 2, a CKM counterterm matrix was proposed in Ref. [2]:

$$\delta V = i\left(Vh^D_L - h^U_L V\right),$$

with $h^{D,U}_L$ given by Eq. (3). Both $V_0 = V + \delta V$ and V satisfy the unitarity condition and are explicitly gauge independent.

Some years later an alternative approach based on a gauge-independent quark mass counterterm expressed directly in terms of the Lorentz-invariant self-energy functions was proposed [3]. The mass counterterms so defined obey three important properties: (i) they are gauge independent, (ii) they automatically satisfy the Hermiticity constraint of Eq. (1) and thus are flavour-democratic, and (iii) they are expressed in terms of the invariant self-energy functions and thus useful for practical applications.

A comparative analysis of the W-boson hadronic widths in various CKM renormalization schemes, including the ones discussed above, and the study of the implications of flavour-mixing renormalization on the determination of the CKM parameters are presented in Ref. [4].

We have also considered the mixing of leptons in a minimal, renormalizable extension of the SM that can naturally accommodate heavy Majorana neutrinos. Here mixing appears both in charged- and neutral-current interactions and is described by the bare mixing matrices B_0 and C_0. Following ones more the prescription of Sec. 2, we found that the charged-lepton mass counterterm is identical to that of quarks, up to particle content. However, in the case of the Majorana-neutrino there are two important modifications due to the Majorana condition $\nu = \nu^C$ (here C denotes charge conjugation): (i) in addition to the Hermiticity constraint of Eq. (1) the mass counterterm should be symmetric, and (ii) now only one unitary transformation, $U^\nu = 1 + ih^\nu$, is needed to diagonalize the complete mass matrix $\hat{M}^\nu$. Keeping in mind the two changes, the mixing counterterm matrices are [5]

$$\delta B = i\left(Bh^\nu - h^l_L B\right) \qquad \text{and} \qquad \delta C = i\left(Ch^\nu - h^\nu C\right).$$

Once δB is fixed, δC is fixed as well. Note that both, the bare and renormalized mixing matrices, are gauge independent and preserve the basic properties of the theory.

4 Conclusions

We proposed an explicit on-shell framework to renormalize the fermion-flavour mixing matrices in the SM and its extensions, at one-loop level. It is based on a novel procedure to separate the external-leg mixing corrections into gauge-independent sm and gauge-dependent wfr contributions. An important property is that this formulation complies with UV finiteness and gauge-parameter independence, and also preserves the basic structure of the theory.

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