$\mathcal{O}(\alpha)$ QED Corrections to Polarized Elastic μe and Deep Inelastic lN Scattering

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Abstract

Two computer codes relevant for the description of deep inelastic scattering off polarized targets are discussed. The code μ**e***la* deals with radiative corrections to elastic μe scattering, one method applied for muon beam polarimetry. The code `HECTOR` allows to calculate both the radiative corrections for unpolarized and polarized deep inelastic scattering, including higher order QED corrections.

1 Introduction

The exact knowledge of QED, QCD, and electroweak (EW) radiative corrections (RC) to the deep inelastic scattering (DIS) processes is necessary for a precise determination of the nucleon structure functions. The present and forthcoming high statistics measurements of polarized structure functions in the SLAC experiments, by HERMES, and later by COMPASS require the knowledge of the RC to the DIS polarized cross-sections at the percent level.

Several codes based on different approaches for the calculation of the RC to DIS experiments, mainly for non-polarized DIS, were developped and thoroughly compared in the past, cf. [1]. Later on the radiative corrections for a vast amount of experimentally relevant sets of kinematic variables were calculated [2], including also semi-inclusive situations as the RC's in the case of tagged photons [3]. Furthermore the radiative corrections to elastic μ-e scattering, a process to monitor (polarized) muon beams, were calculated [4]. The corresponding codes are :

- `HECTOR 1.00`, (1994-1995) [5], by the Dubna-Zeuthen Group. It calculates QED, QCD and EW corrections for variety of measuremets for unpolarized DIS.
- μ**e***la* `1.00`, (March 1996) [4], calculates $\mathcal{O}(\alpha)$ QED correction for polarized μe elastic scattering.
- `HECTOR 1.11`, (1996) extends `HECTOR 1.00` including the radiative corrections for polarized DIS [6], and for DIS with tagged photons [3]. The beta-version of the code is available from `http://www.ifh.de/`.

2 The Program μela

Muon beams may be monitored using the processes of μ decay and μe scattering in case of atomic targets. Both processes were used by the SMC experiment. Similar techniques will be used by the COMPASS experiment. For the cross section measurement the radiative corrections to these processes have to be known at high precision. For this purpose a renewed calculation of the radiative corrections to $\sigma(\mu e \to \mu e)$ was performed [4].

The differential cross-section of polarized elastic μe scattering in the Born approximation reads, cf. [7],

$$\frac{d\sigma^{\rm BORN}}{dy} = \frac{2\pi\alpha^2}{m_e E_\mu}\left[\frac{(Y-y)}{y^2 Y}\left(1 - yP_eP_\mu\right) + \frac{1}{2}\left(1 - P_eP_\mu\right)\right], \tag{1}$$

where $y = y_\mu = 1 - E_\mu^{'}/E_\mu = E_e^{'}/E_\mu = y_e$, $Y = (1 + m_\mu/2/E_\mu)^{-1} = y_{max}$, m_μ, m_e – muon and electron masses, E_μ, $E_\mu^{'}$, $E_e^{'}$ the energies of the incoming and outgoing muon, and outgoing electron respectively, in the laboratory frame. P_μ and P_e denote the longitudinal polarizations of muon beam and electron target. At Born level y_μ and y_e agree. However, both quantities are different under inclusion of radiative corrections due to bremsstrahlung. The correction factors may be rather different depending on which variables (y_μ or y_e) are used.

In the SMC analysis the y_μ-distribution was used to measure the electron spin-flip asymmetry $A_{\mu e}^{\rm exp}$. Since previous calculations, [8, 9], referred to y_e, and only ref. [9] took polarizations into account, a new calculation was performed, including the complete $\mathcal{O}(\alpha)$ QED correction for the y_μ-distribution, longitudinal polarizations for both leptons, the μ-mass effects, and neglecting m_e wherever possible. Furthermore the present calculation allows for cuts on the electron recoil energy (35 GeV), the energy balance (40 GeV), and angular cuts for both outgoing leptons (1 mrad). The default values are given in parentheses.

Up to order $\mathcal{O}(\alpha^3)$, 14 Feynman graphs contribute to the cross-section for μ-e scattering, which may be subdivided into $12 = 2 \times 6$ pieces, which are separately gauge invariant

$$\frac{d\sigma^{\rm QED}}{dy_\mu} = \sum_{l=1}^{2}\sum_{k=1}^{6}\frac{d\sigma_k^l}{dy_\mu}\ . \tag{2}$$

One may express (2) also as

$$\frac{d\sigma^{\rm QED}}{dy_\mu} = \sum_k\left(\frac{d\sigma_k^{\rm unpol}}{dy_\mu} + P_eP_\mu\frac{d\sigma_k^{\rm pol}}{dy_\mu}\right). \tag{3}$$

The indices l and k in the combinations lk have the meaning

$l =$ 1 – unpolarized contribution, $l =$ unpol;
2 – polarized contribution (terms with P_eP_μ in $d\sigma^{\rm BORN}$), $l =$ pol.
$k =$ 1 – Born cross-section, $k = b$;
2 – RC for the muonic current: vertex + bremsstrahlung, $k = \mu\mu$;
3 – amm contribution from muonic current, $k =$ amm;
4 – RC for the electronic current: vertex + bremsstrahlung, $k = ee$;
5 – μe interference: two-photon exchange +
muon-electron bremsstrahlung interference, $k = \mu e$;
6 – vacuum polarization correction, running α, $k =$ vp.

The FORTRAN code for the scattering cross section (2) μ**e***la* was used in a recent analysis of the SMC collaboration.

The RC, $\delta^A_{y_\mu}$, to the asymmetry $A^{\rm QED}_{\mu e}$ shown in figures 1 and 2 is defined as

$$\delta^A_{y_\mu} = \frac{A^{\rm QED}_{\mu e}}{A^{\rm BORN}_{\mu e}} - 1 \quad (\%), \quad \text{where} \quad A_{\mu e} = \frac{d\sigma^{\rm pol}}{d\sigma^{\rm unpol}}. \tag{4}$$

The results may be summarized as follows. The $\mathcal{O}(\alpha)$ QED RC to polarized elastic μe scattering were calculated for the first time using the variable y_μ. A rather general FORTRAN code μ**e***la* for this process was created allowing for the inclusion of kinematic cuts. Since under the conditions of the SMC experiment the corrections turn out to be small our calculation justifies their neglection.

3 Program HECTOR

3.1 Different approaches to RC for DIS

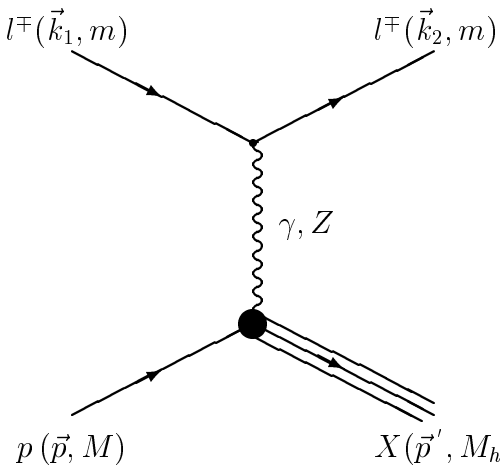
The radiative corrections to deep inelastic scattering are treated using two basic approaches. One possibility consists in generating events on the basis of matrix elements including the RC's. This approach is suited for detector simulations, but requests a very hughe number of events to obtain the corrections at a high precision. Alternatively, semi-analytic codes allow a fast and very precise evaluation, even including a series of basic cuts and flexible adjustment to specific phase space requirements, which may be caused by the way kinematic variables are experimentally measured, cf. [2, 5]. Recently, a third approach, the so-called deterministic approach, was followed, cf. [10]. It treats the RC's completely exclusively combining features of fast computing with the possibility to apply any cuts. Some elements of this approach were used in μ**e***la* and in the branch of HECTOR 1.11, in which DIS with tagged photons is calculated.

Concerning the theoretical treatment three approaches are in use to calculate the radiative corrections: 1) the model-independent approach (MI); 2) the leading-log approximation (LLA); and 3) an approach based on the quark-parton model (QPM) in evaluating the radiative corrections to the scattering cross-section.

In the model-independent approach the QED corrections are only evaluated for the leptonic tensor. Strictly it applies only for neutral current processes. The hadronic tensor can be dealt with in its most general form on the Lorentz-level. Both lepton-hadron corrections as well as pure hadronic corrections are neglected. This is justified in a series of cases in which these corrections turn out to be very small. The leading logarithmic approximation is one of the semi-analytic treatments in which the different collinear singularities of $O((\alpha \ln(Q^2/m_l^2))^n)$ are evaluated and other corrections are neglected. The QPM-approach deals with the full set of diagrams on the quark level. Within this method, any corrections (lepton-hadron interference, EW) can be included. However, it has limited precision too, now due to use of QPM-model itself. Details on the realization of these approaches within the code HECTOR are given in ref. [5, 11].

3.2 $O(\alpha)$ QED Corrections for Polarized Deep Inelastic Scattering

To introduce basic notation, we show the Born diagram



and the Born cross-section, which is presented as the product of the leptonic and hadronic tensor

$$d\sigma_{Born} = \frac{2\pi\alpha^2}{Q^4} y \Big[L^{\mu\nu} W_{\mu\nu} \Big] dx dy, \tag{5}$$

with

$$q = k_1 - k_2, \qquad Q^2 = -q^2, \qquad S = 2(p.k_1), \tag{6}$$

and the Bjorken scaling variables

$$y = \frac{p.q}{p.k_1}, \qquad x = \frac{Q^2}{Sy}. \tag{7}$$

For the hadronic tensor, we use the representation of ref. [12]

$$\begin{aligned}
W_{\mu\nu} &= p^0 (2\pi)^6 \sum \int \langle p^{'} | \mathcal{J}_\mu | p \rangle \langle p | \mathcal{J}_\nu | p^{'} \rangle \delta^4 (\sum_i p_i^{'} - p^{'}) \prod_i dp_i^{'} \\
&= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \mathcal{F}_1(x, Q^2) + \frac{\widehat{p_\mu} \widehat{p_\nu}}{p.q} \mathcal{F}_2(x, Q^2) - i e_{\mu\nu\lambda\sigma} \frac{q^\lambda p^\sigma}{2p.q} \mathcal{F}_3(x, Q^2) \\
&\quad + i e_{\mu\nu\lambda\sigma} \frac{q^\lambda s^\sigma}{p.q} \mathcal{G}_1(x, Q^2) + i e_{\mu\nu\lambda\sigma} \frac{q^\lambda (p.q s^\sigma - s.q p^\sigma)}{(p.q)^2} \mathcal{G}_2(x, Q^2) \\
&\quad + \left[\frac{\widehat{p_\mu} \widehat{s_\nu} + \widehat{s_\mu} \widehat{p_\nu}}{2} - s.q \frac{\widehat{p_\mu} \widehat{p_\nu}}{p.q} \right] \frac{1}{p.q} \mathcal{G}_3(x, Q^2) \\
&\quad + s.q \frac{\widehat{p_\mu} \widehat{p_\nu}}{(p.q)^2} \mathcal{G}_4(x, Q^2) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \frac{s.q}{p.q} \mathcal{G}_5(x, Q^2),
\end{aligned} \tag{8}$$

where

$$\widehat{p_\mu} = p_\mu - \frac{p.q}{q^2} q_\mu, \qquad \widehat{s_\mu} = s_\mu - \frac{s.q}{q^2} q_\mu,$$

and s is the four vector of nucleon polarization, which is given by $s = \lambda_p M(0, \vec{n})$ in the nucleon rest frame.

The combined structure functions in eq. (8)

$$\begin{aligned}
\mathcal{F}_{1,2}(x,Q^2) &= Q_e^2 F_{1,2}^{\gamma\gamma}(x,Q^2) + 2|Q_e|\,(v_l - p_e\lambda_l a_l)\,\chi(Q^2)F_{1,2}^{\gamma Z}(x,Q^2) \\
&\quad + \left(v_l^2 + a_l^2 - 2p_e\lambda_l v_l a_l\right)\chi^2(Q^2)F_{1,2}^{ZZ}(x,Q^2), \\
\mathcal{F}_3(x,Q^2) &= 2|Q_e|\,(p_e a_l - \lambda_l v_l)\,\chi(Q^2)F_3^{\gamma Z}(x,Q^2), \\
&\quad + \left[2p_e v_l a_l - \lambda_l\left(v_l^2 + a_l^2\right)\right]\chi^2(Q^2)F_3^{ZZ}(x,Q^2), \\
\mathcal{G}_{1,2}(x,Q^2) &= -Q_e^2\lambda_l g_{1,2}^{\gamma\gamma}(x,Q^2) + 2|Q_e|\,(p_e a_l - \lambda_l v_l)\,\chi(Q^2)g_{1,2}^{\gamma Z}(x,Q^2), \\
&\quad + \left[2p_e v_l a_l - \lambda_l\left(v_l^2 + a_l^2\right)\right]\chi^2(Q^2)g_{1,2}^{ZZ}(x,Q^2), \\
\mathcal{G}_{3,4,5}(x,Q^2) &= 2|Q_e|\,(v_l - p_e\lambda_l a_l)\,\chi(Q^2)g_{3,4,5}^{\gamma Z}(x,Q^2), \\
&\quad + \left(v_l^2 + a_l^2 - 2p_e\lambda_l v_l a_l\right)\chi^2(Q^2)g_{3,4,5}^{ZZ}(x,Q^2),
\end{aligned} \tag{9}$$

are expressed via the hadronic structure functions, the Z-boson-lepton couplings v_l, a_l, and the ratio of the propagators for the photon and Z-boson

$$\chi(Q^2) = \frac{G_\mu}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha}\frac{Q^2}{Q^2+M_Z^2}. \tag{10}$$

Furthermore we use the parameter p_e for which $p_e = 1$ for a scattered lepton and $p_e = -1$ for a scattered antilepton. The hadronic structure functions can be expressed in terms of parton densities accounting for the twist-2 contributions only, see [12]. Here, a series of relations between the different structure functions are used in leading order QCD.

The DIS cross-section on the Born-level

$$\frac{d^2\sigma_{\rm Born}}{dxdy} = \frac{d^2\sigma_{\rm Born}^{\rm unpol}}{dxdy} + \frac{d^2\sigma_{\rm Born}^{\rm pol}}{dxdy}, \tag{11}$$

contains two contributions, the unpolarized part

$$\frac{d\sigma_{Born}^{unpol}}{dxdy} = \frac{2\pi\alpha^2}{Q^4}S\sum_{i=1}^{3} S_i^U(x,y)\mathcal{F}_i(x,Q^2), \tag{12}$$

with the kinematic functions

$$\begin{aligned}
S_1^U(y,Q^2) &= 2xy^2, \\
S_2^U(y,Q^2) &= 2\left[(1-y) - \frac{xyM^2}{S}\right], \\
S_3^U(y,Q^2) &= x\left[1-(1-y)^2\right],
\end{aligned} \tag{13}$$

and the polarized part

$$\frac{d\sigma_{Born}^{pol}}{dxdy} = \frac{2\pi\alpha^2}{Q^4}\lambda_N^p f^p S\sum_{i=1}^{5} S_{gi}^p(x,y)\mathcal{G}_i(x,Q^2). \tag{14}$$

Here, $S_{gi}^p(x,y)$ are functions, similar to (13), and may be found in [6]. Furthermore we used the abbrevations

$$\begin{aligned}
f^{^L} &= 1, \qquad \vec{n}^{^L} = \lambda_N^p\frac{\vec{k}_1}{|\vec{k}_1|}, \\
f^{^T} &= \cos\phi\,\frac{d\phi}{2\pi}\sqrt{\frac{4M^2x}{Sy}\left(1-y-\frac{M^2xy}{S}\right)} = \cos\phi\,\frac{d\phi}{2\pi}\frac{1-y}{y}\sin\theta_2, \\
\vec{n}^{^T} &= \lambda_N^{^T}\vec{n}_\perp, \qquad \text{with } \vec{k_1}.\vec{n}_\perp = 0.
\end{aligned} \tag{15}$$

The $O(\alpha)$ DIS cross-section reads

$$\frac{d^2\sigma_{\rm QED,1}}{dx_l dy_l} = \frac{\alpha}{\pi}\delta_{\rm VR}\frac{d^2\sigma_{\rm Born}}{dx_l dy_l} + \frac{d^2\sigma_{\rm Brems}}{dx_l dy_l} = \frac{d^2\sigma^{\rm unpol}_{\rm QED,1}}{dx_l dy_l} + \frac{d^2\sigma^{\rm pol}_{\rm QED,1}}{dx_l dy_l}. \tag{16}$$

All partial cross-sections have a form similar to the Born cross-section and are expressed in terms of kinematic functions and combinations of structure functions. In the $O(\alpha)$ approximation the measured cross-section, $\sigma_{\rm rad}$, is define as

$$\frac{d^2\sigma_{\rm rad}}{dx_l dy_l} = \frac{d^2\sigma_{\rm Born}}{dx_l dy_l} + \frac{d^2\sigma_{\rm QED,1}}{dx_l dy_l} = \frac{d^2\sigma^{\rm unpol}_{\rm rad}}{dx_l dy_l} + \frac{d^2\sigma^{\rm pol}_{\rm rad}}{dx_l dy_l}. \tag{17}$$

In the four following figures we illustrate the RC-factor

$$\delta = \frac{d^2\sigma_{\rm rad}}{d^2\sigma_{\rm Born}} - 1. \tag{18}$$

The radiative corrections calculated for leptonic variables grow towards high y and smaller values of x. The figures compare the results obtained in LLA, accounting for initial (i) and final state (f) radiation, as well as the Compton contribution ($c2$) with the result of the complete calculation of the leptonic corrections. In most of the phase space the LLA correction provides an excellent description, except of extreme kinematic ranges.

A comparison of the radiative corrections for polarized deep inelastic scattering between the codes `HECTOR` and `POLRAD` [17] was carried out. It had to be performed under *simplified* conditions due to the restrictions of `POLRAD`. Corresponding results may be found in [11, 13, 14].

3.3 Conclusions

For the evaluation of the QED radiative corrections to deep inelastic scattering of polarized targets two codes `HECTOR` and `POLRAD` exist. The code `HECTOR` allows a completely general study of the radiative corrections in the model independent approach in $O(\alpha)$ for neutral current reactions including Z-boson exchange. Furthermore, the LLA corrections are available in 1st and 2nd order, including soft-photon resummation and for charged current reactions. `POLRAD` contains a branch which may be used for some semi-inclusive DIS processes. The initial state radiative corrections (to 2nd order in LLA + soft photon exponentiation) to these (and many more processes) can be calculated in detail with the code `HECTOR`, if the corresponding user-supplied routine `USRBRN` is used together with this package. This applies both for neutral and charged current processes as well as a large variety of different measurements of kinematic variables. Aside the leptonic corrections, which were studied in detail already, further investigations may concern QED corrections to the hadronic tensor as well as the interference terms.

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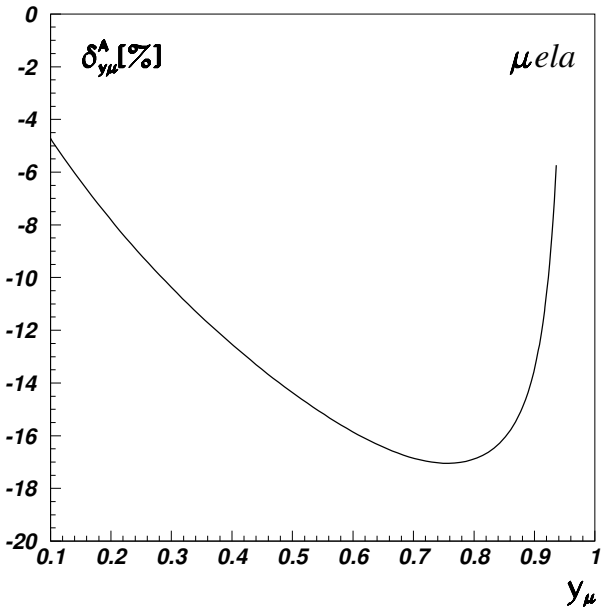

Figure 1: The QED radiative corrections to asymmetry without experimental cuts

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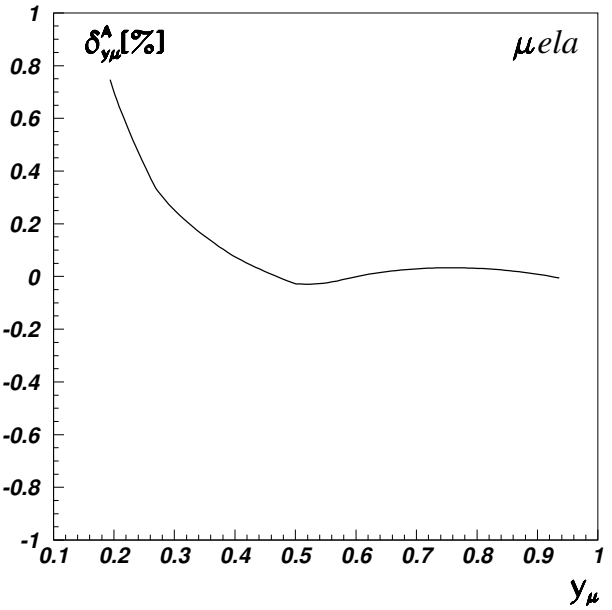

Figure 2: The QED radiative corrections to asymmetry with experimental cuts.

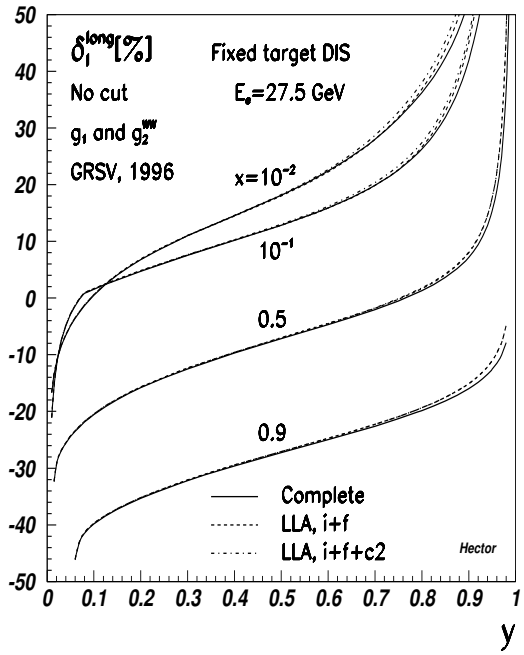

Figure 3: A comparison of complete and LLA RC's in the kinematic regime of HERMES for neutral current longitudinally polarized DIS in leptonic variables. The polarized parton densities [15] are used. The structure function g_2 is calculated using the Wandzura–Wilczek relation. $c2$ stands for the Compton contribution, see [6] for details.

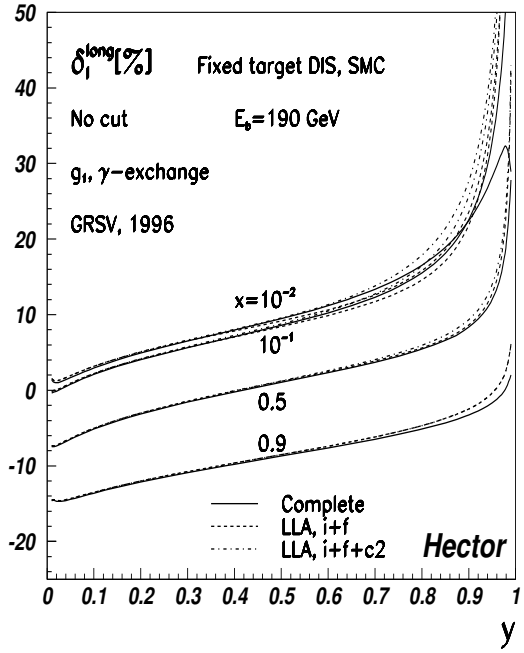

Figure 4: The same as in fig. 3, but for energies in the range of the SMC-experiment.

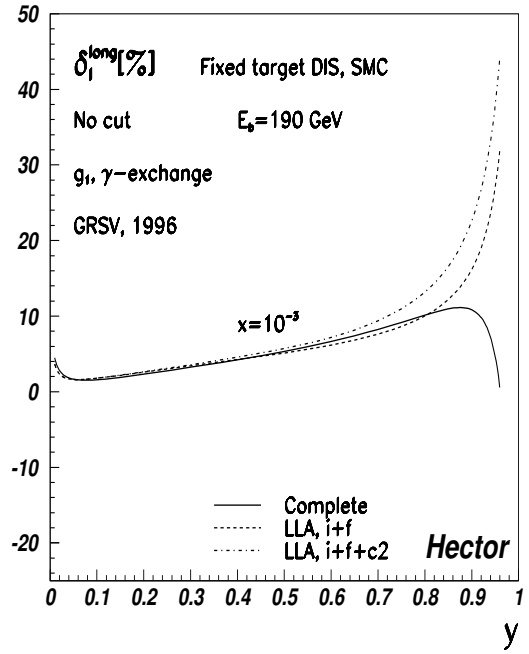

Figure 5: The same as in fig. 4 for $x = 10^{-3}$.

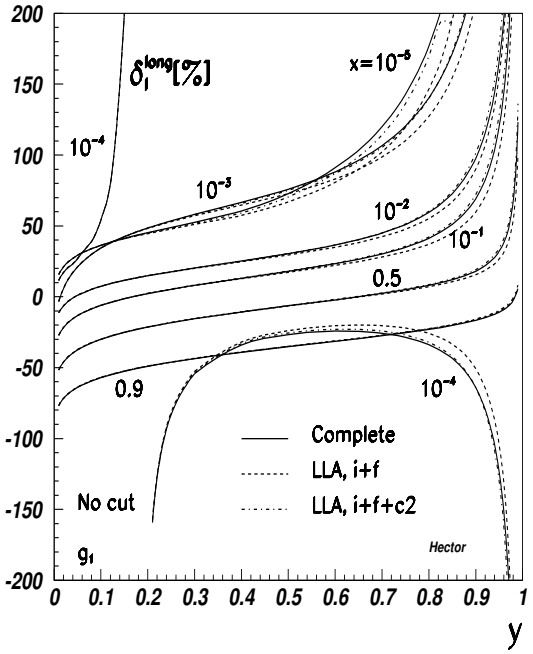

Figure 6: A comparison of complete and LLA RC's at HERA collider kinematic regime for neutral current deep inelastic scattering off a longitudinally polarized target measuring the kinematic variables at the leptonic vertex.