Non-Forward Balitsky-Kovchegov Equation and Vector Mesons

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Considering the Balitsky-Kovchegov QCD evolution equation in full momentum space, we derive the travelling wave solutions expressing the nonlinear saturation constraints on the dipole scattering amplitude at non-zero momentum transfer. A phenomenological application to elastic vector meson production shows the compatibility of data with the QCD prediction: an *enhanced* saturation scale at intermediate momentum transfer.

1 Motivation

The saturation of parton densities at high energy has been mainly studied for the forward dipole-target scattering amplitude $\mathcal{T}(r, q = 0, Y)$, where r, q, Y are, respectively, the dipole size, the momentum transfer and the total rapidity of the process. For instance, the corresponding QCD Balitsky-Kovchegov (BK) equation [2] has been shown to provide a theoretical insight on the "geometric scaling" properties [3] of the related γ^*-proton cross-sections. Indeed, it can be related to the existence of a scaling for $\mathcal{T}(r, q = 0, Y) \sim \mathcal{T}(r^2Q^2(Y))$ where the saturation scale is $Q^2(Y) \sim \exp cY$ and the constant c can be interpreted as the critical speed of "travelling wave" solutions of the nonlinear BK equation [4]. Our theoretical and phenomenological subjects are the extension of these properties to the non-forward amplitude $\mathcal{T}(r, q \neq 0, Y)$, which is phenomenologically relevant for the elastic production of vector mesons in deep inelastic scattering.

2 BK equation in full momentum space

In order to study the properties of $\mathcal{T}(r, q \neq 0, Y)$, one has first to deal with both conceptual and technical difficulties. It is known that the BK formalism has been originally derived in impact parameter b but then its validity especially at large b is questionable, since it leads to non physical power-law tails. Hence we start with the formulation of the BK equation in momentum q, which is more *local* but has a non-trivial nonlinear form [5]. In fact, despite this problem, the general method of travelling wave solutions can be extended in the non-forward domain [6]. It consists in 3 steps: first, one solves the equation restricted to its linear part which is related to the non-forward Balitsky Fadin Kuraev Lipatov (BFKL) equation

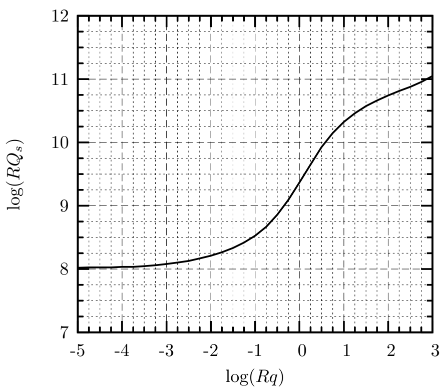

Figure 1: q^2-dependent saturation scale

[7] for the dipole-dipole amplitude *via* factorisation and whose solution takes the form of a linear superposition of waves. Second, one finds that the nonlinearities act by selecting the travelling wave with *critical* speed c, in a way which, interestingly, is independent of the specific structure of the nonlinear damping terms. Third, one obtains after enough rapidity evolution, a solution which appears independent from initial conditions ($\mathcal{T}_0 \sim r^{2\gamma_0}$) , provided these are sharper than the critical travelling wave front profile $\mathcal{T} \sim r^{2\gamma_c}$, with $\gamma_0 > \gamma_c$. Interestingly enough, QCD color transparency satisfies this criterium. Applying these gen-

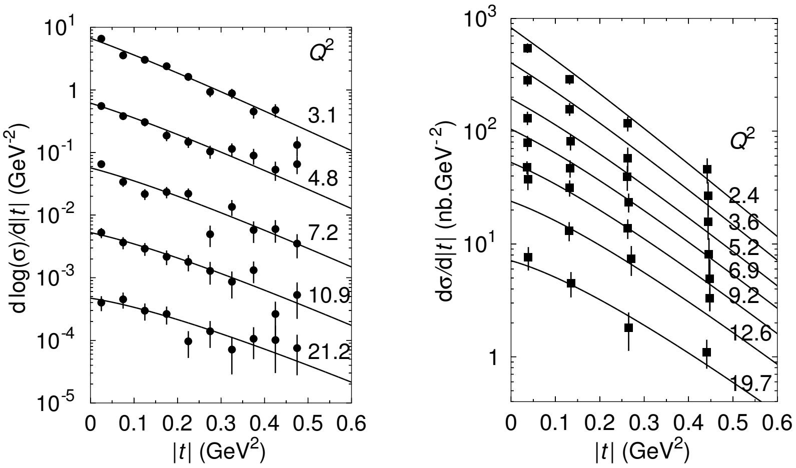

Figure 2: ρ (H1) and ϕ (ZEUS) differential cross-sections at $W = 75$ GeV

eral results on the non-forward case one finds the following QCD predictions, depending on the relative magnitude of three scales involved in the process, namely q, k_T^{-1} (the target size) and $k_P^{-1} \equiv r$ (the projectile *i.e.* dipole size).

- Near-Forward region $q \ll k_T \ll k_P$: $Q_s^2(Y) \sim k_T^2 \exp cY$
- Intermediate transfer region $k_T \ll q \ll k_P$: $Q_s^2(Y) \sim q^2 \exp cY$
- High transfer region $q \ll k_T \ll k_P$: No saturation.

Our main prediction is thus the validity of the forward travelling wave solution extended in the non-forward intermediate-transfer domain but with an *enhanced* saturation scale by the ratio q^2/k_T^2, where k_T is a typically small, nonperturbative scale. Hence we are led to predict *geometric scaling* properties with a purely perturbative initial saturation scale given by the transverse momentum. This saturation scale *enhancement* prediction is confirmed by numerical simulations of the BK solutions as shown in Fig.1.

3 QCD Saturation Model for Exclusive VM production

The differential cross-section for exclusive vector meson (VM) production at HERA, see Fig.2, can be theoretically obtained from the non-forward dipole-proton amplitude and from $\Phi_{T,L}^{\gamma^* V}$, the overlap functions between the (longitudinal and transverse) virtual photon and vector meson wave-functions [8]. For completion, we used two different VM wave-functions of the literature, without noticeable difference in our conclusions. One writes

$$\frac{d\sigma_{T,L}^{\gamma^* p\to Vp}}{dq^2} = \frac{1}{16\pi}\left|\int d^2r \int_0^1 dz\ \Phi_{T,L}^{\gamma^* V}(z,r;Q^2,M_V^2)\ e^{-izq\cdot r}\ \mathcal{T}(r,q,Y)\right|^2 ,$$

Following theoretical prescriptions, we consider a forward dipole-proton amplitude $\mathcal{N}_{IIM}$ satisfactorily describing the total DIS cross-sections in a saturation model [9]. We just make the saturation scale varying with q^2, following the trend shown in Fig.1 and starting from the forward model one $Q_s^2(Y)$, one writes

$$T(r,q;Y) = 2\pi R_p^2\ e^{-Bq^2}\mathcal{N}_{IIM}(r^2\ Q_s^2(Y,q))\ ;\ Q_s^2(q,Y) = Q_s^2(Y)\ (1+c\ q^2)\ .$$

The factor $2\pi R_p^2\ e^{-Bq^2}$ comes from the non-perturbative proton form factor. For clarity of the analysis, we considered only B and c as free parameters of the non-forward parametrisations, the others being independently fixed by the forward analysis.

Cross-sections	q^2-Sat.	fixed-Sat.
$\rho,\ \sigma_{\rm el}$	1.156	1.732
$\rho,\ \frac{d\sigma}{dt}$	1.382	1.489
$\phi,\ \sigma_{\rm el}$	1.322	2.247
$\phi,\ \frac{d\sigma}{dt}$	1.076	0.931
Total	1.212	1.480

Table 1: Comparison of the χ^2/points

In Table 1, one displays the χ^2/point obtained by a fit of ρ (47 data points) and ϕ (34 points) total elastic production cross-sections and of ρ (50 data points) and ϕ (70 points) differential cross-sections. The Table compares the saturation fits for fixed and q^2-dependent scales, with a favour for the enhanced-scale model in the total. The model gives a comparable fit with a more conventional non-saturation model using a Q^2-dependent slope $B \propto M_V^2 + Q^2$. Some of our results for the cross-sections are displayed in the figures. In Fig.2, one shows the results of the fit for ρ-production (H1) and ϕ-production (ZEUS) differential cross-sections for a total $\gamma^* - p$ energy $W =$ 75GeV and different Q^2 values. Let us finally present our predictions for the

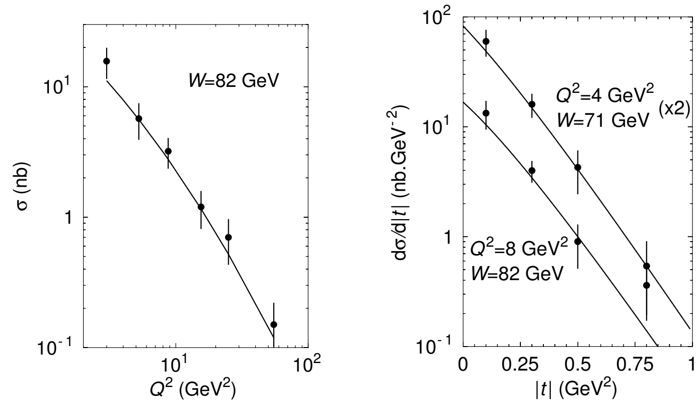

Figure 3: Predictions for the DVCS measurements. Left plot: cross-section, right plot: differential cross-section.

DVCS cross-section, which is obtained without any free parameter from our analysis. In

Fig.3, they are compared with the available data and the agreement is good in the simple chosen parametrisation.

4 Conclusions

Let us summarize our new results

• *Saturation at non-zero transfer:* The Balitsky-Kovchegov QCD evolution equation involving full momentum transfer predicts (besides the known $q = 0$ case) saturation in the *intermediate* transfer range, namely for $Q_0 < q < Q$, where Q_0 (resp. Q) is the target (resp. projectile) typical scale.

• *Characterisation of the universality class:* The universality class of the corresponding travelling-wave solutions is governed by a purely perturbative saturation scale $Q_s(Y) \equiv q^2\Omega(Y)$, where $\Omega(Y) \sim e^{cY}$ is the same rapidity evolution factor as in the forward case. Consequently the *intermediate transfer* saturation scale gets enhanced by a factor q^2/Q_0^2.

• *Phenomenology of Vector mesons:* The QCD predictions are applied in the experimentally accessible *intermediate transfer* range of vector meson production. The model uses an interpolation between the forward and non-forward saturation scale together with a parameter-frozen forward saturation model. It fits better the data on ρ (H1) and ϕ (ZEUS) cross-sections than for a non-enhanced saturation.

• *Prospects:* The next phenomenological prospect is to add charm to the discussion, both with the modification of the forward case by including the charm contribution [10] and by also considering the production of Ψ mesons. On a theoretical ground, it would be interesting to go beyond the mean-field approximation of the BK equation.

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