# Novel Master Formula for Twist-3 Soft-Gluon-Pole Mechanism to Single Transverse-Spin Asymmetry

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We prove that twist-3 soft-gluon-pole (SGP) cross section for single spin asymmetries is determined by a certain "primordial" twist-2 cross section up to kinematic and color factors in the leading order perturbative QCD. This unveils universal structure behind the SGP cross sections in a variety of hard processes, and also the special role of the scale invariance in the corresponding primordial cross section, which leads to remarkable simplification of the SGP cross sections for the production of massless particle, such as those for pion production $p^{\uparrow}p \to \pi X$ and direct-photon production $p^{\uparrow}p \to \gamma X$.

The single transverse-spin asymmetry (SSA) is observed as "T-odd" effect proportional to $\vec{S}_{\perp} \cdot (\vec{p} \times \vec{q})$ in the cross section for the scattering of transversely polarized proton with momentum $p$ and spin $S_{\perp}$, off unpolarized particle with momentum $p'$, producing a particle with momentum $q$ which is observed in the final state. Famous examples [2] are $p^{\uparrow}p \to \pi X$ with the large asymmetry $A_N \sim 0.3$ observed in the forward direction, and semi-inclusive deep inelastic scattering (SIDIS), $ep^{\uparrow} \to e\pi X$, by HERMES and COMPASS Collaborations. The Drell-Yan (DY) process, $p^{\uparrow}p \to \ell^+\ell^- X$, and the direct $\gamma$ production, $p^{\uparrow}p \to \gamma X$, at RHIC, J-PARC, etc. are also expected to play important roles for the study of SSA.

The SSA requires, (i) nonzero $q_{\perp}$ originating from transverse motion of quark or gluon; (ii) proton helicity flip; and (iii) interaction beyond Born level to produce the interfering phase for the cross section. For processes with $q_{\perp} \sim \Lambda_{\rm QCD}$, all (i)-(iii) may be generated nonperturbatively from the T-odd, transverse-momentum-dependent parton distribution/fragmentation functions [3]. For large $q_{\perp} \gg \Lambda_{\rm QCD}$, (i) should come from perturbative mechanism, while the nonperturbative effects can participate in the other two, (ii) and (iii), allowing us to obtain large SSA. This is realized as the twist-3 mechanism in QCD for the SSA. Recently we have thoroughly discussed the collinear-factorization property and gauge invariance in the twist-3 mechanism in the context of the SSA in SIDIS [4]. We have also revealed universal structure behind the twist-3 mechanism [5, 6], which we discuss here.

As an example, consider the DY production of the dilepton with $q_{\perp} \gg \Lambda_{\rm QCD}$: The large $q_{\perp}$ of (i) is provided by hard interaction as the recoil from a hard final-state parton, as illustrated in Fig. 1. Proton helicity flip (ii) is provided by the participation of the coherent, nonperturbative gluon from the transversely polarized proton, the lower blob in Fig. 1, associated with the twist-3 quark-gluon correlation functions such as $G_F(x_1, x_2)$ with $x_1$ ($x_2$) the lightcone momentum fraction of the quark leaving from

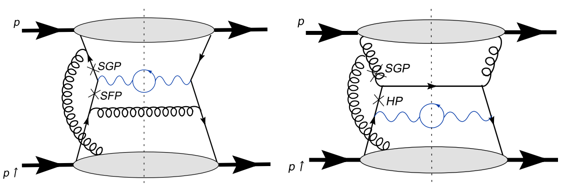


Figure 1: Typical diagrams for DY process with $q_{\perp} \gg \Lambda_{\rm QCD}$. The cross $\times$ denotes the pole contribution of the parton propagator.

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(entering into) the proton [4]. Due to the coupling of this coherent gluon, some parton propagators in the partonic subprocess can be on-shell, and this produces the imaginary phase of (iii) as the pole contribution using $1/(k^2 + i\varepsilon) = P(1/k^2) - i\pi\delta(k^2)$. Depending on the value of the coherent-gluon's momentum $k_g$ at the corresponding poles, these are soft-gluon pole (SGP) for $k_g = 0$, and soft-fermion pole (SFP) and hard pole (HP) for $k_g \neq 0$.

Among these three-types of poles, the SGP deserves special attention; indeed, the SGP is considered to give dominant twist-3 mechanism in many phenomenological applications (see e.g. [4, 5]). We have derived the *master formula* for the SGP cross section, which embodies the remarkable structure that the SGP contributions from many Feynman diagrams of the type of Fig. 1 are united into a derivative of the 2-to-2 partonic Born diagrams without the coherent-gluon line: The SSA for the DY process can be expressed as [5]

$$\frac{d\sigma^{\rm DY}_{\text{tw-3,SGP}}}{[d\omega]} = \frac{\pi M_N}{2C_F}\epsilon^{\sigma p n S_\perp} \sum_{j=\bar{q},g} \mathcal{B}_j \int \frac{dx'}{x'} \int \frac{dx}{x} f_j(x') \frac{\partial H_{jq}(x',x)}{\partial(x'p'^\sigma)} G_F^q(x,x), \tag{1}$$

where $j = \bar{q}$ and $g$ represent the "$q\bar{q}$-annihilation" and "$qg$-scattering" channels, respectively, corresponding to the left and right diagrams in Fig. 1. $f_j(x')$ denotes the twist-2 parton distribution functions for the unpolarized proton, and $M_N$ is the nucleon mass representing nonperturbative scale associated with the twist-3 correlation function $G_F^q(x,x)$ for the flavor $q$. The sum over quark and antiquark flavors is implicit for the index $q$ as $q = u, \bar{u}, d, \bar{d}, \cdots$. $[d\omega] = dQ^2 dy d^2 q_\perp$ denotes the relevant differential elements with $Q^2 = q^2$ and $y$ the rapidity of the virtual photon. The color factors are introduced as $\mathcal{B}_q = 1/(2N_c)$ and $-1/(2N_c)$ for quark and antiquark flavors, respectively, $\mathcal{B}_g = N_c/2$, and $C_F = (N_c^2 - 1)/(2N_c)$. The derivative with respect to $p'$ is taken under $p'^2 = 0$, and $H_{jq}(x',x)$ denote the partonic hard-scattering functions which participate in the unpolarized twist-2 cross section for DY process as

$$\frac{d\sigma^{\rm unpol,DY}_{\text{tw-2}}}{[d\omega]} = \sum_{j=\bar{q},g} \int \frac{dx'}{x'} \int \frac{dx}{x} f_j(x') H_{jq}(x',x) f_q(x). \tag{2}$$

Namely, in order to obtain the explicit formula for the twist-3 SGP contributions to the SSA, knowledge of the twist-2 unpolarized cross section is sufficient.

A proof of (1) is described in detail in [5]: Only the *initial-state interaction* (ISI) diagrams like Fig. 1, where the coherent gluon couples to an "external parton" associated with an initial-state hadron, survive as the SGP contributions for DY process, while the SGPs from the other diagrams cancel out combined with the corresponding "mirror" diagrams [7]. For such ISI diagrams, the coherent-gluon insertion and the associated SGP structure can be disentangled from the partonic subprocess, keeping the remaining factors mostly intact. For the *scalar-polarized* coherent-gluon, this is performed using Ward identity; moreover, also for the transversely-polarized coherent-gluons that are relevant at the twist-3 level, this can be performed exactly through the logic different from the scalar-polarized case [5]. Using the background field gauge, the three-gluon coupling relevant to the $qg$-scattering channel can be disentangled similarly as the quark-gluon coupling case. After disentangling ISI with the coherent gluons, the collinear expansion in terms of the parton transverse momenta gives the twist-3 cross section (1) at the SGP, as the response of 2-to-2 partonic subprocess to the change of the transverse momentum carried by the external parton, to which the coherent gluon had coupled. Note that $\mathcal{B}_{\bar{q}}$ and $\mathcal{B}_g$ in (1) come from the insertion of the color matrix $t^a$ in the fundamental and adjoint representations, respectively, into the 2-to-2 subprocess, as the remnant of the coherent-gluon insertion via the quark-gluon and three-gluon vertices.

The hard-scattering functions in the twist-2 factorization formula (2) are expressed as $H_{jq}(x',x) = (\alpha_{em}^2 \alpha_s e_q^2/3\pi N_c s Q^2)\widehat{\sigma}_{jq}(\hat{s},\hat{t},\hat{u})\delta\left(\hat{s}+\hat{t}+\hat{u}-Q^2\right)$, where $s=(p+p')^2$, and explicit form of $\widehat{\sigma}_{jq}(\hat{s},\hat{t},\hat{u})$ in terms of the partonic Mandelstam variables $\hat{s},\hat{t}$ and $\hat{u}$ can be found in Eq. (28) in [5]. The derivative in (1) can be performed through that for the $\hat{u}$, and this may act on either $\widehat{\sigma}_{jq}$ or the delta function in $H_{jq}(x',x)$, where the latter case produces the derivative of the twist-3 correlation function, $dG_F^q(x,x)/dx$, as well as the non-derivative term $\propto G_F^q(x,x)$, by partial integration with respect to $x$. Our master formula (1) yields [5]

$$\frac{d\sigma_{\text{tw-3,SGP}}^{\text{DY}}}{dQ^2 dy d^2 q_\perp} = \frac{\alpha_{em}^2 \alpha_s e_q^2}{3\pi N_c s Q^2}\frac{\pi M_N}{C_F}\epsilon^{pnS_\perp q_\perp}\sum_{j=\bar{q},g}\mathcal{B}_j\int\frac{dx'}{x'}\int\frac{dx}{x}\delta\left(\hat{s}+\hat{t}+\hat{u}-Q^2\right)f_j(x')$$
$$\times\left\{\frac{\widehat{\sigma}_{jq}}{-\hat{u}}x\frac{dG_F^q(x,x)}{dx}+\left[\frac{\widehat{\sigma}_{jq}}{\hat{u}}-\frac{\partial\widehat{\sigma}_{jq}}{\partial\hat{u}}-\frac{\hat{s}}{\hat{u}}\frac{\partial\widehat{\sigma}_{jq}}{\partial\hat{s}}-\frac{\hat{t}-Q^2}{\hat{u}}\frac{\partial\widehat{\sigma}_{jq}}{\partial\hat{t}}\right]G_F^q(x,x)\right\}. \quad (3)$$

This novel expression not only reproduces the known pattern [7] that the partonic hard scattering functions associated with the derivative term are directly proportional to those participating in the twist-2 unpolarized process, $\widehat{\sigma}_{jq}$, but also reveals the structure that was hidden in the corresponding formula in the literature [7]: the partonic hard-scattering functions associated with the non-derivative term are also completely determined by $\widehat{\sigma}_{jq}$.

We obtain the SSA for the direct $\gamma$ production in the real-photon limit, $Q^2\to 0$; here only the massless particles participate in the 2-to-2 Born subprocess, so that the corresponding partonic cross sections $\widehat{\sigma}_{jq}$ are scale-invariant as $(\hat{u}\partial/\partial\hat{u}+\hat{s}\partial/\partial\hat{s}+\hat{t}\partial/\partial\hat{t})\widehat{\sigma}_{jq}=0$. Consequently, (3) reduces to the compact structure where $-\widehat{\sigma}_{jq}/\hat{u}$ appears both for derivative and non-derivative terms, as the coefficient for the combination, $xdG_F^q(x,x)/dx - G_F^q(x,x)$ [5].

The DY process can be formally transformed into SIDIS, $ep^\uparrow \to e\pi X$, *crossing* the initial unpolarized proton into the final-state pion with momentum $P_h$, and the virtual photon into the initial-state one. The proof of (1) discussed above is unaffected by the analytic continuation corresponding to this "crossing transformation": $p'\to -P_h$, $x'\to 1/z$, $f_{\bar{q}}(x')\to D_q(z)$, $f_g(x')\to D_g(z)$, and $q^\mu\to -q^\mu$, where $D_j(z)$ denote the twist-2 parton fragmentation functions for the final-state pion, and the new $q^\mu$ gives $Q^2=-q^2$. Our master formula (1) now gives the SGP contribution to the SSA in SIDIS, which is actually associated with the corresponding *final-state interaction* (FSI) diagrams, as $(\mathcal{C}_q\equiv\mathcal{B}_{\bar{q}},\mathcal{C}_g\equiv\mathcal{B}_g)$ [5]

$$\frac{d\sigma_{\text{tw-3,SGP}}^{\text{SIDIS}}}{[d\omega]} = \frac{\pi M_N}{C_F z_f^2}\epsilon^{pnS_\perp P_{h\perp}}\left.\frac{\partial}{\partial q_T^2}\frac{d\sigma_{\text{tw-2}}^{\text{unpol,SIDIS}}}{[d\omega]}\right|_{f_q(x)\to G_F^q(x,x),\ D_j(z)\to\mathcal{C}_j z D_j(z)}, \quad (4)$$

in a frame where the 3-momenta $\vec{q}$ and $\vec{p}$ of the virtual photon and the transversely polarized nucleon are collinear along the $z$ axis. $[d\omega]=dx_{bj}dQ^2dz_f dq_T^2 d\phi$, where, as usual, $x_{bj}=Q^2/(2p\cdot q)$, $z_f=p\cdot P_h/p\cdot q$, $q_T=P_{h\perp}/z_f$, and $\phi$ is the azimuthal angle between the lepton and hadron planes. The twist-2 unpolarized cross section in the RHS of (4) is known to have several terms with different $\phi$-dependence, proportional to $1,\cos\phi$, and $\cos 2\phi$, respectively (see [4]). Performing the derivative in (4) explicitly, the result obeys the similar pattern as (3) with derivative and non-derivative terms, for each azimuthal dependence, and unveils the structure behind the complicated formula obtained by direct evaluation of the diagrams [4].

Our master formula can be extended to "QCD-induced" $pp$ collisions, like $p^\uparrow p\to\pi X$ [6]. For example, the $qq\to qq$ scattering subprocess induce the twist-2 unpolarized cross section,

$$E_h\frac{d^3\sigma_{\text{tw-2}}^{pp\to\pi X}}{d^3P_h} = \frac{\alpha_s^2}{s}\int\frac{dz}{z^2}\frac{dx'}{x'}\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u})D_q(z)f_q(x')f_q(x)\widehat{\sigma}_U(\hat{s},\hat{t},\hat{u}), \quad (5)$$

for $p(p) + p(p') \to \pi(P_h) + X$, where $E_h \equiv P_h^0$, and $\widehat{\sigma}_U(\hat{s}, \hat{t}, \hat{u}) = (C_F/N_c)(\hat{s}^2 + \hat{u}^2)/\hat{t}^2 + (C_F/N_c)(\hat{s}^2 + \hat{t}^2)/\hat{u}^2 + (-2C_F/N_c^2)\hat{s}^2/(\hat{t}\hat{u})$ is the $qq \to qq$ unpolarized cross section [8]. The SGP contribution from the interference between $qqg \to qq$ and $qq \to qq$ is generated by FSI and ISI with the coherent gluon as in Fig. 2 (a) and (b), which can be treated similarly as the SIDIS and DY cases, respectively, and yields [6] the twist-3 cross section for $p^{\uparrow}p \to \pi X$:

$$E_h \frac{d^3\sigma^{pp\to\pi X}_{\text{tw-3,SGP}}}{d^3P_h} = \frac{\pi M_N \alpha_s^2}{s} \int \frac{dz}{z^2} \frac{dx'}{x'} \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u}) D_q(z) f_q(x') \left[ x \frac{dG_F^q(x,x)}{dx} - G_F^q(x,x) \right] \times \left[ \frac{1}{z} \epsilon^{S_\perp P_h p n} + \frac{x'\hat{t}}{\hat{s}} \epsilon^{S_\perp p' p n} \right] \left( \frac{\hat{s}\ \widehat{\sigma}_F(\hat{s},\hat{t},\hat{u})}{\hat{t}\hat{u}} - \frac{\widehat{\sigma}_I(\hat{s},\hat{t},\hat{u})}{\hat{u}} \right), \tag{6}$$

where the hard cross sections from the FSI and ISI diagrams, $\widehat{\sigma}_W = A_{W,1}(\hat{s}^2 + \hat{u}^2)/\hat{t}^2 + A_{W,2}(\hat{s}^2 + \hat{t}^2)/\hat{u}^2 + A_{W,3}\hat{s}^2/(\hat{t}\hat{u})$ for $W = F$ and $I$, are the same as the above $\widehat{\sigma}_U$, except for the associated color factors $A_{W,i}$ that come from the *color insertion* of $t^a$, similarly as $\mathcal{B}_j$ of (1). Note that the combination, $xdG_F^q(x,x)/dx - G_F^q(x,x)$, in (6) is a consequence of the scale invariance, $\widehat{\sigma}_U(\hat{s},\hat{t},\hat{u}) = \widehat{\sigma}_U(\lambda\hat{s},\lambda\hat{t},\lambda\hat{u})$, similarly as in $p^{\uparrow}p \to \gamma X$ discussed below (3). These remarkable structures arise universally for all the other relevant channels associated with the "primordial" 2-to-2 partonic subprocesses, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to gg$, $qg \to qg$, etc. (see also [8]).

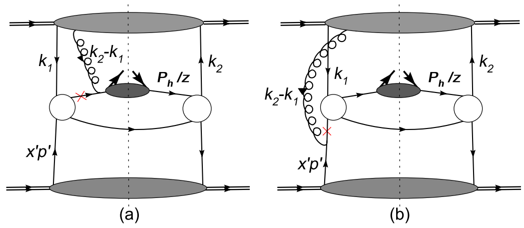


Figure 2: (a) and (b) as FSI and ISI diagrams for SGP mechanism, respectively. White circles denote hard scattering between quarks.

We have derived the novel master formula which gives the twist-3 SGP contributions to the SSA entirely in terms of the knowledge of the "primordial" twist-2 cross section. This is based on the new approach, which allows us to disentangle ISI as well as FSI with the soft coherent-gluon, irrespective of the details of the corresponding partonic subprocess. Thus our single master formula is applicable universally to all processes relevant to SSA, including QED-induced processes like DY process, direct $\gamma$ production, SIDIS, etc., and also QCD-induced processes like $p^{\uparrow}p \to \pi X$, $p^{\uparrow}p \to$ 2jets $X$, $pp \to \Lambda^{\uparrow}X$, etc. For SSA associated with the chiral-even twist-3 function $G_F^q(x,x)$, the primordial twist-2 process is unpolarized as discussed above, while for SSA associated with the chiral-odd functions, the primordial process is the polarized one [6]. The primordial twist-2 structure behind the SGP mechanism manifests gauge invariance of the results, and unveil the remarkable role of scale invariance.

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