$\Lambda_{\rm QCD}$ and $\alpha_s(M_Z^2)$ from DIS Structure Functions

Johannes Blümlein[1] *

Deutsches Elektronen-Synchrotron, DESY, Platanenallee 6, D-15738 Zeuthen, Germany

A brief summary is given on recent determinations of $\Lambda_{\rm QCD}$ and $\alpha_s(M_Z^2)$ from deeply inelastic structure functions.

Various QCD analyzes of the world unpolarized and polarized deep-inelastic data on charged lepton–nucleon and neutrino–nucleus scattering were performed in order to measure the QCD scale $\Lambda_{\rm QCD}$, resp. $\alpha_s(M_Z^2)$, from the scaling violations of the nucleon structure functions. In this note we give a brief overview on the status of these analyzes.[a] Most of the analyzes performed in the past were of next-to-leading order, see Table 1. Here the values of $\alpha_s(M_Z^2)$ range between 0.1171–0.1148 mainly with the exception of the old, very low BCDMS value [7] and Ref. [8] obtaining $\alpha_s(M_Z^2) = 0.112$. The typical theory error is estimated by varying the renormalization and factorization scales between $Q^2/4$ and $4 \cdot Q^2$ amounts to $\sim$5% for $\alpha_s(M_Z^2)$, a theoretical uncertainty too large to cope with the experimental uncertainty of $O(1..2\%)$ after the completion of the HERA programme. The analyzes of polarized nucleon data still yield rather large errors [10, 11] due to the present accuracy reached for polarization asymmetries. Moreover, these analyzes include data down to $Q^2 \sim 1\,{\rm GeV}^2$, which is not unproblematic w.r.t. higher twist terms, the scaling violations of which are yet unknown. The unpolarized analyzes at the present level of accuracy require rigorous cuts for potential higher twist effects, which can be achieved demanding $W^2 > 12...15\,{\rm GeV}^2$. Furthermore, we will consider only proton and deuteron data, to avoid potential interference with nuclear effects.

With the advent of the 3-loop anomalous dimensions [16] in the unpolarized case one may extend the analysis to next-to-next-to-leading order, where the remaining theory error is of $O(1\%)$ or less, see below. To cope with the present experimental errors 3–loop analyzes are mandatory. A theoretically consistent analysis can be performed at least in the non-singlet case, where the heavy flavor effects known to $O(\alpha_s^2)$, are negligibly small. 3–loop valence analyzes were performed in [8, 9]. One even may extend the non-singlet analysis to the 4–loop level [9]. A closer numerical study of the potential effect of the yet missing 4–loop anomalous dimension, performing a comparison with the recently calculated second moment in [17] shows that the overwhelming effect at 4–loops is due to the 3–loop Wilson coefficient. To see the convergence of the perturbative expansion we list the values for $\alpha_s(M_Z^2)$ obtained in the NLO, N^{2}LO, and N^{3}LO analyzes :

$$\alpha_s(M_Z^2) = 0.1148 \rightarrow 0.1134 \rightarrow 0.1142 \pm 0.0021. \tag{1}$$

The change from the N^{2}LO to the N^{3}LO value is found deeply inside the current experimental error. The N^{3}LO value corresponds to

$$\Lambda_{\rm QCD}^{\overline{\rm MS}, {\rm N_f}=4} = 234 \pm 26\,{\rm MeV}\,. \tag{2}$$

A drawback of the valence analysis are small, remaining contributions of sea-quark densities in the region $x > 0.4$, the effect of which can finally only be studied in combined singlet/non-singlet analyzes.

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*This paper was supported in part by SFB-TR-9: Computergestütze Theoretische Teilchenphysik.

[a] For a recent survey on the status of deep-inelastic scattering see [2].

In the singlet case the 3–loop heavy flavor corrections are yet missing. Still analyzes may be performed to determine $\Lambda_{\rm QCD}$ under an assumption for these terms. The results are summarized in Table 1. Compared to the respective NLO analyzes, the values of $\alpha_s(M_Z^2)$ turn out to be lower by 1–2% in case comparable values are available. Three independent analyses using different codes and methods to solve the evolution equations agree [8,9,12] at the 1σ level and better. These analyzes were performed using the world structure function data for deep-inelastic charged lepton proton and deuteron scattering. The analysis in [12] is a combined singlet and non–singlet analysis and fully confirms the value of $\alpha_s(M_Z^2)$ obtained in the non–singlet analysis Ref. [9], showing that the remaining uncertainties there do not affect the value of $\Lambda_{\rm QCD}$. Alternatively to the standard $\overline{\rm MS}$-analysis one may perform factorization scheme-invariant analyzes [14], based on observables only. This method is free of shape–assumptions, in particular for the gluon density. A slightly higher value of $\alpha_s(M_Z^2)$ was found in an earlier analysis [13] using the method of Bernstein polynomials. A recent analysis [15], including also jet data from colliders, reports a much higher value of $\alpha_s(M_Z^2)$.

$\Lambda_{\rm QCD}^{\overline{\rm MS}}$ was measured also in two recent lattice simulations based on two active flavors ($N_f = 2$). These investigations paid special attention to non-perturbative renormalization and kept the systematic errors as small as possible.

$$\Lambda_{N_f=2}^{\rm latt} = 245 \pm 16 \pm 16\,{\rm MeV} \quad [17], \qquad \Lambda_{N_f=2}^{\rm latt} = 261 \pm 17 \pm 26\,{\rm MeV} \quad [18] \tag{3}$$

NLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
CTEQ6	0.1165	±0.0065		[3]
A02	0.1171	±0.0015	±0.0033	[4]
ZEUS	0.1166	±0.0049		[5]
H1	0.1150	±0.0017	±0.0050	[6]
BCDMS	0.110	±0.006		[7]
GRS	0.112			[8]
BBG	0.1148	±0.0019		[9]
BB (pol)	0.113	±0.004	$^{+0.009}_{-0.006}$	[10]
N^{2}LO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
A02m	0.1141	±0.0014	±0.0009	[12]
SY01(ep)	0.1166	±0.0013		[13]
MSTW	0.1191	±0.002	±0.003	[15]
GRS	0.111			[8]
A06	0.1128	+0.0015		[12]
BBG	0.1134	+0.0019/ − 0.0021		[9]
N^{3}LO				
BBG	0.1142	± 0.0021		[9]

Table 1: Summary of $\alpha_s(M_Z^2)$ values determined from deep-inelastic scattering.

A direct comparison with the case $N_f = 4$ in the above data analyzes is not yet possible. However, the difference between the earlier $N_f = 0$ and the present result in $\Lambda_{\rm QCD}$ amounts to $O(10\,{\rm MeV})$ only. We have to wait and see what is obtained for $N_f = 4$ in coming analyzes.

More global analyzes were performed using also semi-inclusive *ep*– and *pp*–data from jet measurement, mostly aiming on a global determination of the quark and gluon densities.

As shown in [20–23] the $\alpha_s(M_Z^2)$ values obtained in analyzing the jet data and other data sets beyond those of the structure functions differ significantly in their χ^2–profiles and fitted value for the strong coupling constant pointing to systematic differences. The jet data prefer a higher value of $\alpha_s(M_Z^2)$ than the inclusive DIS data. This effect deserves further detailed studies before one is allowed to combine these data sets for a precision determination of $\Lambda_{\rm QCD}$.

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