

Three-Jet Event-Shapes: First NLO+NLL+1/Q Results

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Three-jet event-shape distributions can be exploited to investigate the dependence of hadronisation effects on the colour and the geometry of the underlying hard event. We present here the first comparison of data in e^+e^- annihilation and state-of-the-art theoretical predictions, including resummation of large logarithms at next-to-leading logarithmic accuracy matched to exact next-to-leading order and leading non-perturbative power corrections.

1 Power corrections to multi-jet event shapes

The remarkable success of the QCD description of two-jet event shape distributions has made these observables one of the most useful tools to test our understanding of the dynamics of strong interactions, both in the perturbative (PT) and non-perturbative (NP) regime. This is because event-shape distributions span a wide range of physical scales, from the region where the event shape V is large, described well by fixed-order QCD, to the exclusive $V \rightarrow 0$ region where hadronisation effects dominate, through the intermediate region where one needs to resum large infrared and collinear logarithms. The combination of next-to-leading order (NLO) predictions and next-to-leading logarithmic (NLL) resummation, supplemented with non-perturbative (NP) hadronisation corrections provided by Monte Carlo (MC) event generators, has led to one of the most precise determinations of the QCD coupling α_s [2].

In view of the fact that hadronisation corrections are suppressed by inverse power of the process hard scale Q , in recent years it has been attempted to describe two-jet event shape distributions at hadron level by simply adding to the NLL resummation the NP shift $\langle\delta V\rangle$ originated by leading $1/Q$ power corrections, which is a reliable approximation as long as $\langle\delta V\rangle \ll V$. The shift has a remarkably simple structure, being the product of a calculable coefficient c_V , which depends on the considered shape variable, and a genuine NP quantity $\langle k_t \rangle_{\text{NP}}$, the mean transverse momentum of large-angle hadrons produced in the collision, which is variable independent (*universal*). The universality of $\langle k_t \rangle_{\text{NP}}$, and hence of $1/Q$ power corrections, has been thoroughly tested both in e^+e^- annihilation and DIS, and is found to hold within 20% (see [3] for a recent review).

This universality property is based on two facts. The first is that particles responsible for leading power corrections are low transverse momentum hadrons in a central rapidity region, away from the hard jets. Any of these hadrons k contributes to a two-jet event shape V with an extra $\delta V(k)Q \simeq k_t f_V(\eta)$, with k_t and η the hadron transverse momentum and rapidity with respect to the jet axis. The second is that central hadrons are distributed uniformly in rapidity. This ensures that in the region $\langle\delta V\rangle \ll V$, where only leading power corrections are important, the dependence of $\langle\delta V\rangle$ on rapidity and transverse momentum gets factorised [4]:

$$\langle\delta V\rangle \simeq \langle k_t \rangle_{\text{NP}} c_V, \quad c_V = \int d\eta f_V(\eta). \quad (1)$$

Among all models that, for two-jet events, predict a uniform rapidity distribution of central hadrons, the dispersive DMW approach [5] makes it possible to extend eq. (1) to

multi-jet event shapes, where there is no natural way to identify k_t and η . The starting point is the probability $dw(k)$ of emitting a soft dressed gluon k from a quark-antiquark pair (whose momenta are p and \bar{p}) in a colour singlet:

$$dw(k) = C_F \frac{dk_t^2}{k_t^2} d\eta \frac{d\phi}{2\pi} \frac{\alpha_s(k_t)}{\pi}, \quad \eta = \frac{1}{2} \ln \frac{\bar{p}k}{pk}, \quad k_t^2 = \frac{(2pk)(2k\bar{p})}{2p\bar{p}}, \quad (2)$$

where α_s is the physical CMW coupling [6]. The CMW coupling is then extended at low transverse momenta via a dispersion relation, and the very same probability $dw(k)$ is exploited to compute NP corrections [5]. The resulting shift $\langle \delta V \rangle$ has the same form as in eq. (1), where the c_V coefficient is identical and the NP parameter $\langle k_t \rangle_{\text{NP}}$ can be expressed in terms of $\alpha_0(\mu_I)$, the average of the dispersive coupling below the merging scale μ_I , as follows [7]:

$$\langle k_t \rangle_{\text{NP}} = \frac{4\mu_I}{\pi^2} C_F \mathcal{M} (\alpha_0(\mu_I) - \alpha_s(Q) + \mathcal{O}(\alpha_s^2)), \quad \alpha_0(\mu_I) = \int_0^{\mu_I} \frac{dk}{\mu_I} \alpha_s(k). \quad (3)$$

Here the Milan factor \mathcal{M} accounts for non-inclusiveness of event-shape variables.

One can now naturally extend the above analysis to multi-jet event shapes, where the soft dressed gluon probability is given by

$$dw(k) = \sum_{i<j} (-\vec{T}_i \cdot \vec{T}_j) \frac{d\kappa_{ij}^2}{\kappa_{ij}^2} d\eta_{ij} \frac{d\phi_{ij}}{2\pi} \frac{\alpha_s(\kappa_{ij})}{\pi}, \quad \eta_{ij} = \frac{1}{2} \ln \frac{p_j k}{p_i k}, \quad \kappa_{ij}^2 = \frac{(2p_i k)(2k p_j)}{2p_i p_j}, \quad (4)$$

with \vec{T}_i the colour charge of hard parton p_i , and κ_{ij} and η_{ij} the invariant transverse momentum and rapidity with respect to the emitting dipole ij . This gives the following result for the shift:

$$\langle \delta V \rangle = \frac{4\mu_I}{\pi^2} \mathcal{M} (\alpha_0(\mu_I) - \alpha_s(Q) + \mathcal{O}(\alpha_s^2)) \sum_{i<j} (-\vec{T}_i \cdot \vec{T}_j) c_V^{(ij)}. \quad (5)$$

The above expression states that NP corrections to multi-jet event shapes depend on the same parameter $\alpha_0(\mu_I)$ encountered for two-jet shapes. Moreover, they depend in a non-trivial way on the colour of the underlying hard event through the correlation matrices $\vec{T}_i \cdot \vec{T}_j$ and on the event geometry (the angles between the jets) through the calculable coefficients $c_V^{(ij)}$ [8].

The simplest environment in which the validity of eq. (5) can be tested is three-jet events. Here colour conservation ensures that the colour matrices $\vec{T}_i \cdot \vec{T}_j$ are in fact proportional to the identity, thus simplifying considerably both the PT and the NP analysis.

2 Results for three-jet event shapes in e^+e^- annihilation

Two three-jet event shapes have been studied in e^+e^- annihilation, the D -parameter [9] and the thrust minor T_m [10]. Both variables are small when the three hard jets are in a near-to-planar configuration, and measure radiation outside the event plane.

We present here the first ever comparison of theoretical predictions for D and T_m differential distributions and existing data provided by the ALEPH collaboration [11]. Theoretical predictions are at the state-of-the-art level, that is NLL resummation matched to the NLO

calculation obtained with NLOJET++ [12], and leading $1/Q$ NP corrections computed with the dispersive method [9, 10]. Events with three separated jets are selected by requiring the three-jet resolution parameter y_3 in the Durham algorithm to be larger than y_{cut} . It is then clear that different values of y_{cut} correspond to different event geometries.

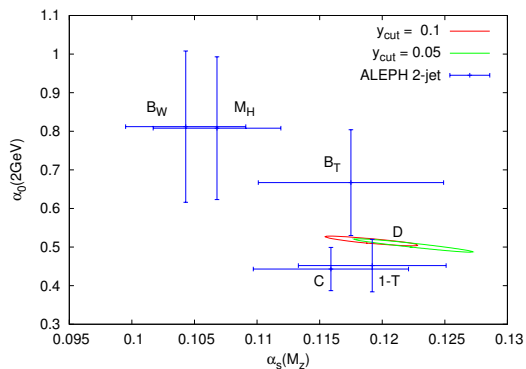


Figure 1: Contour plots in the α_s - α_0 plane for the D -parameter differential distributions corresponding to two different values of y_{cut} .

The comparison to data is less satisfactory for T_m , as can be seen from Fig. 2. There one notices a discrepancy between theory and data at large values of T_m . To track down the origin of the problem, one can look at hadronisation corrections produced by MC programs, defined as the ratio of the MC results at hadron and parton level. From the plots in [11] one can see that hadronisation corrections for the D -parameter are always larger than one, corresponding to a positive shift, consistent with our predictions. On the contrary, hadronisation corrections for T_m become smaller than one at large T_m , a feature that will never be predicted by a model based on a single dressed gluon emission from a three hard parton system. This issue is present also in the heavy-jet mass and wide-jet broadening distributions, and requires further theoretical investigation.

3 Extension to other hard processes

Observables that measure the out-of-event-plane radiation in three-jet events can be introduced also in other hard processes.

In DIS two observables have been already measured. One is a variant of T_m [13], where

Figure 1 shows the result of a simultaneous fit of $\alpha_s(M_Z)$ and $\alpha_0(\mu_I=2\text{GeV})$ for the D -parameter distribution at $Q = M_Z$ corresponding to $y_{\text{cut}} = 0.1$ and $y_{\text{cut}} = 0.05$. The $1\text{-}\sigma$ contour plots in the α_s - α_0 plane are plotted together with results for other distributions of two-jet event shapes. There is a remarkable consistency among the various distributions, thus strongly supporting the idea that universality of $1/Q$ power corrections holds also for three-jet variables. This leads to the non-trivial implication that leading power corrections are indeed sensitive to the colour and the geometry of the hard underlying event, and moreover this dependence is the one predicted by eq. (5).

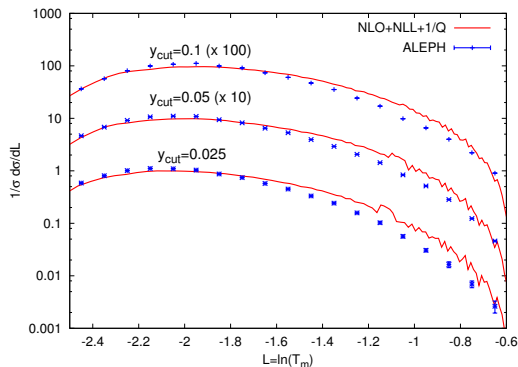


Figure 2: Theoretical predictions for T_m distribution plotted against ALEPH data for three different values of y_{cut} .

all momenta are in the Breit frame, and the event plane is formed by the virtual photon direction and the thrust major axis, defined as the direction that maximises the projection of transverse momenta. Differential T_m distributions have been measured both by the H1 [14] and ZEUS collaboration [15], and fits of experimental data are currently in progress. The other observable is the distribution in the transverse energy correlation $E_T E_T C(\chi)$, defined as [16]

$$E_T E_T C(\chi) = \sum_{i,j} p_{ti} p_{tj} \delta(\chi - (\pi - |\phi_i - \phi_j|)) . \quad (6)$$

The interesting features of the $E_T E_T C(\chi)$ distribution are that it approaches a constant for small χ and that it has fractional power corrections.

In hadron-hadron collisions one can consider for instance the production of a Z boson q in association with a hard jet p_{jet} . The event plane is determined by the beam and the Z direction, and one can study [17]

$$T_m = \sum_i \frac{|\vec{p}_{ti} \times \vec{q}_t|}{p_{t,\text{jet}} q_t} \Theta(\eta_0 - |\eta_i|) , \quad (7)$$

where the sum is extended to all hadrons not too close to the beam pipe, and the normalisation is fixed so as to cancel systematic uncertainties in the jet energy scale. In order to compare data with existing predictions, η_0 should be taken as large as is experimentally possible. The interest in this variable is that its distribution can take large corrections from the underlying event, thus making it a useful tool to tune MC models of minimum bias and multiple hard collisions. We look forward to experimental investigations in this direction.

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