Two-Loop Massive Operator Matrix Elements for Polarized and Unpolarized Deep-Inelastic Scattering

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The $O(\alpha_s^2)$ massive operator matrix elements for unpolarized and polarized heavy flavor production at asymptotic values $Q^2 >> m^2$ are calculated in Mellin space without applying the integration-by-parts method. We confirm previous results given in Refs. [5, 6], however, obtain much more compact representations.

1 Introduction

The heavy-flavor corrections to deeply inelastic structure functions are very important for the range of small values of x and do contribute there on the level of 20–40%. They have to be known at the same level of accuracy as the light-flavor contributions for precision measurements of $\Lambda_{\rm QCD}$ [2] and the parton distributions. The next-to-leading order corrections were given semi-analytically in [3] for the general kinematic range. Fast and accurate implementations of these corrections in Mellin-space were given in [4]. In the region $Q^2 >> m^2$, the heavy flavor Wilson coefficients were derived analytically to $O(\alpha_s^2)$ [5, 6]. Here Q^2 denotes the virtuality of the gauge boson exchanged in deeply–inelastic scattering and m is the mass of the heavy quark. In this note we summarize the results of a first re-calculation of the operator matrix elements (OMEs) in [7,8]. The calculation is being performed in Mellin-space using harmonic sums [9, 10] without applying the integration-by-parts technique. In this way, we can significantly compactify both, the intermediary and final results. We agree with the results in [5, 6]. The unpolarized and polarized $O(\alpha_s^2)$ massive OMEs can be used to calculate the asymptotic heavy-flavor Wilson coefficients for $F_2(x, Q^2)$ and $g_1(x, Q^2)$ to $O(\alpha_s^2)$ [5–8], and for $F_L(x, Q^2)$ to $O(\alpha_s^3)$ [11].

2 The Method

In the limit $Q^2 >> m^2$ the heavy quark contributions to the twist-2 Wilson coefficients are determined by universal massive operator matrix elements $\langle i|A_l|j\rangle$ between partonic states. The process dependence is due to the corresponding massless Wilson coefficients [12]. This separation is obtained by applying the renormalization group equation(s) to the (differential) scattering cross sections, cf. [5]. In this way all logarithmic and the constant contribution in m^2/Q^2 can be determined. The operator matrix elements are calculated applying the operator insertions due to the light-cone expansion in the respective amplitudes. One obtains the following representation

$$H^{\rm S,NS}_{(2,L),i}\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) = \underbrace{A^{\rm S,NS}_{k,i}\left(\frac{m^2}{\mu^2}\right)}_{\text{massive OMEs}} \otimes \underbrace{C^{\rm S,NS}_{(2,L),k}\left(\frac{Q^2}{\mu^2}\right)}_{\text{light Wilson coefficients}},$$

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with $\otimes$ denoting the Mellin convolution. The OMEs contain ultraviolet and collinear divergences. The collinear singularities are absorbed into the parton distribution functions while the ultraviolet divergences are removed through renormalization. To 2–loop order, the renormalized OMEs read :

$$\begin{aligned} A_{Qg}^{(2)} &= \frac{1}{8}\left\{\widehat{P}_{qg}^{(0)} \otimes \left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right]\right\}\ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{1}{2}\widehat{P}_{qg}^{(1)}\ln\left(\frac{m^2}{\mu^2}\right) \\ &\quad + \overline{a}_{Qg}^{(1)}\left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right] + a_{Qg}^{(2)} \ , \end{aligned}$$

and similar for the quarkonic contributions. Here, μ^2 denotes the factorization and renormalization scale, $P_{ij}^{(k-1)}$ are the kth loop splitting functions and β_0 denotes the lowest expansion coefficient of the β–function. $a_{ij}^{(k)}$ and $\bar{a}_{ij}^{(k)}$ are the $O(\varepsilon^0)$ resp. $O(\varepsilon)$-terms in the expansion of the OME, which form the main objective of the present calculation.

3 Results

We calculated the massive operator matrix elements both, for the gluon–heavy quark and light–heavy quark transitions in the flavor non-singlet and singlet cases, for unpolarized and polarized nucleon targets.

The constant contribution to the unpolarized and polarized OMEs for the transition $g \to Q$ are :

$$\begin{aligned} a_{Qg}^{(2,\mathrm{unpol})}(N) = 4C_F T_R\Biggl\{ & \frac{N^2+N+2}{N(N+1)(N+2)}\Biggl[-\frac{1}{3}S_1^3(N-1) + \frac{4}{3}S_3(N-1) - S_1(N-1)S_2(N-1) \\ & - 2\zeta_2 S_1(N-1)\Biggr] + \frac{N^4+16N^3+15N^2-8N-4}{N^2(N+1)^2(N+2)}S_2(N-1) + \frac{3N^4+2N^3+3N^2-4N-4}{2N^2(N+1)^2(N+2)}\zeta_2 \\ & + \frac{2}{N(N+1)}S_1^2(N-1) + \frac{N^4-N^3-16N^2+2N+4}{N^2(N+1)^2(N+2)}S_1(N-1) + \frac{P_1(N)}{2N^4(N+1)^4(N+2)}\Biggr\} \\ & + 4C_A T_R\Biggl\{\frac{N^2+N+2}{N(N+1)(N+2)}\Biggl[4\mathrm{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N+1) + \frac{1}{3}S_1^3(N) + 3S_2(N)S_1(N) \\ & + \frac{8}{3}S_3(N) + \beta''(N+1) - 4\beta'(N+1)S_1(N) - 4\beta(N+1)\zeta_2 + \zeta_3\Biggr] - \frac{N^3+8N^2+11N+2}{N(N+1)^2(N+2)^2}S_1^2(N) \\ & - 2\frac{N^4-2N^3+5N^2+2N+2}{(N-1)N^2(N+1)^2(N+2)}\zeta_2 - \frac{7N^5+21N^4+13N^3+21N^2+18N+16}{(N-1)N^2(N+1)^2(N+2)^2}S_2(N) \\ & - \frac{N^6+8N^5+23N^4+54N^3+94N^2+72N+8}{N(N+1)^3(N+2)^3}S_1(N) - 4\frac{N^2-N-4}{(N+1)^2(N+2)^2}\beta'(N+1) \\ & + \frac{P_2(N)}{(N-1)N^4(N+1)^4(N+2)^4}\Biggr\} \ . \end{aligned}$$

$$\begin{aligned} a_{Qg}^{(2,\mathrm{pol})}(N) = C_F T_R\Biggl\{ & 4\frac{N-1}{3N(N+1)}\Bigl(-4S_3(N) + S_1^3(N) + 3S_1(N)S_2(N) + 6S_1(N)\zeta_2\Bigr) \\ & - 4\frac{N^4+17N^3+43N^2+33N+2}{N^2(N+1)^2(N+2)}S_2(N) - 4\frac{3N^2+3N-2}{N^2(N+1)(N+2)}S_1^2(N) \\ & - 2\frac{(N-1)(3N^2+3N+2)}{N^2(N+1)^2}\zeta_2 - 4\frac{N^3-2N^2-22N-36}{N^2(N+1)(N+2)}S_1(N) - \frac{2P_3(N)}{N^4(N+1)^4(N+2)}\Biggr\} \end{aligned}$$

$$
\begin{aligned}
&+ C_A T_R \Biggl\{ 4\frac{N-1}{3N(N+1)}\Bigl(12\mathrm{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N+1) + 3\beta''(N+1) - 8S_3(N) - S_1^3(N) \\
&- 9S_1(N)S_2(N) - 12S_1(N)\beta'(N+1) - 12\beta(N+1)\zeta_2 - 3\zeta_3\Bigr) - 16\frac{N-1}{N(N+1)^2}\beta'(N+1) \\
&+ 4\frac{N^2+4N+5}{N(N+1)^2(N+2)}S_1^2(N) + 4\frac{7N^3+24N^2+15N-16}{N^2(N+1)^2(N+2)}S_2(N) + 8\frac{(N-1)(N+2)}{N^2(N+1)^2}\zeta_2 \\
&+ 4\frac{N^4+4N^3-N^2-10N+2}{N(N+1)^3(N+2)}S_1(N) - \frac{4P_4(N)}{N^4(N+1)^4(N+2)} \Biggr\} .
\end{aligned}
$$

Here $P_i(N)$ denote polynomials given in [7, 8]. The corresponding quarkonic expressions are given in [7, 8]. The integrals were performed using Mellin-Barnes techniques [13, 14] and applying generalized hypergeometric function representations. The results were further simplified using algebraic relations between harmonic sums [15]. Furthermore, structural relations for harmonic sums [16], which include half–integer relations and differentiation for the Mellin variable N, lead to the observation that the OMEs above depend only on two basic harmonic sums :

$$S_1(N), \qquad\qquad S_{-2,1}(N) .$$

We expressed $S_{-2,1}(N)$ in terms of the Mellin transform $M[\mathrm{Li}_2(x)/(1+x)](N)$ in the above. Here $\beta(N) = (1/2)\cdot[\psi((N+1)/2) - \psi(N/2)]$. Previous analyzes of various other space- and time-like 2–loop Wilson coefficients and anomalous dimensions including also the soft and virtual corrections to Bhabha-scattering [15a,16], showed that six basic functions are needed in general to express these quantities :

$$S_1(N), \quad S_{\pm 2,1}(N), \quad S_{-3,1}(N), \quad S_{\pm 2,1,1}(N) .$$

Non of the harmonic sums occurring contains an index $\{-1\}$ as observed in all other cases being analyzed.

Comparing to the results obtained in Refs. [5, 6] in x–space, there 48 functions were needed to express the final result in the unpolarized case and 24 functions in the polarized case.

To obtain expressions for the heavy flavor contributions to the structure functions in x–space, analytic continuations have to be performed to $N \epsilon \mathbf{C}$ for the basic functions given above, see [16, 18, 19]. Finally a (numeric) contour integral has to be performed around the singularities present.

4 Conclusions

We calculated the unpolarized and polarized massive operator matrix elements to $O(\alpha_s^2)$, which are needed to express the heavy flavor Wilson coefficients contributing to the deep–inelastic structure functions F_2, g_1 and F_L to $O(\alpha_s^2)$ resp. $O(\alpha_s^3)$ in the region $Q^2 >> m^2$. The calculation was performed in Mellin space without using the integration-by-parts technique, leading to nested harmonic sums. We both applied representations through Mellin–Barnes integrals and generalized hypergeometric functions. In course of the calculations, a series of new infinite sums over products of harmonic sums weighted by related functions were evaluated, cf. [7, 8]. These representations were essential to keep the complexity of the intermediary and final results as low as possible. Furthermore, we applied a series of mathematic relations for the harmonic sums to compactify the results further. We confirm the results obtained earlier in Refs. [5, 6] by other technologies.

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