Spin Structure Function g_1 at Small x and Arbitrary Q^2

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The Standard Approach (SA) for the description of the structure function g_1 combines the DGLAP evolution equations with the standard fits for the initial parton densities. The DGLAP equations describe the region of large Q^2 and large x, so there are no theoretical grounds to exploit them at small x. In practice, extrapolation of DGLAP into the region of small x is done by complementing DGLAP with ad hoc, singular ($\sim x^{-a}$) phenomenological fits for the initial parton densities. The factors x^{-a} are wrongly believed to be of non-perturbative origin. Actually, they mimic the summation of logs of x and should not be included in the fits when the summation is accounted for. Contrary to SA, the summation of logarithms of x is a straightforward and natural way to describe g_1 in the small-x region. This approach can be used both at large and small Q^2 where DGLAP cannot be used by definition. Confronting this approach and SA shows that the singular initial parton densities and the power Q^2-corrections (or at least a sizable part of them) do not describe real physical phenomena but they are just artifacts caused by extrapolating DGLAP into the small-x region.

1 Introduction

The *Standard Approach* (**SA**) for description of the structure function g_1 involves the DGLAP evolution equations[2] and standard fits[3] for the initial parton densities δq and δg. The fits are defined from phenomenological considerations at $x \sim 1$ and $Q^2 = \mu^2 \sim 1$ GeV^2. The DGLAP equations are one-dimensional, and describe the Q^2 -evolution only, converting δq and δg into the evolved distributions Δq and Δg. They represent g_1 at the region **A**:

$$\textbf{A:} \quad Q^2 \gg \mu^2 \ , \qquad x \lesssim 1 \ . \tag{1}$$

The x -evolution is supposed to come by convoluting Δq and Δg with the coefficient functions C_{DGLAP}. However, in the leading order $C^{LO}_{DGLAP} = 1$ and the NLO corrections account for one- or two- loop contributions and neglect higher loops. It is the correct approximation in the region **A** but becomes false in the region **B:**

$$\textbf{B:} \quad Q^2 \gg \mu^2 \ , \qquad x \ll 1 \tag{2}$$

where contributions $\sim \ln^k(1/x)$ are large and should be accounted for to all orders in α_s. C_{DGLAP} do no include the *total summation of leading logarithms of x* (**LL**), so there are no theoretical grounds to exploit DGLAP at small x. However, **SA** extrapolates DGLAP

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into the region **B**, invoking special fits for δq and δg. The general structure of such fits (see Refs. [3]) is as follows:

$$\delta q = N x^{-a} \varphi(x) \tag{3}$$

where N is a normalization constant; $a > 0$, so x^{-a} is singular when $x \to 0$, and $\varphi(x)$ is regular in x at $x \to 0$. As we showed in Ref. [4], the factor x^{-a} in Eq. (3) just mimics the result of **LL** performed in Refs [5, 6]. Similarly to **LL**, the factor x^{-a} provides the steep rise to g_1 at small x and sets the Regge asymptotics for g_1 at $x \to 0$, with the exponent a being the intercept. The presence of this factor is very important for extrapolating DGLAP into the region **B:** When the factor x^{-a} is dropped from Eq. (3), DGLAP stops to work at $x \lesssim 0.05$ (see Ref. [4] for detail). Accounting for **LL** is beyond the DGLAP framework because **LL** come from the phase space region not included in the DGLAP ordering. Indeed the DGLAP -ordering is

$$\mu^2 < k_{1\ \perp}^2 < k_{2\ \perp}^2 < ... < Q^2 \tag{4}$$

for the ladder partons. **LL** can be accounted only when the ordering Eq. (4) is lifted and all $k_{i\ \perp}$ obey

$$\mu^2 < k_{i\ \perp}^2 < (p+q)^2 \approx (1-x)2(pq) \approx 2(pq) \tag{5}$$

at small x. Replacing Eq. (4) by Eq. (5) leads inevitably to the change of the DGLAP parametrization

$$\alpha_s^{DGLAP} = \alpha_s(Q^2) \tag{6}$$

by the alternative parametrization of α_s obtained in Ref. [7] and used in Refs. [5, 6] in order to find explicit expressions accounting for **LL** for g_1 in the region **B**. Obviously, those expressions require fits for the initial parton densities without singular factors x^{-a}. Let us note that the replacement of Eq. (4) by Eq. (5) brings a more involved μ -dependence to g_1. Indeed, Eq. (4) makes the contributions of gluon ladder rungs to be infrared (IR) stable, with μ acting as a IR cut-off for the lowest rung and $k_{i\ \perp}$ playing the role of the IR cut-off for the $i+1$-rung. In contrast, Eq. (5) implies that μ acts as the IR cut-off for every rung.

Besides the regions **A** and **B**, it is also necessary to know g_1 in the region **C**:

$$\mathbf{C:} \quad Q^2 < \mu^2 , \qquad x \ll 1 . \tag{7}$$

This region is studied experimentally by the COMPASS collaboration. Obviously, DGLAP cannot be exploited here. Alternatively, in Refs. [8, 9] we obtained expressions for g_1 in the region **C**. In particular, in Ref. [8] we showed that g_1 practically does not depend on x at small x, even at $x \ll 1$. Instead, it depends on the total invariant energy $2(pq)$. Experimental investigation of this dependence is extremely interesting because according to our results g_1, being positive at small $2(pq)$, can turn negative at greater values of this variable. The position of the turning point is sensitive to the ratio between the initial quark and gluon densities, so its experimental detection would enable to estimate this ratio. In Ref. [9] we have analyzed the power contributions $\sim 1/(Q^2)^k$ to g_1 usually attributed to higher twists. We have proved that a great amount of those corrections have a simple perturbative origin and have summed them. Therefore, the genuine impact of higher twists can be estimated only after accounting for the perturbative Q^2 -corrections.

2 Description of g_1 in the regions B and C

The total sum of the double-logarithms (DL) and single-logarithms of x in the region **B** was done in Refs. [5, 6]. In particular, the non-singlet component, g_1^{NS} of g_1 is

$$g_1^{NS}(x,Q^2) = (e_q^2/2)\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\pi\imath}(1/x)^{\omega}C_{NS}(\omega)\delta q(\omega)\exp\left(H_{NS}(\omega)\ln(Q^2/\mu^2)\right)\,, \qquad (8)$$

with the new coefficient function C_{NS} and new anomalous dimension H_{NS}. H_S and C_{NS} account for DL and SL contributions to all orders in α_s and depend on the IR cut-off μ. As is shown in Refs. [5, 6], there exists an optimal scale for fixing μ: $\mu \approx 1$ Gev for g_1^{NS} and $\mu \approx 5$ GeV for g_1^S. The arguments in favor of existence of the optimal scale were given in Ref. [9]. Eq. (8) predicts that g_1 has the power behavior in x and Q^2 when $x \to 0$:

$$g_1^{NS} \sim \left(Q^2/x^2\right)^{\Delta_{NS}/2}\,, \qquad g_1^S \sim \left(Q^2/x^2\right)^{\Delta_S/2} \qquad (9)$$

where the non-singlet and singlet intercepts are $\Delta_{NS} = 0.42$ and $\Delta_S = 0.86$ respectively. The asymptotic expressions (9) should be used with great care: According to Ref. [4], Eq. (9) should not be used at $x \gtrsim 10^{-6}$. So, Eq. (8) should be used instead of Eq. (9) in the region of small x so far available. Expressions accounting **LL** for the singlet g_1 in the region **B** were obtained in Ref. [6]. They are more complicated because involve two coefficient functions and four anomalous dimensions.

Region **C** is defined in Eq. (7). It includes small Q^2, so there are not large contributions $\ln^k(Q^2/\mu^2)$ in this region. In other words, the DGLAP ordering of Eq. (4) does not make sense in the region **C** , which makes impossible exploiting DGLAP here. In contrast, Eq. (4) is not sensitive to the value of Q^2 and therefore **LL** does make sense in the region **C**. In Ref. [8] we suggested that the shift

$$Q^2 \to Q^2 + \mu^2 \qquad (10)$$

allows to extrapolate our previous results obtained in the region **B** to the region **C**. Then in Ref. [9] we proved this suggestion. Therefore, applying Eq. (10) to g_1^{NS} leads to the following expression for g_1^{NS} valid simultaneously in the regions **B** and **C**:

$$g_1^{NS}(x+z,Q^2) = (e_q^2/2)\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\pi\imath}\left(\frac{1}{x+z}\right)^{\omega}C_{NS}(\omega)\delta q(\omega)\exp\left(H_{NS}(\omega)\ln\left((Q^2+\mu^2)/\mu^2\right)\right)\,, \qquad (11)$$

where $z = \mu^2/2(pq)$. Obviously, Eq. (11) reproduces Eq. (8) in the region **B**. Expression for g_1^S looks similarly but more complicated, see Refs. [8, 9] for detail.

3 Prediction for the COMPASS experiments

The COMPASS collaboration now measures the singlet g_1^S at $x \sim 10^{-3}$ and $Q^2 \lesssim 1$ GeV^2, i.e. in the kinematic region beyond the reach of DGLAP. However, our formulae for g_1^{NS} and g_1^S obtained in Refs. [8, 9] cover this region. Although expressions for singlet and non-singlet g_1 are different, with formulae for the singlet being much more complicated, we can explain the essence of our approach, using Eq. (11) as an illustration. According to results of [6],

$\mu \approx 5$ GeV for g_1^S, so in the COMPASS experiment $Q^2 \ll \mu^2$. It means, $\ln^k(Q^2+\mu^2)$ can be expanded into series in Q^2/μ^2, with the first term independent of Q^2:

$$g_1^S(x+z, Q^2, \mu^2) = g_1^S(z, \mu^2) + \sum_{k=1} (Q^2/\mu^2)^k E_k(z) \tag{12}$$

where $E_k(z)$ account for **LL** in z and

$$g_1^S(z, \mu^2) = (< e_q^2/2 >) \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/z)^{\omega} \left[C_S^q(\omega)\delta q(\omega) + C_S^g(\omega)\delta g(\omega)\right] , \tag{13}$$

so that $\delta q(\omega)$ and $\delta g(\omega)$ are the initial quark and gluon densities respectively and $C_S^{q,g}$ are the singlet coefficient functions. Explicit expressions for $C_S^{q,g}$ are given in Refs. [6, 8]. The initial parton densities can be approximated by constants: $\delta q \approx N_q$ and $\delta g \approx N_g$, so

$$g_1(Q^2 \ll \mu^2) \approx (< e_q^2 > /2) N_q G_1(z) , \qquad G_1 = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/z)^{\omega} \left[C_S^q + (N_g/N_q C_S^g)\right] . \tag{14}$$

The results for G_1 for different values of the ratio $r = N_g/N_q$ are shown in Fig. 1.

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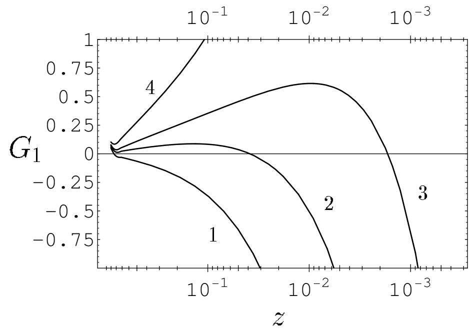

Figure 1: G_1 evolution with decreasing $z = \mu^2/2(pq)$ for different values of ratio $r = \delta g/\delta q$: curve 1 - for $r = 0$, curve 2 - for $r = -5$, curve 3 -for $r = -8$ and curve 4 -for $r = -15$.