Coordinate-Space Picture and $x \to 1$ Singularities at Fixed $k_\perp$

F. Hautmann

CERN, PH-TH Division, Geneva, Switzerland and
Institut für Theoretische Physik, Universität Regensburg, Germany

This talk [1] discussed ongoing progress towards precise characterizations of parton distributions at fixed transverse momentum, focusing on matrix elements in coordinate space and the treatment of endpoint singularities.

Parton distributions unintegrated in transverse momentum are naturally defined for small x via high-energy factorization [2]. This relates off-shell matrix elements with physical cross sections at x $\to$ 0, and gives a well-prescribed method to introduce unintegrated parton distributions in a gauge-invariant manner.

The question of how to characterize gauge-invariantly a $k_\perp$ distribution over the whole phase space, on the other hand, is more difficult and not yet fully answered. Its relevance was already emphasized long ago in the context of Sudakov processes [3], jet physics [4], exclusive production [5], spin physics [6]. Although a complete framework is still missing, much work is currently underway on this subject, see e.g. [7, 8, 9, 10, 11, 12, 13]. The discussion that follows focuses on aspects related to the gauge-invariant operator matrix elements and regularization methods for lightcone divergences.

To ensure gauge invariance, the approach commonly used is to generalize the matrix elements that serve to define ordinary parton distributions to the case of field operators at non-lightcone distances [6, 14]. This leads one to consider the matrix element for the quark distribution (Fig. 1)

$$\widetilde{f}(y) = \langle P | \overline{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle \tag{1}$$

with the quark fields ψ evaluated at distance $y = (0, y^-, y_\perp)$ for arbitrary y^- and $y_\perp$, and the eikonal-line operators V given by

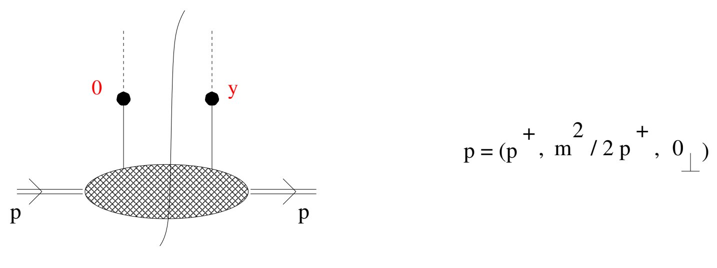

Figure 1: Quark distribution function in the target of momentum p.

$$V_y(n) = \mathcal{P} \exp \left(i g_s \int_0^\infty d\tau \; n^\mu A_\mu(y + \tau \, n) \right) \quad , \tag{2}$$

where n is the direction of the eikonal line and A is the gauge field.

However, while the use of Eq. (1) does not pose major problems at tree level, it becomes more subtle at the level of radiative corrections. Part of the subtleties are associated with incomplete KLN cancellations that come from measuring $k_\perp$ in the initial state [3, 15]. These may appear as uncancelled divergences near the endpoints for certain lightcone momentum components [16]. Another set of issues are associated with the integration over all transverse momenta, and involve the relation of unintegrated parton distributions with the ordinary ones [17, 18, 19] and the treatment of ultraviolet divergences. As observed in [20] for the case of the Sudakov form factor, the choice of a particular regularization method for the lightcone divergences also affects integrated distributions and ultraviolet subtractions.

In [11] these effects are examined by an explicit calculation at one loop using techniques for the expansion of nonlocal operators. The answer for the coordinate-space matrix element is analyzed in powers of y^2, separating logarithmic contributions from long distances and short distances,

$$\begin{aligned} \widetilde{f}_1(y) &= \frac{\alpha_s C_F}{\pi}\, p^+ \int_0^1 dv\, \frac{v}{1-v} \left\{ \left[e^{ip\cdot yv} - e^{ip\cdot y}\right] \Gamma(2-\frac{d}{2})\, (\frac{4\pi\mu^2}{\rho^2})^{2-d/2} \right. \\ &+ \left. e^{ip\cdot yv}\, \pi^{2-d/2}\, \Gamma(\frac{d}{2}-2)\, (-y^2\mu^2)^{2-d/2} + \cdots \right\} , \end{aligned} \tag{3}$$

where μ is the dimensional-regularization scale and ρ is an infrared mass regulator. The lightcone singularity $v \to 1$ corresponds to the exclusive phase-space boundary $x = 1$. The singularity cancels for ordinary parton distributions (first term in the right hand side of Eq. (3)) but it is present, even at $d \neq 4$ and finite ρ, in subsequent terms, which contribute to the unintegrated parton distribution [11]. This is then treated on the same footing as a physical correlation function, to be expanded in terms of the ordinary parton distributions with nontrivial, perturbatively calculable coefficient functions [17, 18].

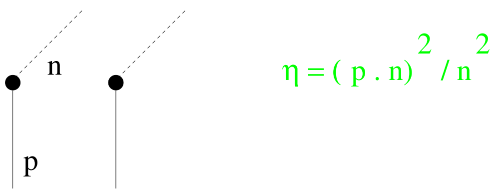

Figure 2: Cut-off regularization for the quark matrix element.

Traditionally the effect of endpoint singularities is suppressed by the use of a cut-off. This is likely the case, for instance, in existing Monte-Carlo event generators that implement unintegrated parton distributions [21, 22, 23, 24, 25]. A cut-off is also implemented in treatments [3, 13, 26] based on regularizing the parton-distribution matrix element by taking the eikonal line n to be non-lightlike (Fig. 2), combined with evolution equations in the cut-off parameter $\eta = (p \cdot n)^2/n^2$ [4, 27]. One-loop formulas in coordinate space corresponding to the regularization method of Fig. 2 are given in [11]. This method leads to a cut-off in x at fixed $k_\perp$ of order

$$1 - x \gtrsim k_\perp/\sqrt{4\eta} \quad . \tag{4}$$

However, cut-off regularization is not very well-suited for applications beyond the leading order. Furthermore, as the two lightcone limits $y^2 \to 0$ and $n^2 \to 0$ do not commute, a residual dependence on the regularization parameter η is left after integrating in $k_\perp$ the distribution defined with the cut-off. The relation with the standard operator product expansion is therefore not so transparent.

An alternative approach is based on the subtractive regularization method [20, 28]. As explained in [15], in this approach the eikonal n is kept in lightlike direction but the singularities are canceled by multiplicative, gauge-invariant factors given by eikonal-line vacuum expectation values. The matrix element with subtraction factors is pictured in Fig. 3, where $\bar{y} = (0, y^-, 0_\perp)$, and u is the direction of an auxiliary (non-lightlike) eikonal that provides a gauge-invariant regulator near $x = 1$ and cancels in the matrix element at $y_\perp = 0$ [11]. The form of the counterterms is simple in coordinate space, where it can be given in terms of compact all-order expressions.

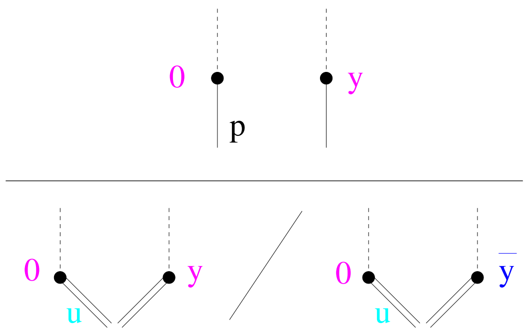

Figure 3: Matrix element with subtractive regularization.

The subtractive method is more systematic than the cut-off, and likely more suitable for using unintegrated parton distributions at subleading-log level. It can be useful for incorporating the unintegrated formulation in parton shower approaches [18, 28, 29]. Also, subleading accuracy is needed for matching large-x contributions with calculations at small x [19, 25, 30] and in the Sudakov region [31, 32].

The techniques discussed above will be instrumental to analyze factorization and evolution for $k_\perp$ parton distributions with increased precision [8, 9]. The issue of soft gluon exchanges with spectator partons is revisited in [8] for hard pp collisions. A potential breakdown of factorization at high order of perturbation theory is discussed (N^{3}LO correction to dihadron production, with two soft and one collinear partons), which would be of interest to verify by calculation. Also, it will be interesting to investigate how the argument of [8] is modified by the inclusion of destructive interference effects due to soft gluon coherence. A better understanding of these issues will help improve the present accuracy in current phenomenological studies of the effects of partons' transverse momentum [25, 30, 31].

References

[1] Slides:
`http://indico.cern.ch/contributionDisplay.py?contribId=290&sessionId=7&confId=9499`

[2] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. **B242** (1990) 97; Nucl. Phys. **B366** (1991) 135.

[3] J.C. Collins, in *Perturbative Quantum Chromodynamics*, ed. A.H. Mueller, World Scientific 1989, p. 573.

[4] J.C. Collins and D.E. Soper, Nucl. Phys. **B193** (1981) 381.

[5] S.J. Brodsky and G.P. Lepage, in *Perturbative Quantum Chromodynamics*, ed. A.H. Mueller, World Scientific 1989, p. 93.

[6] P. Mulders and R.D. Tangerman, Nucl. Phys. **B461** (1996) 197; D. Boer and P. Mulders, Phys. Rev. **D** 57 (1998) 5780.

[7] J.W. Qiu, W. Vogelsang and F. Yuan, arXiv:0706.1196; X. Ji, J.W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. Lett. **97** (2006) 082002.

[8] J.C. Collins and J.W. Qiu, arXiv:0705.2141.

[9] F. Ceccopieri and L. Trentadue, arXiv:0706.4242.

[10] K. Goeke, S. Meissner and A. Metz, hep-ph/0703176.

[11] F. Hautmann, hep-ph/0702196.

[12] A. Idilbi and T. Mehen, hep-ph/0702022.

[13] P. Chen, A. Idilbi and X. Ji, Nucl. Phys. **B763** (2007) 183.

[14] A.V. Belitsky, X. Ji and F. Yuan, Nucl. Phys. **B656** (2003) 165.

[15] J.C. Collins, Acta Phys. Polon. B **34** (2003) 3103.

[16] S.J. Brodsky, D.S. Hwang, B.Q. Ma and I. Schmidt, Nucl. Phys. **B593** (2001) 311.

[17] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. **B307** (1993) 147; S. Catani and F. Hautmann, Nucl. Phys. **B427** (1994) 475.

[18] J.C. Collins and X. Zu, JHEP **0503** (2005) 059.

[19] A.D. Martin, M.G. Ryskin and G. Watt, Eur. Phys. J. C**31** (2003) 73.

[20] J.C. Collins and F. Hautmann, Phys. Lett. B **472** (2000) 129.

[21] G. Marchesini and B.R. Webber, Nucl. Phys. **B386** (1992) 215.

[22] G. Gustafson, L. Lönnblad and G. Miu, JHEP **0209** (2002) 005; L. Lönnblad and M. Sjödahl, JHEP **0402** (2004) 042, JHEP **0505** (2005) 038.

[23] H. Jung, Mod. Phys. Lett. A19 (2004) 1.

[24] K. Golec-Biernat, S. Jadach, W. Placzek, P. Stephens and M. Skrzypek, hep-ph/0703317.

[25] S. Höche, F. Krauss and T. Teubner, arXiv:0705.4577.

[26] X. Ji, J. Ma and F. Yuan, Phys. Rev. D **71** (2005) 034005, JHEP **0507** (2005) 020.

[27] G.P. Korchemsky and G. Marchesini, Phys. Lett. B **313** (1993) 433; G.P. Korchemsky, Phys. Lett. B **220** (1989) 62.

[28] J.C. Collins and F. Hautmann, JHEP **0103** (2001) 016.

[29] C.W. Bauer and M.D. Schwartz, hep-ph/0607296.

[30] H. Jung, A.V. Kotikov, A.V. Lipatov and N.P. Zotov, arXiv:0706.3793.

[31] C.P. Yuan, talk at DIS07 Workshop, Munich, April 2007.

[32] P.M. Nadolsky, N. Kidonakis, F.I. Olness and C.P. Yuan, Phys. Rev. D **67** (2003) 074015; R. Brock, F. Landry, P.M. Nadolsky and C.P. Yuan, Phys. Rev. D **67** (2003) 073016.