QCD and String Theory

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We review recent new relations between string theory and QCD based on the AdS/CFT correspondence and its extensions. We give a brief overview over AdS/CFT and discuss generalizations to field theories with running gauge coupling, partially or fully broken supersymmetry and with added flavor degrees of freedom. Moreover we discuss applications such as chiral symmetry breaking and meson spectra, as well as finite temperature field theories and transport phenomena.

1 Introduction

String theory originated as a theory of hadrons in the 1960's, when it was noticed that hadron spectra coincide with excited states of a rotating string. String theory as a theory of strong interactions was abandoned however since four-dimensional string theory contains tachyonic modes. From the beginning of the 1970's, Quantum Chromodynamics (QCD) has established itself as a very successful quantum field theory of strong interactions, which is by now very well tested experimentally.

String theory took a rather different route due to the fact that it contains a graviton in its spectrum and is by now a very promising candidate for a unified quantum theory of all four fundamental interactions.

Within the last ten years, following the paper by Maldacena [1] introducing the AdS/CFT correspondence, a wealth of interesting new relations between modern string theory and quantum field theory have been found. In this review we discuss a series of examples for these new relations. The slides for this talk may be found at [2].

2 Gauge/gravity duality

2.1 AdS/CFT correspondence

The AdS/CFT correspondence is a *duality* which – in its simplest form – maps a quantum field theory at strong coupling to a gravity theory at weak coupling. The best known example is the map between $\mathcal{N} = 4$ supersymmetric $U(N)$ Yang-Mills theory, which is mapped to type IIB supergravity on the space $AdS_5 \times S^5$. Here AdS_5 denotes five-dimensional Anti-de Sitter space and S^5 denotes the five-sphere. Anti-de Sitter space is a space of constant negative curvature which has a boundary. The metric of $AdS_5 \times S^5$ may be written in the form

$$ds^2 = L^2 \left(\frac{1}{u^2} \eta_{ij} dx^i dx^j + \frac{du^2}{u^2} + d\Omega_5{}^2 \right) , \tag{1}$$

where L is the AdS radius and η_{ij} is the standard $3+1$-dimensional Minkowski metric. There is a boundary of AdS_5 at $u = 0$. It is sometimes convenient to perform a coordinate transformation and to write the metric of AdS_5 in the form

$$ds^2 = e^{2r/L} \eta_{ij} dx^i dx^j + dr^2 , \tag{2}$$

The boundary is then located at $r \to \infty$.

The AdS/CFT correspondence arises from string theory in a particular low-energy limit in which the 't Hooft coupling is large and fixed, while $N \to \infty$, such that the planar limit of the gauge theory is considered. $\mathcal{N} = 4$ Super Yang-Mills theory is a conformal field theory in which the beta function vanishes to all orders in perturbation theory. Therefore it has a $SO(4, 2)$ conformal symmetry, which coincides exactly with the isometry of AdS_5. Similarly, the $SU(4) \simeq SO(6)$ R symmetry of the field theory coincides with the isometry of the five-sphere S^5.

The AdS/CFT correspondence has been developed further in [3, 4] where a *field-operator map* has been established: There is a one-to-one correspondence between gauge-invariant operators in the field theory and supergravity fields on AdS_5. This maps gauge invariant operators of $\mathcal{N} = 4$ Yang-Mills theory in a particular irreducible representation of $SU(4)$ to supergravity fields in the same representation. These five-dimensional supergravity fields are obtained by Kaluza-Klein reduction of the original ten-dimensional supergravity fields on the five-sphere S^5. There is a precise relation between the Kaluza-Klein mass m_{sugra} of the supergravity fields and the dimension Δ of the dual operator. For scalars this relation is $m^2_{\text{sugra}} = \Delta(\Delta - d)$, with d the dimension of the AdS boundary. For our purposes, $d = 4$. The asymptotic behaviour of the supergravity fields at the AdS boundary is of central importance. For a given supergravity field ϕ of Kaluza-Klein mass m it is given by

$$\phi(u) \sim u^{d-\Delta}\phi_0 \, + \, u^{\Delta}\langle\mathcal{O}\rangle \tag{3}$$

for $u \to 0$. As discussed in [4], the boundary value ϕ_0 may be identified with the source of the gauge theory-operator $\mathcal{O}$, and $\langle\mathcal{O}\rangle$ is the VEV of $\mathcal{O}$. – The AdS/CFT correspondence has been tested in numerous examples, among which the calculation of correlation functions [5, 6] and of the conformal anomaly [7].

The string-theoretical origin of the AdS/CFT correspondence arises from the two different interpretations of D3 branes, i. e. 3+1-dimensional hyperplanes within 9+1-dimensional space. On the one hand, D3 branes are hyperplanes on which open strings can end. In the low-energy limit where only massless string excitations are taken into account, the degrees of freedom on a stack of N D3 branes correspond to $\mathcal{N} = 4$ $U(N)$ Super Yang-Mills theory in four-dimensional Minkowski space. On the other hand, D3 branes are a solitonic solution of ten-dimensional IIB supergravity. As such they are massive extended objects which curve the space around them. In the *near-horizon* limit, which is also a low-energy limit, this curved space is just $AdS_5 \times S^5$. The excitations in this curved-space background are closed strings whose massless mode corresponds to gravitons. In the *Maldacena limit* in which the 't Hooft coupling is large and fixed, while $N \to \infty$, the string modes decouple, such that only supergravity, i.e. pointlike particles, survive.

2.2 Generalizations of AdS/CFT

It is an appealing idea to generalize this gauge/gravity duality to less symmetric quantum field theories which at least in some respects are similar to QCD. A number of avenues have been pursued over the last few years. These are listed in the following:

- **Holographic RG flows:** By considering more involved metrics than $AdS_5 \times S^5$ with a reduced degree of symmetry, it is possible to construct gravity duals of field theories with running gauge coupling. Important examples are [8, 9]. In many cases these correspond to $\mathcal{N} = 4$ theory perturbed by relevant operators, for instance in [10, 11]. The fifth dimension – perpendicular to the boundary on which the field theory lives – may be interpreted as an energy scale. Some of these holographic renormalization group flows flow to confining field theories in the infrared, as may be shown by calculating the Wilson loop within the dual gravity theory, which follows an area law in this case.

- **Adding flavor:** In $\mathcal{N} = 4$ theory, all fields are in the adjoint representation of the gauge group, since all fields are in the same supermultiplet as the gauge field. Matter in the fundamental representation of the gauge group, i.e. quarks, may be added by adding further D-branes to the original stack of N D3 branes. The prototype example is the addition of D7 brane probes. This corresponds to adding $\mathcal{N} = 2$ hypermultiplets in the fundamental representation of the gauge group. By combining the addition of brane probes with the deformation of the gravity background, a gravity dual description of spontaneous chiral symmetry breaking by a quark condensate is obtained, as well as meson spectra involving Goldstone bosons. An alternative approach which provides a gravity dual realization of $U(N_f) \times U(N_f) \to U(N_f)$ chiral symmetry breaking uses D4, D8 and $\bar{\text{D}}$8 branes.
- **Quark-gluon plasma:** By considering the field-theory dual of the AdS black hole background, a strongly coupled field theory at finite temperature is obtained. By virtue of relevant Kubo formulae, this allows to calculate hydrodynamic quantities for a strongly coupled field theory. This may be of relevance for describing the quark-gluon plasma. A particular virtue is the natural formulation in Minkowski space which allows for the description of non-equilibrium transport processes. A central result is the ratio of shear viscosity and entropy density, which provides a very small lower bound.
- **Integrability:** Another avenue of generalizing AdS/CFT in a different direction is to go beyond the Maldacena supergravity limit discussed in section 2.1, and to consider classical string configurations in $AdS_5 \times S^5$. The energy levels of these string modes are mapped to anomalous scaling dimensions of local operators in $\mathcal{N} = 4$ super Yang-Mills theory. Due to their integrability properties these operators may be described with the Bethe ansatz [12, 13]. Similar integrability methods have been used in QCD for some time [14]. Using classical string configurations, it is possible to test the AdS/CFT correspondence at higher loop order in a perturbative expansion on the field theory side. Relevant calculations on the field theory side have been performed for instance in [15].
- **Hard scattering:** In a similar approach, hard scattering of glueballs is mapped to string amplitudes in $AdS_5 \times S^5$. This provides in particular a unified description of the soft and hard pomeron [16]. Recently, a classical string configuration was proposed for gluon scattering itself [17].
- **AdS/QCD:** Whereas this review is devoted to the 'top-down' approach where string-theory models are developed fo describing field theory features, it should also be mentioned that there are important efforts in a 'bottom-up' approach of constructing phenomenological five-dimensional models for describing QCD phenomenology, for instance [18, 19]. These models are quite successful in describing QCD masses and decay constants. It remains an open question though about how to realize these models

within string theory.

3 Flavor and Chiral symmetry breaking

3.1 Adding Flavor

The original AdS/CFT correspondence only involves fields in the adjoint representation of the gauge group. For generalising the correspondence to quark degrees of freedom, which are in the fundamental representation of the gauge group, additional ingredients are necessary. The simplest way to obtain quark bilinear operators within gauge/gravity duality is to add a D7 brane probe [20]. This is done in such a way that the D7 brane probe extends in space-time as given in Table 1, where 0 is the time direction.

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		

Table 1: Embedding of the D7 brane probe into 9+1 dimensional space relatively to the D3 branes.

The term 'brane probe' refers to the fact that only a very small number of D7 branes is added, while the number of D3 branes, N, which also determines the rank of the gauge group $U(N)$, goes to infinity. In this limit we neglect the backreaction of the D7 branes on the geometry.

The field theory corresponding to this brane set-up is a $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory, which in addition to the degrees of freedom of $\mathcal{N} = 4$ Super Yang-Mills contains N_f hypermultiplets in the fundamental representation of the gauge group, where N_f is given by the number of D7 branes. N_f must be small in the probe limit.

On the supergravity side of the duality, the $\mathcal{N} = 4$ degrees of freedom are described by supergravity on $AdS_5 \times S^5$ as before. However in addition, there are new degrees of freedom corresponding to the D7 brane probe within the ten-dimensional curved space. The low-energy degrees of freedom of this brane are described by the Dirac-Born-Infeld action. These correspond to open string fluctuations on the D7 probe. It turns out that the minimum action configuration for the D7 brane probe corresponds to the probe wrapping an $AdS_5 \times S^3$ subspace of $AdS_5 \times S^5$.

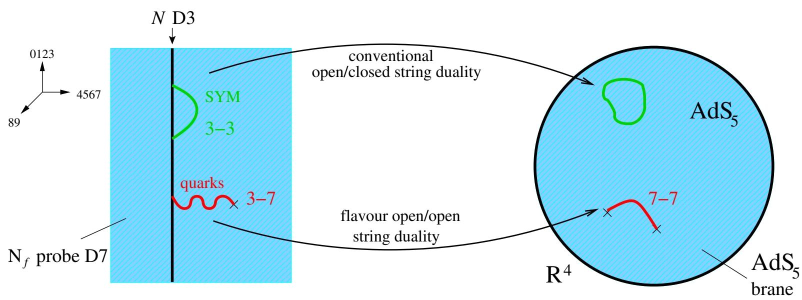

Figure 1: Schematic representation of the AdS/CFT duality with added flavour. In addition to the original AdS/CFT duality, open string degrees of freedom representing quarks are mapped to open strings beginning and ending on the D7 probe wrapping $AdS_5 \times S^3$ inside $AdS_5 \times S^5$. For simplicity, the five-sphere is not drawn in this picture.

The new duality conjectured in [20] is an open-open string duality, as opposed to the original AdS/CFT correspondence which is an open-closed string duality. The duality states that in addition to the original AdS/CFT duality, gauge invariant field theory operators involving fundamental fields are mapped to fluctuations of the D7 brane probe on $AdS_5 \times S^3$ within $AdS_5 \times S^5$. This is shown in Figure 1.

A particularly interesting feature arises if the $D7$ brane probe is separated from the stack of D3 branes in either the x_8 or x_9 directions, where the indices refer to the coordinates given in Table 1. This corresponds to giving a mass to the fundamental hypermultiplet. In this case the radius of the S^3 becomes a function of the radial coordinate r in AdS_5. At a radial distance from the deep interior of the AdS space given by the hypermultiplet mass, the radius of the S^3 shrinks to zero. From a five-dimensional AdS point of view, the D7 brane probe seems to 'end' at this value of the AdS radial coordinate.

The scalar mode of the D7 brane probe embedding with dimension $\Delta = 3$ (i.e. supergravity mass $m^2_{\rm sugra} = -3$) maps to the fermion bilinear $\tilde{\psi}\psi$ in the dual field theory. This mode corresponds to a imaginary AdS mass. However this mass is above the Breitenlohner-Freedman bound [21] for AdS_5 ($m^2_{\rm BF} = -4$) and thus guarantees stability. For this is important that the D7 branes do not carry any net charge from the five-dimensional point of view, since they wrap a topologically trivial cycle with zero flux.

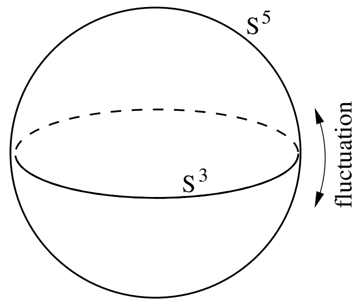

Figure 2: Fluctuations of the S^3 wrapped by the D7 probe inside S^5. These modes give rise to the meson masses.

Fluctuations of the D7 brane give rise to meson masses [22]. This is similar to previously studied supergravity fluctuations which give rise to glueball masses [23]. For this, fluctuations of the D7 brane probe of the form $\delta w(\rho, x) = f(\rho) sin(k \cdot x)$ are considered. Here ρ is the radial direction in the four-dimensional space spanned by the cartesian $4, 5, 6, 7$ directions (see Table 1), and x denote the coordinates on $3+1$-dimensional Minkowski space at the boundary of the five-dimensional Anti-de Sitter space. The meson masses are defined by $M^2 = -k^2$ for the wavevector k.

3.2 Chiral symmetry breaking

To obtain a gravity dual of spontaneous chiral symmetry breaking by a quark condensate [24], the addition of a D7 brane probe is combined with an appropriate deformation of the $AdS_5 \times S^5$ space. A suitable background is the one introduced by Constable-Myers [25]. This background has a single deformation parameter b, which may be related to $\Lambda_{\rm QCD}$ on the field theory side, and a singularity at $r = b$. On the field theory side, the $\mathcal{N} = 4$ Super-Yang-Mills theory is deformed to a non-supersymmetric, confining field theory, as can be seen by a Wilson loop analysis.

By adding a D7 brane probe to this background, we obtain a gravity dual of a confining $U(N)$ gauge theory with one flavor [24]. This theory has an $U(1)_{\rm A}$ axial symmetry. In the $N \to \infty$ limit, this symmetry is non-anomalous. It can thus be spontaneously broken by the quark condensate $\langle \bar{\psi}\psi \rangle$.

The $U(1)$ rotational invariance in the two space directions perpendicular to the D7 brane (see Table 1) corresponds to the field theory' s $U(1)_A$. The fluctuations in the two directions transverse to the probe are associated to the quark bilinear operator. The UV asymptotic behaviour of the embedding scalars is of the form $|w| \propto m\, e^{-r} + c\, e^{-3r}$. m and c fix the boundary conditions for the second order supergravity equations of motion.

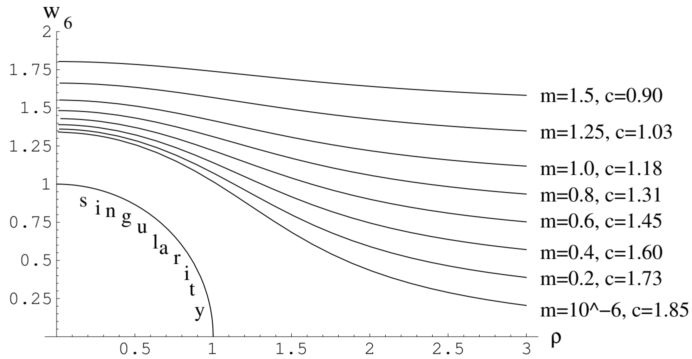

Figure 3: Regular solutions for D7 embeddings in the Constable-Myers background (Scale $b = 1$, $b \sim \Lambda_{\rm QCD}$). w_6 denotes one of the coordinates perpendicular to the D7 probe, ρ given by $r^2 = \rho^2 + {w_6}^2$ is related to the radial coordinate r which corresponds to an energy scale: In the IR r is small and in the UV r is large. The fact that the D7 branes bend corresponds to a geometric realization of spontaneous $U(1)$ symmetry breaking. From [24].

Following the standard AdS/CFT prescription (see Section 2.1 above), we associate the coefficient m with the quark mass and c with the quark condensate $\langle\bar{\psi}\psi\rangle$. Solutions with $m \neq 0$ explicitly breaks the $U(1)_A$ symmetry. The solution with $m = 0$, but $c \neq 0$ realizes spontaneous breaking of chiral symmetry.

Imposing the regularity of the solution in the IR selects a condensate c for each given quark mass m. Figure 3 shows regular solutions of the D7-brane equation of motion for different values of m. There are three important features of the solutions. The first is the presence of a screening effect: All regular solutions end before reaching the singularity. The presence of a condensate ($c \neq 0$) is essential for this behavior. Moreover we see a geometrical realization of the $U(1)_A$ spontaneous breaking, since for $m \to 0$ we still have $c \neq 0$. Finally, at large m we have $c \sim 1/m$, as expected from field theory.

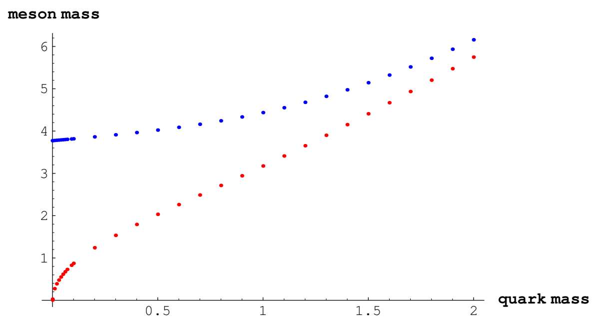

Figure 4: Meson mass versus quark mass as obtained from D7 probe brane fluctuations. The upper curve corresponds to the radial fluctuation, the bottom one to the $U(1)$ symmetric fluctuation. In units of $\Lambda_{\rm QCD}$. From [24].

Since there is spontaneous symmetry breaking for $m \to 0$, we expect a Goldstone boson in the meson spectrum. Solving the supergravity equation of motion for D7 probe brane fluctuations in the two directions transverse to probe, ($\delta w_5 = f(r)\sin(k{\cdot}x)\,, \quad \delta w_6 = h(r)\sin(k{\cdot}x)$) around the D7 brane probe embedding shown in Figure 3, the meson masses are given by $M^2 = -k^2$. There are indeed two distinct mesons (see Figure 4): One is massive for every m, and corresponds to fluctuations in the radial transverse direction, the other, corresponding to the $U(1)$ symmetric fluctuation, is massless for $m = 0$ and is thus a Goldstone boson. It may be identified with the η', which becomes a $U(1)_A$ Goldstone boson for $N \to \infty$. At finite N, pure stringy corrections will give the η' a non-zero mass in the gravity picture, similarly to instantons in the field theory dual [26].

Another important property of the model of [24] is the small quark mass behaviour of the meson mass, proportional to the square root of m, thus satisfying the Gell-Mann–Oakes–Renner relation [27] of chiral QCD. Also the linear asymptotics for large m correctly reproduce the field theory results.

The model has many remarkable QCD-like features. For instance heavy light mesons, involving a heavy and a light quark, have been studied in [28, 29]. Note however that in the UV, it flows again to strongly coupled $\mathcal{N} = 2$ theory with the degrees of freedom of $\mathcal{N} = 4$ theory plus one additional fundamental hypermultiplet, and thus is not asymptotically free.

A similar model based on a $D4$ brane background in which one of the space directions wrapped by the D4 branes is compacitified on a circle was studied in [30]. There the flavor degrees of freedom are provided by D6 and $\bar{\text{D}}6$ brane probes. Chiral symmetry breaking is seen in this model too. It has the advantage of not displaying a singularity in the interior of the curved space. On the hand, the dual gauge theory becomes five-dimensional in the UV.

3.3 $D4/D8/\bar{\text{D}}8$ brane model

In a further development of the model of [30], Sakai and Sugimoto [31] have considered a model of D4 branes with D8 and $\bar{\text{D}}8$ probes, distributed as shown in Table 2, with the 4-direction again compactified on a circle.

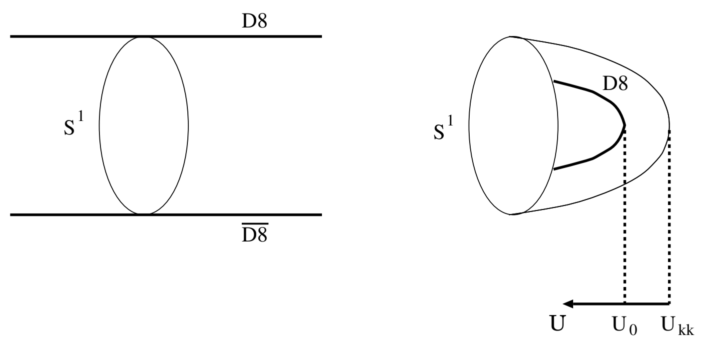

Figure 5: Brane configuration in the Sakai-Sugimoto model. From [31].

The brane configuration in the probe limit is given by N_c D4-branes compactified on a supersymmetry-breaking S^1 and N_f D8-$\bar{\text{D}}8$ pairs transverse to this S^1. Anti-periodic boundary conditions are imposed for the fermions on the D4-branes in order to break SUSY and to cause unwanted fields to become massive. The $U(N_f)_L \times U(N_f)_R$ chiral symmetry in QCD is realized as the gauge symmetry of the N_f D8-$\bar{\text{D}}8$ pairs. The existence of the compact direction plays a crucial role in obtaining a holographic picture of chiral symmetry breaking. The radial coordinate U transverse to the D4-branes is known to be bounded from below due to the existence of a horizon $U \geq U_{\text{KK}}$ in the supergravity background. As $U \to U_{\text{KK}}$, the radius of the S^1 shrinks to zero. It is found through the study of the D8 and $\bar{\text{D}}8$ probe brane action that the D8 and $\bar{\text{D}}8$ branes merge at some point $U = U_0$ to form a single component of the D8-branes, yielding, in general, a one-parameter family of solutions (See Fig. 5). On the resultant D8-brane, only a single factor of $U(N_f)$ survives. This mechanism is interpreted as the gravity dual of spontaneous breaking of $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

	0	1	2	3	(4)	5	6	7	8	9
D4	X	X	X	X	X					
D8 - $\bar{\text{D}}$8	X	X	X	X		X	X	X	X	X

Table 2: Embedding of the D8 and $\bar{\text{D}}$8 brane probes into 9+1 dimensional space relatively to the D4 branes.

This construction provides a model of $U(N)$ QCD with N_f massless flavors. It allows for the calculation of a number of observables, such as meson masses, which may be compared with experiment. A striking example is the ratio of the ρ and a_1 meson masses, for which Sakai and Sugimoto find

$$\frac{{M_\rho}^2}{{M_{a_1}}^2} = 2.4\,. \tag{4}$$

The experimental value according to the Review of Particle Physics [32] is 2.51. The theoretical result is strikingly close, though of course it has to be stressed that it is hard to estimate the error of the prediction.

4 Quark-gluon plasma

A particular feature of gauge/gravity dualities is that they are naturally formulated for field theories in Minkowski space. They are thus useful for describing non-equilibrium processes. In particular, the kinetic coefficients of hydrodynamics may be calculated for strongly coupled thermal field theory.

This is potentially of use for describing the quark-gluon plasma. Significant evidence for the presence of the quark-gluon plasma at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven was accumulated [a]. The quark-gluon plasma created at RHIC is not described by a weakly coupled gas of quarks and hadrons. Its temperature is approximately 170 MeV, which is close to the confinement scale of QCD, i.e. in the non-perturbative regime of QCD. A review of the relation between generalizations of AdS/CFT and hydrodynamics is found in [33].

It was noted already in [34] that $\mathcal{N} = 4$ Super Yang-Mills theory at finite temperature (where supersymmetry is broken) may be viewed as being dual to the AdS-Schwarzschild black hole background., A notable result in this context is the calculation of the ratio between shear viscosity η and entropy density s [35]. This gives

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}\,, \tag{5}$$

which provides a lower bound. The calculation is performed using the Kubo formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d^3x\, e^{i\omega t}\, \langle [T_{xy}(t, \mathbf{x}) T_{xy}(0,0)] \rangle\,. \tag{6}$$

The stress tensor correlator is obtained according to the standard AdS/CFT procedure from graviton propagation in AdS-Schwarzschild space.

[a] See talk by B. Zajc at DIS 2007.

Jet quenching, i.e. medium-induced modification of high-p_T parton fragmentation, may also be described within the gauge/gravity approach. There are several ansätze for calculating the jet quenching parameter, for instance one using a Wilson loop [36], and another one using the drag force on a heavy quark. This drag force is described by the force necessary to pull a string moving through the AdS-Schwarzschild background. The string begins on a D7 brane probe and extends to the black hole horizon. From this calculation, a jet quenching of

$$\frac{d}{dt}\langle(\vec{p}_\perp)^2\rangle = 2\pi\sqrt{\lambda}T^3 \tag{7}$$

is obtained, where the jet quenching parameter $\hat{q}$ is given by $d/dt\,\langle(\vec{p}_\perp)^2\rangle$ divided by the velocity v of the quark. – Recently, also the sonic boom expected to be present in the quark-gluon plasma has been explored using gauge/gravity duality [38].

The description of finite temperature field theories using gauge/gravity duality may be combined with the addition of flavor via brane probes. The embedding of a D7 brane probe into the AdS Schwarzschild black hole background gives rise to an interesting first order phase transition of geometrical nature, where the two phases are distinguished by whether or not the D7 brane probe reaches the black hole horizon [24, 39, 40]. If they do, the meson masses associated with the fluctuations of the brane probe become unstable. This may be interpreted as meson melting [41]. – Moreover, the presence of flavor branes give rise to an additional contribution to the shear viscosity relation (5) of the form [40] $\eta_{\text{fund}} \propto \lambda N N_f T^3$.

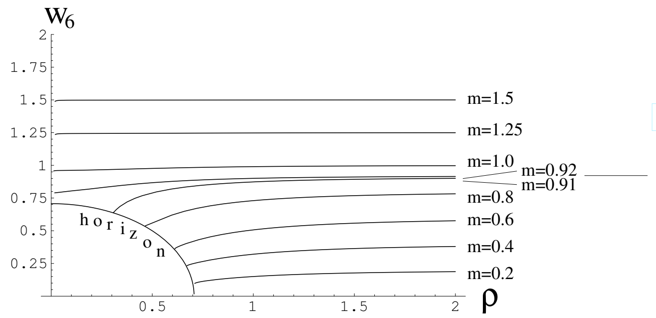

Figure 6: D7 brane probe embeddings in the AdS-Schwarzschild black hole background. There are two phases distinguished by whether or not the brane probes end on the black hole horizon. The phase transition is first order. w_6 is one of the two coordinates perpendicular to the D7 probe. ρ, given by $r^2 = \rho^2 + {w_6}^2$, is related to the the radial (energy) coordinate r and runs from IR (left) to UV (right). All dimensionful quantities are normalized to the temperature. From [24].

Transport processes in the presence of D7 brane probes have also been studied. Here, a chemical potential is introduced by giving a VEV to the time component of the gauge field on the D7 brane probe. In [42], an isospin chemical potential is introduced by considering the $SU(2)$ gauge field on two coincident D7 brane probes. This gives rise to a memory function and frequency-dependent diffusion. Transport processes in presence of a baryon chemical potential for the $U(1)$ symmetry on a single brane probe have been studied in detail in [43].

5 Hard scattering

In the context of hard scattering, the AdS/CFT has been used to give a unified description of the soft and hard pomeron. The pomeron is the coherent excitation that dominates hadronic elastic scattering in a large N gauge theory at large s and small t. At large N, the pomeron contributes the leading singularity in the angular momentum plane. In [16], the calculation of the field theory glueball scattering amplitude is calculated from the ten-dimensional string amplitude in $AdS_5 \times S^5$ with a cut-off in the AdS radial direction. Four-dimensional scattering is obtained from a coherent sum over the six transverse directions.

The holographic encoding of the gauge theory physics is central to this calculation, the AdS radial direction r corresponding to an energy scale. Low-energy states are mapped to states at small r, i.e. in the interior of AdS space. High-energy states are mapped to states at large r, ie. near the boundary. For the momenta there is the relation

$$p_\mu = \frac{r}{L}\tilde{p}_\mu , \tag{8}$$

where the conserved four-momentum p_μ corresponds to the invariance under translation of the boundary coordinates x^μ. $\tilde{p}_\mu$ is the momentum in local inertial coordinates for momenta localized at r. L is the AdS radius. The amplitudes depend on r,

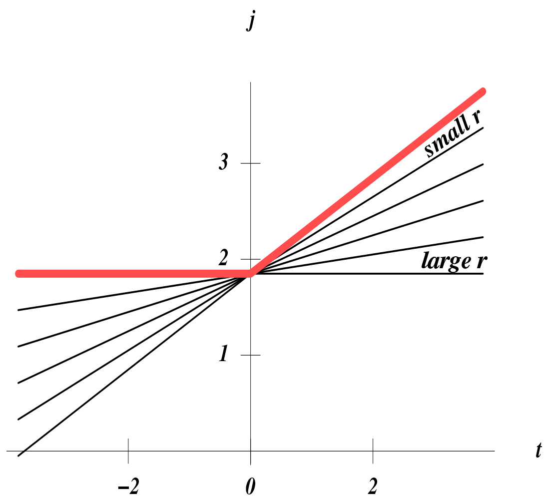

Figure 7: Unified description of soft and hard pomeron from [16].

$$\mathcal{A}(s,t) \propto s^{\alpha(t,r)} , \quad \alpha(t,r) = 2 + \alpha' L^2 \frac{t}{2r^2}. \tag{9}$$

This gives a unified description of soft and hard pomeron, as shown in Figure 7: At large s, highest trajectory dominates. For t positive, this is the case for r small. Thus there is a soft (Regge) pomeron, whose properties are determined by confining dynamics. This corresponds to a glueball. On the other hand, for t negative, the r large case gives the highest trajectory: There is a hard (BFKL) pomeron, i.e. a two-gluon perturbative small object.

6 Conclusion

The examples given show that the AdS/CFT correspondence and its gauge/gravity duality generalizations are useful tools for describing strongly-coupled gauge theories. They give remarkable agreement with QCD for instance as far as meson masses are concerned. Moreover they are particularly successful in making predictions for non-equilibrium processes in strongly coupled field theories.

The challenge which remains of course is to explain why the gauge/gravity dual description works so well, and to investigate whether it is possible to make progress towards understanding the microscopic dynamics in the field theory. A further important challenge is to use the recent results in gauge/gravity duality for gaining further insight into the structure of string theory itself.

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