Soft Gluon Resummation and a Novel Asymptotic Formula for Double-Spin Asymmetries in Dilepton Production at Small Transverse Momentum

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We discuss the double-spin asymmetries $\mathcal{A}_{TT}(Q_T)$ in transversely polarized Drell-Yan process at small transverse momentum Q_T of the produced dilepton. Soft gluon radiations relevant for small Q_T are resummed to all orders in α_s, up to the next-to-leading logarithmic accuracy. We show that the soft gluon contributions to polarized and unpolarized cross sections mostly cancel in the asymmetries, but significant corrections still remain. We propose a novel asymptotic formula for $\mathcal{A}_{TT}(Q_T)$ at small Q_T, which provides a new approach to extract the transversity $\delta q(x)$ from the experimental data.

Transversly polarized Drell-Yan (tDY) process, $p^\uparrow p^\uparrow \to l^+ l^- X$, is one of the processes where we can measure the chiral-odd transversity distributions, $\delta q(x)$. The NLO cross sections of tDY, with the transverse momentum Q_T of the final dilepton unobserved (integrated), has been studied at RHIC kinematics in [2], and it turned out that the corresponding double transverse-spin asymmetries A_{TT} are small because, at RHIC, the sea-quark distributions are probed at small partonic momentum fraction x. Here, we consider the double-spin asymmetries $\mathcal{A}_{TT}(Q_T)$ for the Q_T-observed case, especially at small Q_T, where the bulk of dileptons is produced. For Q_T smaller than the invariant mass Q of the dilepton, soft gluon emissions contributing as $\alpha_s^n \log^m(Q^2/Q_T^2)/Q_T^2$ $(m \leq 2n-1)$ bring dominant corrections in each order in α_s. We perform all-order resummation of them at the next-to-leading logarithmic (NLL) accuracy, i.e., of the LL $(m = 2n-1)$ and NLL $(m = 2n-2, 2n-3)$ terms. The parton distributions at the low scale Q_T can participate in $\mathcal{A}_{TT}(Q_T)$, while A_{TT} in [2] is determined solely by the distributions at the scale Q; $\mathcal{A}_{TT}(Q_T)$ may be larger than A_{TT}.

The spin-dependent part of the Q_T-differential cross section can be expressed as [3, 4]

$$\frac{\Delta_T d\sigma}{dQ^2 dQ_T^2 dy d\phi} = \cos(2\phi) \frac{\alpha^2}{3\, N_c\, S\, Q^2} \left[\Delta_T \tilde{X}^{\rm NLL}(Q_T^2, Q^2, y) + \Delta_T \tilde{Y}(Q_T^2, Q^2, y) \right], \qquad (1)$$

where $\sqrt{S}$ and y denote the energy of the the incoming protons and rapidity of dilepton in the proton-proton CM system, and ϕ is the azimuthal angle of one of the outgoing leptons with respect to the proton's spin axis. The first term $\Delta_T \tilde{X}^{\rm NLL}$ denotes the NLL resummed cross section which is given by the integral over the impact parameter b, according to the general formalism of Collins-Soper-Sterman [5] combined with varous kinds of elaboration [3, 4, 6]:

$$\Delta_T \tilde{X}^{\rm NLL}(Q_T^2, Q^2, y) = \int_{\mathcal{C}} db \frac{b}{2} J_0(bQ_T) e^{S(b,Q) - g_{NP} b^2} \left[\delta H \left(x_1^0, x_2^0; \frac{b_0^2}{b^2} \right) \right. \qquad (2)$$

$$\left. + \frac{\alpha_s(Q^2)}{2\pi} \left\{ \int_{x_1^0}^1 \frac{dz}{z} \Delta_T C_{qq}^{(1)}(z) \delta H \left(\frac{x_1^0}{z}, x_2^0; \frac{b_0^2}{b^2} \right) + \int_{x_2^0}^1 \frac{dz}{z} \Delta_T C_{qq}^{(1)}(z) \delta H \left(x_1^0, \frac{x_2^0}{z}; \frac{b_0^2}{b^2} \right) \right\} \right],$$

*Deceased.

where $x_{1,2}^0 = \sqrt{Q^2/S}e^{\pm y}$ is the Drell-Yan scaling variables, $J_0(bQ_T)$ is a Bessel function, $b_0 = 2e^{-\gamma_E}$ with the Euler's constant γ_E, and

$$\delta H(x_1, x_2; \mu^2) = \sum_q e_q^2 \left[\delta q(x_1, \mu^2)\delta\bar{q}(x_2, \mu^2) + \delta\bar{q}(x_1, \mu^2)\delta q(x_2, \mu^2)\right]. \quad (3)$$

Using $\lambda = \beta_0\alpha_s(Q^2)\log(Q^2b^2/b_0^2+1) \equiv \beta_0\alpha_s(Q^2)\tilde{L}$ with $\beta_0 = (11N_c-2N_f)/(12\pi)$, the large logarithmic corrections are resummed into the Sudakov factor $e^{S(b,Q)} = e^{h^{(0)}(\lambda)/\alpha_s(Q^2)+h^{(1)}(\lambda)}$, where $h^{(0)}(\lambda) = (A_q^{(1)}/2\pi\beta_0^2)[\lambda+\log(1-\lambda)]$ with $A_q^{(1)} = 2C_F = (N_c^2-1)/N_c$ collects the LL contributions, and $h^{(1)}(\lambda)$ corresponds to the NLL contributions; the explicit form of $h^{(1)}(\lambda)$, as well as another perturbatively calculable function $\Delta_T C_{qq}^{(1)}(z)$, is found in [3, 4]. $\tilde{L}$ plays a role of the large logarithmic expansion parameter in the b space, as $b \sim 1/Q_T$. We have introduced the Gaussian smearing factor $e^{-g_{NP}b^2}$ in (2), with a nonperturbative parameter g_{NP} [5], to take care of the long-distance behavior in the extremely large $|b|$ region. For the detail of elaboration of (2) beyond CSS, including the choice of the b-integration contour $\mathcal{C}$, see [3, 4]. The second term in (1), $\Delta_T\tilde{Y}$, is of $\mathcal{O}(\alpha_s)$, and does not contain the singular terms to be resummed, such as $\sim \log(Q^2/Q_T^2)/Q_T^2$ and $1/Q_T^2$; $\Delta_T\tilde{Y}$ is determined [3] such that the expansion of (1) to $\mathcal{O}(\alpha_s)$ reproduces the LO cross section for finite Q_T, which is of $\mathcal{O}(\alpha_s)$. Accordingly, we refer to (1) as the "NLL+LO" cross section. The NLL+LO cross section for unpolarized DY process is obtained similarly as (1), with $\tilde{X}^{\rm NLL}$ and $\tilde{Y}$ as the counterparts of $\Delta_T\tilde{X}^{\rm NLL}$ and $\Delta_T\tilde{Y}$, so that the NLL+LO asymmetry reads [4]

$$\mathcal{A}_{TT}(Q_T) = \frac{\cos(2\phi)}{2}\frac{\Delta_T\tilde{X}^{\rm NLL}(Q_T^2, Q^2, y) + \Delta_T\tilde{Y}(Q_T^2, Q^2, y)}{\tilde{X}^{\rm NLL}(Q_T^2, Q^2, y) + \tilde{Y}(Q_T^2, Q^2, y)}. \quad (4)$$

In the following Figs. 1 and 2, we show the asymmetries $\mathcal{A}_{TT}(Q_T)$ for $\phi = 0$, using a model of the NLO transversity distributions constructed as in [2], and $g_{NP} = 0.5$ GeV^2 for the nonperturbative parameter of (2). Figure 1 shows [4] the asymmetries at RHIC kinematics, $\sqrt{S} = 200$ GeV, $Q = 5$ GeV, and $y = 2$. The solid line shows the NLL+LO result (4), the dot-dashed line shows the NLL result $\mathcal{A}_{TT}^{\rm NLL}(Q_T)$, obtained by omitting $\Delta_T\tilde{Y}$ and $\tilde{Y}$ in (4), and the two-dot-dashed line shows the LL result $\mathcal{A}_{TT}^{\rm LL}(Q_T)$, which is obtained by retaining only the LL terms in $\mathcal{A}_{TT}^{\rm NLL}(Q_T)$:

$$\mathcal{A}_{TT}^{\rm LL}(Q_T) = \frac{\cos(2\phi)}{2}\frac{\delta H(x_1^0, x_2^0; Q^2)}{H(x_1^0, x_2^0; Q^2)}, \quad (5)$$

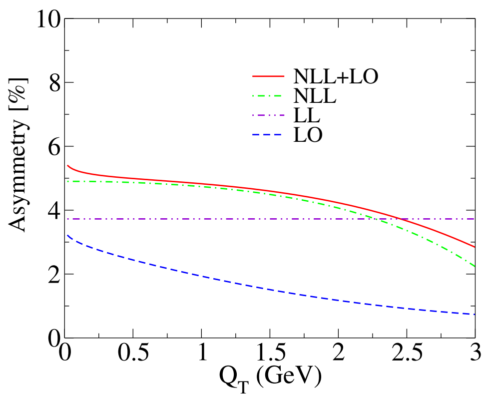

Figure 1: The asymmetries at RHIC, using $\sqrt{S} = 200$ GeV, $Q = 5$ GeV, $y = 2$ and $\phi = 0$.

where H is obtained from δH of (3) by replacing $\delta q(x, \mu^2)$ with the density distributions $q(x, \mu^2)$, and (5) is independent of Q_T [4]. The dashed line shows the LO asymmetry as the ratio of the LO cross sections. The NLL+LO result is flat and close to $\mathcal{A}_{TT}^{\rm NLL}(Q_T)$ in the small Q_T region around $Q_T \simeq 1$ GeV: in this region, the NLL+LO cross section (1) and the corresponding unpolarized one are dominated by the resummed contributions $\Delta_T\tilde{X}^{\rm NLL}$ and

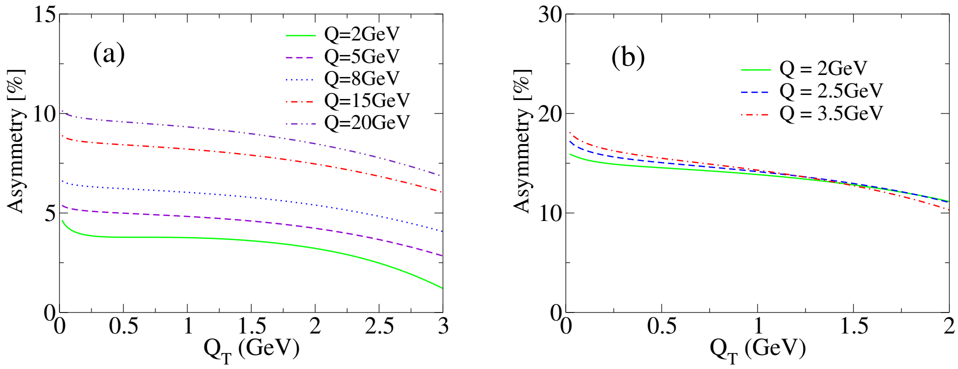

Figure 2: The NLL+LO asymmetries (4) for $\phi = 0$ at (a) RHIC kinematics with $\sqrt{S} = 200$ GeV and $y = 2$, and (b) J-PARC kinematics with $\sqrt{S} = 10$ GeV and $y = 0$.

$\tilde{X}^{\rm NLL}$, and form a well-developed peak [3, 4]; moreover, the Sudakov factor $e^{S(b,Q)}$ of (2) due to soft gluon resummation is universal up to the NLL level, so the dominant contributions cancel between $\Delta_T\tilde{X}^{\rm NLL}$ and $\tilde{X}^{\rm NLL}$ in $\mathcal{A}_{TT}^{\rm NLL}(Q_T)$. However, remarkably, some effects at the NLL level survive the cancellation, and raise $\mathcal{A}_{TT}^{\rm NLL}(Q_T)$ at small Q_T significantly compared with $\mathcal{A}_{TT}^{\rm LL}(Q_T)$ that coincides with the conventional asymmetry A_{TT} [2] using Q_T-integrated cross sections, up to the NLO ($\mathcal{O}(\alpha_s)$) corrections. On the other hand, the LO result is much smaller than the other asymmetries and decreases as Q_T increases, indicating that the soft gluon resummation is crucial for the prediction of the asymmetries.

The NLL+LO asymmetries $\mathcal{A}_{TT}(Q_T)$ of (4) at RHIC kinematics, $\sqrt{S} = 200$ GeV, $y = 2$, and various values of Q, are presented in Fig. 2 (a), which shows that $\mathcal{A}_{TT}(Q_T)$ increases for increasing Q. This Q dependence is a result of the small-x behavior of the relevant parton distributions, in particular, the steep rise of the unpolarized sea-distributions for small $x_{1,2}^0 = \sqrt{Q^2/S}e^{\pm y}$, which enhances the denominator of (4) for small Q. Figure 2 (b) is same as Fig. 2 (a), but for possible polarized pp experiment at J-PARC, $\sqrt{S} = 10$ GeV, $y = 0$, and $Q = 2, 2.5, 3.5$ GeV, where the distributions at moderate x are probed and $\mathcal{A}_{TT}(Q_T)$ at the flat region are around 15%, irrespective of the value of Q. We find [4] that all cases of Figs. 2 (a) and (b) in fact obey the similar mechanism as shown in Fig. 1, resulting in the values of $\mathcal{A}_{TT}(Q_T)$ larger by 20-30% than the corresponding NLO A_{TT}.

The NLL+LO asymmetry (4) in the flat region as in Figs. 1, 2 can be generically approximated, to good accuracy, as $\mathcal{A}_{TT}(Q_T) \approx \mathcal{A}_{TT}^{\rm NLL}(Q_T = 0)$, which is completely expressed by $\Delta_T\tilde{X}^{\rm NLL}$ and $\tilde{X}^{\rm NLL}$ at $Q_T = 0$. The b-integration in those quantities can be evaluated analytically by the saddle-point method: for (2) at $Q_T = 0$, we get [4]

$$\Delta_T\tilde{X}^{\rm NLL}(0, Q^2, y) = \left(\frac{b_0^2}{4Q^2\beta_0\alpha_s(Q^2)} \sqrt{\frac{2\pi}{\zeta''(\lambda_{SP})}} e^{-\zeta(\lambda_{SP}) + h^{(1)}(\lambda_{SP})} \right) \delta H\left(x_1^0, x_2^0;\ b_0^2/b_{SP}^2\right), \tag{6}$$

where $\zeta(\lambda) = -\lambda/(\beta_0\alpha_s(Q^2)) - h^{(0)}(\lambda)/\alpha_s(Q^2) + (g_{NP}b_0^2/Q^2)e^{\lambda/(\beta_0\alpha_s(Q^2))}$, and $b_{SP} = (b_0/Q)e^{\lambda_{SP}/(2\beta_0\alpha_s(Q^2))}$, with λ_{SP} satisfying $\zeta'(\lambda_{SP}) = 0$, i.e.,

$$1 - \frac{A_q^{(1)}}{2\pi\beta_0}\frac{\lambda_{SP}}{1-\lambda_{SP}} = \frac{g_{NP}b_0^2}{Q^2} e^{\frac{\lambda_{SP}}{\beta_0\alpha_s(Q^2)}}. \tag{7}$$

	$\sqrt{S} = 200$ GeV, $y = 2$					$\sqrt{S} = 10$ GeV, $y = 0$		
Q	2GeV	5GeV	8GeV	15GeV	20GeV	2GeV	2.5GeV	3.5GeV
SP-I	4.3%	5.4%	6.6%	8.7%	9.8%	14.1%	14.5%	14.8%
SP-II	7.3%	8.7%	9.8%	11.8%	12.7%	14.7%	14.8%	14.2%

Table 1: $\mathcal{A}_{TT}^{\rm NLL}(Q_T = 0)$ for $\phi = 0$ using the saddle-point formula (8).

Here (6) gives the saddle-point formula in the NLL accuracy, and corresponds to extension of that in the LL level in the literature [5]: the solution of (7) formally determines the saddle point at the LL level combined with the contribution due to the Gaussian factor $e^{-g_{NP}b^2}$ in (2), but we find [4] that the "shift" of the saddle point at the NLL level from λ_{SP} yields only the NNLL corrections to (6); note that the NNLL contributions are of $\mathcal{O}(\alpha_s)$, according to the counting of the relevant logarithms in the region $Q_T \approx 0$ (see also [5]). The saddle-point formula for $\tilde{X}^{\rm NLL}(0, Q^2, y)$ can be obtained similarly, and the result is given by the above result (6) with the replacement $\delta H(x_1^0, x_2^0;\ b_0^2/b_{SP}^2) \to H(x_1^0, x_2^0;\ b_0^2/b_{SP}^2)$. The common factor, in the parentheses of (6), involves "very large perturbative effects" due to the universal Sudakov factor, but this factor cancels out for the asymmetry. We get [4]

$$\mathcal{A}_{TT}^{\rm NLL}(Q_T = 0) = \frac{\cos(2\phi)}{2} \frac{\delta H\left(x_1^0, x_2^0;\ b_0^2/b_{SP}^2\right)}{H\left(x_1^0, x_2^0;\ b_0^2/b_{SP}^2\right)}\ , \tag{8}$$

which is exact up to the NNLL ($\mathcal{O}(\alpha_s)$) corrections for $Q_T \approx 0$. This clarifies the mechanism discussed in Fig. 1: the contributions surviving the cancellation in (8) are entirely absorbed into the unconventional scale b_0/b_{SP} for the relevant distribution functions. Compared with (5), participation of the new scale b_0/b_{SP} is the effect at the NLL level, and, remarkably, b_0/b_{SP} using (7) depends weakly on Q, as $b_0/b_{SP} \simeq 1$ GeV for all cases in Figs. 1 and 2 [4]. This explains why $\mathcal{A}_{TT}(Q_T)$ at small Q_T is always larger than (5), or the NLO A_{TT} in [2]. Also $\mathcal{A}_{TT}^{\rm NLL}(Q_T)$ does not approach to $\mathcal{A}_{TT}^{\rm LL}(Q_T)$ even in the $Q \to \infty$ limit.

In Table 1, both "SP-I" and "SP-II" show $\mathcal{A}_{TT}^{\rm NLL}(Q_T = 0)$ using (8) with (7), but these two cases differ by the contributions at the NNLL level, reflecting mismatch to classify the terms between NLL and NLO (for the detail of SP-I, II, see [4]). SP-I reproduces $\mathcal{A}_{TT}(Q_T = 0)$ in the flat region in Fig. 2 to the 10% accuracy, i.e., to the canonical size of $\mathcal{O}(\alpha_s)$ corrections associated with the NLL accuracy. However, SP-II overestimates for RHIC, demonstrating that certain NNLL corrections would grow at the small-x region, the edge region of the phase space, beyond the canonical size. To this accuracy, our simple formula (8) is applicable in order to extract the NLO transversity distributions directly from the data.

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