Evolution Equations for Di-hadron Fragmentation Functions

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Di-hadron Fragmentation Functions describe the probability that a quark hadronizes into two hadrons plus anything else, i.e. the process $q \to h_1 \, h_2 \, X$. Via a suitable single-spin asymmetry in semi-inclusive deep inelastic scattering (SIDIS), they can be used to extract the quark transversity distribution h_1 in the nucleon, a missing cornerstone of the nucleon partonic spin structure. I will discuss their evolution equations when they are explicitly depending on the invariant mass of the two hadrons. The equations are necessary to connect two-particle-inclusive measurements at different energies.

1 Introduction

Di-hadron Fragmentation Functions (DiFF) have been introduced for the first time in the context of $e^+e^- \to h_1 h_2 X$ reaction [2], and later have been recognized to be necessary in order to guarantee factorization of all collinear singularities [3]. However, in all these studies DiFF were always considered as functions only of the energy fractions z_1 and z_2 delivered to the two hadrons, while most of the experimental information consists of their invariant mass distribution M_h. Also the twist analysis of the quark-quark correlator for two-hadron inclusive production (see Fig. 1) indicates that the extracted DiFF are in general functions also of the pair relative momentum $R = P_1 - P_2$ [4], whose transverse spatial component $\mathbf{R}_T$ is related to M_h [5]. In this case, I will refer to the socalled extended DiFF (extDiFF).

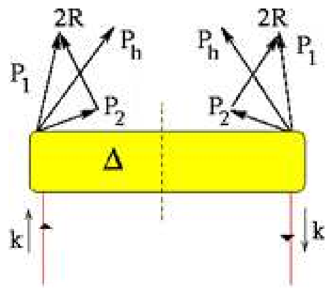

Figure 1: The quark-quark correlator for fragmentation in two hadrons.

ExtDiFF can act as spin analyzers of the fragmenting quark; in particular the transverse polarization $\mathbf{s}_T$ of the latter can be related to the azimuthal orientation of the plane containing $\mathbf{P}_1$ and $\mathbf{P}_2$ via the mixed product $\mathbf{s}_T \cdot \mathbf{P}_1 \times \mathbf{P}_2$ (see Fig. 2). The strength of this effect is described by the extDiFF $H_1^{\triangleleft}$. In SIDIS on transversely polarized targets, this function appears in combination with the transversity function, a leading-twist partonic distribution yet undetermined. The unknown extDiFF can be extracted from e^+e^- annihilations in two hadron pairs [6].

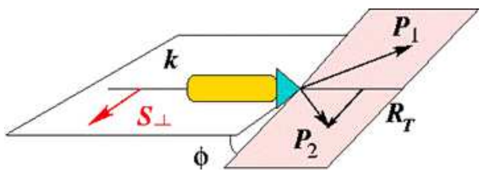

Figure 2: The nonperturbative effect $\mathbf{s}_T \cdot \mathbf{P}_1 \times \mathbf{P}_2$ generating the single-spin asymmetry.

The HERMES and COMPASS collaborations have recently reported preliminary measurements of the asymmetry induced by the $\mathbf{s}_T \cdot \mathbf{P}_1 \times \mathbf{P}_2$ effect at the average scale $\langle Q^2 \rangle = 2.53$

GeV^2 [7, 8]. The BELLE collaboration is planning to extract $H_1^{\triangleleft}$ in e^+e^- annihilation but at the higher scale $s \approx 100$ GeV^2 [9]. In this talk I will discuss the evolution equations for extDiFF.

2 Evolution equations for DiFF

As already anticipated in Sec. 1, DiFF are necessary to get a finite cross section for the $e^+e^- \to h_1 h_2 X$ process at NLO order [3]. The reason relies in the indistinguishability of the two mechanisms depicted in Fig. 3, which both lead to the observed hadron pair, either through DiFF or through separate single-hadron fragmentations after a partonic branching occurring at an arbitrary scale, intermediate between the hard Q^2 one and the soft Q_0^2 one. The consequence is the appearance of an inhomogenous term in the evolution equations for DiFF [3].

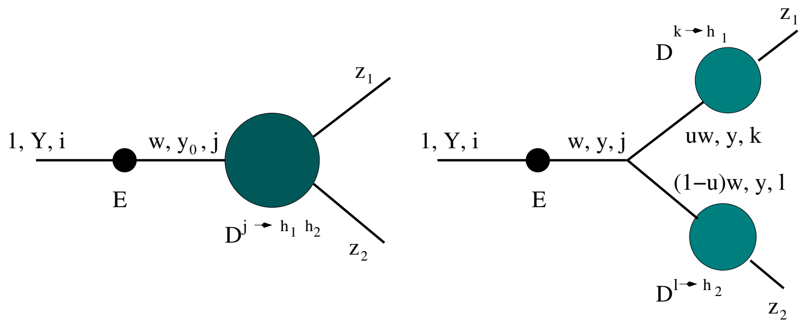

Figure 3: Double- and single-hadron fragmentations; the momentum fractions are indicated along with the scales and the parton indices; the black dots represent the parton evolution function E (see text).

Making use of the techniques of jet calculus [2], the result of Ref. [3] can be easily reproduced when the two hadrons are emitted close in phase space (*i.e.*, inside the same jet) and wide-angle hard partons are neglected. The phase-space structure of collinear singularities singled out in the fixed-order calculation of Ref. [3] can be translated in jet calculus as a degeneracy in all possible competing mechanisms that could realize the desired final state.

It is convenient to introduce the evolution variable

$$y = \frac{1}{2\pi\beta} \ln \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right] , \tag{1}$$

between some two arbitrary hard Q^2 and soft Q_0^2 scales. If working at Leading Log Approximation (LLA), α_s and β are the usual strong coupling constant and β function, both at one loop. We can introduce also the parton evolution function $E_j^i(x, y)$, which expresses the probability of finding a parton j at scale Q_0^2 with a momentum fraction x of the parent parton i at scale Q^2. It satisfies standard DGLAP evolution equations [2] and can be shown to resum all collinear leading logarithms of the kind $\alpha_s^n \ln^n(Q^2/Q_0^2)$ [10]. The evolution variable Y corresponding to the initial hard scale Q^2 is not zero, as one could deduce from Eq. 1, but can be defined by replacing Q_0^2 with the renormalization scale μ_R^2 in Eq. 1 itself.

In this picture, the fragmentation process of Fig. 3 is described by

$$\begin{aligned}\frac{1}{\sigma_{\rm jet}}\frac{d\sigma^{i\to h_1h_2}}{dz_1dz_2dY} &= \int_{z_1+z_2}^{1}\frac{dw}{w^2}\,E_j^i(w,Y-y_0)\,D^{j\to h_1h_2}\left(\frac{z_1}{w},\frac{z_2}{w},y_0\right) \\ &+\int_{y_0}^{Y}dy\int_{z_1+z_2}^{1}\frac{dw}{w^2}\int_{\frac{z_1}{w}}^{1-\frac{z_2}{w}}\frac{du}{u(1-u)}\,E_j^i(w,Y-y)\,\hat{P}_{kl}^j(u) \\ &\qquad\times D^{k\to h_1}\left(\frac{z_1}{wu},y\right)\,D^{l\to h_2}\left(\frac{z_2}{w(1-u)},y\right)\ , \end{aligned} \quad (2)$$

where $\hat{P}$ are the usual real Altarelli-Parisi splitting functions, $D^{i\to h}$ are single-hadron fragmentation functions, and $D^{i\to h_1h_2}$ are DiFF. Taking the derivative d/dY of Eq. 2, and further transforming the dependence upon Y back to the one on Q^2, it easy to recover the inhomogeneous evolution equation for $D^{i\to h_1h_2}$ in the jet calculus language [10].

3 Evolution equations for extDiFF

The difference between extDiFF and DiFF is the explicit dependence of the former upon the transverse component of the hadron pair relative momentum, $\mathbf{R}_T$, or, equivalently, upon their invariant mass M_h through the relation [5, 10]

$$R_T^2=\frac{z_1z_2}{z_1+z_2}\left[\frac{M_h^2}{z_1+z_2}-\frac{M_1^2}{z_1}-\frac{M_2^2}{z_2}\right]\ . \quad (3)$$

Knowledge of the R_T^2 scale makes the scale of the partonic branching no longer arbitrary. In fact, at LLA the virtualities of the involved partons are related by [10]

$$k_j^2=\frac{k_k^2}{u}+\frac{k_l^2}{1-u}+\frac{r_T^2}{u(1-u)}\approx r_T^2\approx R_T^2\ , \quad (4)$$

where r_T^2 is the relative momentum of the partons k and l carrying momentum fractions u and $1-u$ of the parent parton j, respectively.

Consequently, the arbitrary intermediate scale y appearing in the second term of Eq. 2 collapses to y_T, defined similarly to Y but with the replacement $Q^2\leftrightarrow R_T^2$. The analogue of Eq. 2 for extDiFF becomes, therefore,

$$\begin{aligned}\frac{1}{\sigma_{\rm jet}}\frac{d\sigma^{i\to h_1h_2}}{dz_1dz_2dR_T^2dY} &= \int_{z_1+z_2}^{1}\frac{dw}{w^2}\,E_j^i(w,Y-y_0)\,D^{j\to h_1h_2}\left(\frac{z_1}{w},\frac{z_2}{w},R_T^2,y_0\right) \\ &+\frac{\alpha_s(R_T^2)}{2\pi R_T^2}\int_{z_1+z_2}^{1}\frac{dw}{w^2}\int_{\frac{z_1}{w}}^{1-\frac{z_2}{w}}\frac{du}{u(1-u)}\,E_j^i(w,Y-y_T)\,\hat{P}_{kl}^j(u) \\ &\qquad\times D^{k\to h_1}\left(\frac{z_1}{wu},y_T\right)\,D^{l\to h_2}\left(\frac{z_2}{w(1-u)},y_T\right)\ . \end{aligned} \quad (5)$$

Taking the derivative d/dY of the previous expression, and further transforming back to the usual Q^2, we finally get [10]

$$\frac{d}{d\ln Q^2}D^{i\to h_1h_2}(z_1,z_2,R_T^2,Q^2)=\frac{\alpha_s(Q^2)}{2\pi}\int_{z_1+z_2}^{1}\frac{du}{u^2}\,D^{j\to h_1h_2}\left(\frac{z_1}{u},\frac{z_2}{u},R_T^2,Q^2\right)\,P_{ji}(u)\ , \quad (6)$$

where P_{ji} are the complete Altarelli-Parisi splitting functions, including the virtual contributions.

The same result holds also for the polarized fragmentation function $H_1^{\sphericalangle}$, provided that the splitting kernels δP for transversely polarized partons are used [10]. Equation 6 can also be conveniently diagonalized using a double Mellin transformation [10].

On the basis of Eq. 6, we argue that the cross section at NLO order for the inclusive production of two hadrons h_1 and h_2, inside the same jet and with invariant mass M_h, can be expressed in the factorized form

$$\frac{d\sigma^{h_1 h_2}}{dz_1 dz_2 dR_T^2 dQ^2} = \sum_i \sigma^i \otimes D^{i \to h_1 h_2}(R_T^2, Q^2) \; , \tag{7}$$

where σ^i are the same coefficient functions found in the case of single-hadron inclusive production.

4 Acknowledgments

I warmly thank F.A. Ceccopieri and A. Bacchetta, without whom this work would not have been possible.

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