Evolution Equations for Di-hadron Fragmentation Functions
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Di-hadron Fragmentation Functions describe the probability that a quark hadronizes into two hadrons plus anything else, i.e. the process $q \rightarrow h_1 h_2 X$. Via a suitable single-spin asymmetry in semi-inclusive deep inelastic scattering (SIDIS), they can be used to extract the quark transversity distribution $h_1$ in the nucleon, a missing cornerstone of the nucleon partonic spin structure. I will discuss their evolution equations when they are explicitly depending on the invariant mass of the two hadrons. The equations are necessary to connect two-particle-inclusive measurements at different energies.

1 Introduction

Di-hadron Fragmentation Functions (DiFF) have been introduced for the first time in the context of $e^+e^- \rightarrow h_1 h_2 X$ reaction [2], and later have been recognized to be necessary in order to guarantee factorization of all collinear singularities [3]. However, in all these studies DiFF were always considered as functions only of the energy fractions $z_1$ and $z_2$ delivered to the two hadrons, while most of the experimental information consists of their invariant mass distribution $M_h$. Also the twist analysis of the quark-quark correlator for two-hadron inclusive production (see Fig. 1) indicates that the extracted DiFF are in general functions also of the pair relative momentum $R = P_1 - P_2$ [4], whose transverse spatial component $R_T$ is related to $M_h$ [5]. In this case, I will refer to the so-called extended DiFF (extDiFF).

ExtDiFF can act as spin analyzers of the fragmenting quark; in particular the transverse polarization $s_T$ of the latter can be related to the azimuthal orientation of the plane containing $P_1$ and $P_2$ via the mixed product $s_T \cdot P_1 \times P_2$ (see Fig. 2). The strength of this effect is described by the extDiFF $H_T^{s_T}$. In SIDIS on transversely polarized targets, this function appears in combination with the transversity function, a leading-twist partonic distribution yet undetermined. The unknown extDiFF can be extracted from $e^+e^-$ annihilations in two hadron pairs [6].

The HERMES and COMPASS collaborations have recently reported preliminary measurements of the asymmetry induced by the $s_T \cdot P_1 \times P_2$ effect at the average scale $<Q^2> = 2.53$
The BELLE collaboration is planning to extract $H^<\gamma$ in $e^+e^-$ annihilation but at the higher scale $s \approx 100$ GeV$^2$ [9]. In this talk I will discuss the evolution equations for extDiFF.

## 2 Evolution equations for DiFF

As already anticipated in Sec. 1, DiFF are necessary to get a finite cross section for the $e^+e^- \rightarrow h_1 h_2 X$ process at NLO order [3]. The reason relies in the indistinguishability of the two mechanisms depicted in Fig. 3, which both lead to the observed hadron pair, either through DiFF or through separate single-hadron fragmentations after a partonic branching occurring at an arbitrary scale, intermediate between the hard $Q^2$ one and the soft $Q^2_0$ one. The consequence is the appearance of an inhomogenous term in the evolution equations for DiFF [3].

![Figure 3: Double- and single-hadron fragmentations](image)

Making use of the techniques of jet calculus [2], the result of Ref. [3] can be easily reproduced when the two hadrons are emitted close in phase space (i.e., inside the same jet) and wide-angle hard partons are neglected. The phase-space structure of collinear singularities singled out in the fixed-order calculation of Ref. [3] can be translated in jet calculus as a degeneracy in all possible competing mechanisms that could realize the desired final state.

It is convenient to introduce the evolution variable

$$y = \frac{1}{2\pi\beta} \ln \left[ \frac{\alpha_s(Q^2_0)}{\alpha_s(Q^2)} \right],$$

between some two arbitrary hard $Q^2$ and soft $Q^2_0$ scales. If working at Leading Log Approximation (LLA), $\alpha_s$ and $\beta$ are the usual strong coupling constant and $\beta$ function, both at one loop. We can introduce also the parton evolution function $E_i(x, y, z)$, which expresses the probability of finding a parton $j$ at scale $Q^2_0$ with a momentum fraction $x$ of the parent parton $i$ at scale $Q^2$. It satisfies standard DGLAP evolution equations [2] and can be shown to resum all collinear leading logarithms of the kind $\alpha_s^n \ln^n(Q^2/Q^2_0)$ [10]. The evolution variable $Y$ corresponding to the initial hard scale $Q^2$ is not zero, as one could deduce from Eq. 1, but can be defined by replacing $Q^2_0$ with the renormalization scale $\mu^2_R$ in Eq. 1 itself.
In this picture, the fragmentation process of Fig. 3 is described by

\[
\frac{1}{\sigma_{\text{jet}}} \frac{d\sigma^{i \rightarrow h_1 h_2}}{dz_1 dz_2 dY} = \int_{z_1 + z_2}^1 \frac{dw}{w^2} E_j^i(w, Y - y_0) D_j^{i \rightarrow h_1 h_2} \left( \frac{z_1}{w}, \frac{z_2}{w}, y_0 \right) \\
+ \int_{y_0}^Y dy \int_{z_1 + z_2}^1 \frac{dw}{w^2} \int_{z_1}^{1-z_2} \frac{du}{u(1-u)} E_j^i(w, Y - y) \hat{P}_{kl}(u) \\
\times D^{k \rightarrow h_1} \left( \frac{z_1}{wu}, y \right) D^{l \rightarrow h_2} \left( \frac{z_2}{wu(1-u)}, y \right),
\]

where \( \hat{P} \) are the usual real Altarelli-Parisi splitting functions, \( D_i^{i \rightarrow h} \) are single-hadron fragmentation functions, and \( D_i^{i \rightarrow h_1 h_2} \) are DiFF. Taking the derivative \( d/dY \) of Eq. 2, and further transforming the dependence upon \( Y \) back to the one on \( Q^2 \), it easy to recover the inhomogeneous evolution equation for \( D_i^{i \rightarrow h_1 h_2} \) in the jet calculus language [10].

3 Evolution equations for extDiFF

The difference between extDiFF and DiFF is the explicit dependence of the former upon the transverse component of the hadron pair relative momentum, \( R_T \), or, equivalently, upon their invariant mass \( M_0 \) through the relation [5, 10]

\[
R_T^2 = \frac{z_1 z_2}{z_1 + z_2} \left[ \frac{M_0^2}{z_1 + z_2} - \frac{M_0^2}{z_1} - \frac{M_0^2}{z_2} \right].
\]

Knowledge of the \( R_T^2 \) scale makes the scale of the partonic branching no longer arbitrary. In fact, at LLA the virtualities of the involved partons are related by [10]

\[
k_j^2 = \frac{k_j^2}{u} + \frac{k_j^2}{1-u} + \frac{r_j^2}{u(1-u)} \approx r_j^2 \approx R_T^2, \tag{4}
\]

where \( r_j^2 \) is the relative momentum of the partons \( k \) and \( l \) carrying momentum fractions \( u \) and \( 1-u \) of the parent parton \( j \), respectively.

Consequently, the arbitrary intermediate scale \( y \) appearing in the second term of Eq. 2 collapses to \( y_T \), defined similarly to \( Y \) but with the replacement \( Q^2 \leftrightarrow R_T^2 \). The analogue of Eq. 2 for extDiFF becomes, therefore,

\[
\frac{1}{\sigma_{\text{jet}}} \frac{d\sigma^{i \rightarrow h_1 h_2}}{dz_1 dz_2 dR_T^2 dY} = \int_{z_1 + z_2}^1 \frac{dw}{w^2} E_j^i(w, Y - y_0) D_j^{i \rightarrow h_1 h_2} \left( \frac{z_1}{w}, \frac{z_2}{w}, R_T^2, y_0 \right) \\
+ \frac{\alpha_s(R_T^2)}{2\pi R_T^2} \int_{z_1 + z_2}^1 \frac{dw}{w^2} \int_{z_1}^{1-z_2} \frac{du}{u(1-u)} E_j^i(w, Y - y_T) \hat{P}_{kl}(u) \\
\times D^{k \rightarrow h_1} \left( \frac{z_1}{wu}, y_T \right) D^{l \rightarrow h_2} \left( \frac{z_2}{wu(1-u)}, y_T \right). \tag{5}
\]

Taking the derivative \( d/dY \) of the previous expression, and further transforming back to the usual \( Q^2 \), we finally get [10]

\[
\frac{d}{d\ln Q^2} D_j^{i \rightarrow h_1 h_2}(z_1, z_2, R_T^2, Q^2) = \frac{\alpha_s(\ln Q^2)}{2\pi} \left( \frac{z_1}{u}, \frac{z_2}{u}, R_T^2, Q^2 \right) P_{kl}(u), \tag{6}
\]

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where $P_{ji}$ are the complete Altarelli-Parisi splitting functions, including the virtual contributions.

The same result holds also for the polarized fragmentation function $H_{1}^{<}$, provided that the splitting kernels $\delta P$ for transversely polarized partons are used [10]. Equation 6 can also be conveniently diagonalized using a double Mellin transformation [10].

On the basis of Eq. 6, we argue that the cross section at NLO order for the inclusive production of two hadrons $h_{1}$ and $h_{2}$, inside the same jet and with invariant mass $M_{h}$, can be expressed in the factorized form

$$\frac{d\sigma^{h_{1}h_{2}}}{dz_{1}dz_{2}dR_{T}^{2}dQ^{2}} = \sum_{i}^{\sigma_{i}} \otimes D^{i-h_{1}h_{2}}(R_{T}^{2}, Q^{2}), \quad (7)$$

where $\sigma_{i}$ are the same coefficient functions found in the case of single-hadron inclusive production.

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References

[1] Slides: http://indico.cern.ch/contributionDisplay.py?contribId=167&sessionId=4&confId=9499


