On Diffraction and JIMWLK Evolution

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A systematic approach towards description of semi-inclusive processes at low x and with multiple rescatterings taken into account is highlighted. We discuss diffractive processes and their evolution with respect to relevant rapidity intervals.

This talk is based on Ref. [2].

• We develop a general formalism to address semi-inclusive processes at high energies and including multiple rescatterings. Part of formalism is independent of underlying high energy evolution. However, most of applications considered are within DIS framework and assume BK-JIMWLK evolution.
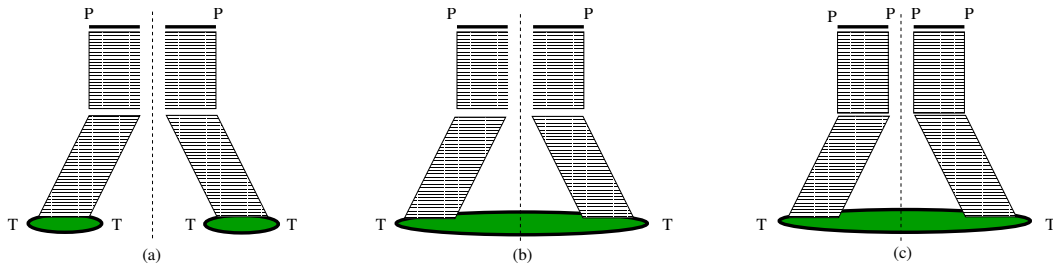

Figure 1: Diffraction and fan diagrams

• We apply our general formalism to High energy diffractive processes. We attempt to derive results not relying on the dipole (large N_c and target factorization) approximation. We reproduce and extend the result of Ref. [3] for the process of projectile diffraction with target scattered elastically. We also obtain results for projectile diffraction with target diffracting in a small rapidity interval and elastic scattering.

• We consider high energy diffraction with multiple gaps. For various diffractive processes we derive evolution equations with respect to total rapidity and gap(s).
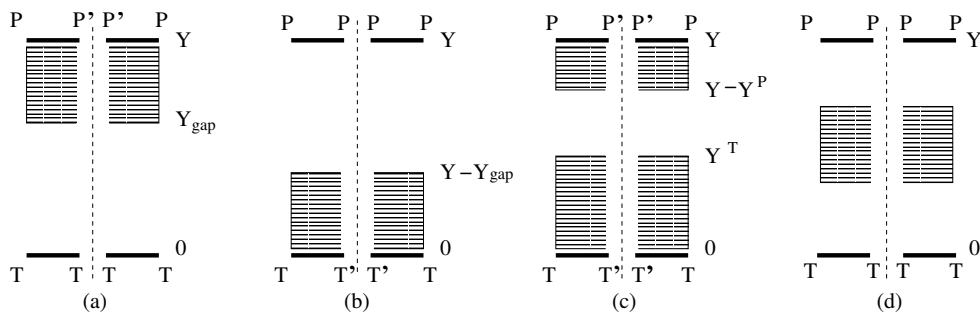

Figure 2: Various types of diffractive processes

Let me flash the formalism which is based on the evolution of hadronic wavefunction. Hadron wave function in the gluon Fock space is

$$|\Psi\rangle \;=\; \Psi[a_i^{\dagger a}(x)]|0\rangle \qquad\qquad |\Psi\rangle \;=\; |v\rangle$$

After rapidity evolution the evolved wave function becomes

$$|\Psi_{in}\rangle \;=\; \Omega_Y(\rho,\,a)\,|v\rangle\,; \qquad\qquad |v\rangle \;=\; |v\rangle \otimes |0_a\rangle$$

Here Ω is the most general evolution kernel. It is known for arbitrary dense hadron [4]. We will however concentrate on the most simple case of dilute hadron. In this limit, Ω reduces to the gluon cloud operator

$$C_Y \;\equiv\; \Omega_Y(\rho \to 0) \;=\; \mathrm{Exp}\left[i \int d^2z\, b_i^a(z) \int_\Lambda^{e^Y\,\Lambda} \frac{dk^+}{\pi^{1/2}|k^+|^{1/2}} \left[a_i^a(k^+,z) + a_i^{\dagger a}(k^+,z))\right]\right].$$

with the classical WW field

$$b_i^a(z) \;=\; \frac{g}{2\pi}\ \int d^2x \frac{(z-x)_i}{(z-x)^2}\ \rho^a(x)$$

The projectile's gluon scattering of a dense target has the eikonal propagator given by the Wilson line

$$S(x) \;=\; \mathcal{P}\,\exp\left\{i \int dx^-\, T^a\, \mathbf{A}_t^a(x, x^-)\right\}.$$

with A_t characterizing the target external field. The evolution of the diagonal element of the S-matrix operator $\Sigma^P \equiv \langle \Psi_{out}|\Psi_{in}\rangle$ reads

$$\partial_Y\, \Sigma^P \;=\; -\,H^{JIMWLK}\, \Sigma^P\,; \qquad\qquad H^{JIMWLK} \;=\; \int_z Q_i^a(z)\, Q_i^a(z)$$

where we introduced the gluon production (and scattering) amplitude

$$Q_i^a(z) \;=\; g\, \int_x \frac{(x-z)_i}{(x-z)^2}\, \left[J_L^a(x) - S^{ab}(z)\, J_R^b(x)\right]$$

The generators of the left/right color rotations are Lie derivatives

$$J_R^a(x) \;=\; -\,\mathrm{tr}\left\{S(x)\, T^a\, \frac{\delta}{\delta S^\dagger(x)}\right\}, \qquad\qquad J_L^a(x) \;=\; -\,\mathrm{tr}\left\{T^a\, S(x)\, \frac{\delta}{\delta S^\dagger(x)}\right\}$$

This operator is visualized in Fig. 3.

Now we turn to discussion of semi-inclusive reactions. The system emerges from the collision at $t = 0$ and keeps evolving to the asymptotic time $t \to +\infty$, at which point the measurement of an observable $\hat{\mathcal{O}}$ is made

$$\langle \hat{\mathcal{O}} \rangle \;=\; \langle v|\Omega_Y^\dagger\, (1 - \hat{S}^\dagger)\, \Omega_Y\ \hat{\mathcal{O}}\ \Omega_Y^\dagger\, (1 - \hat{S})\, \Omega_Y\, |v\rangle$$

We find it convenient to introduce two targets - one for the amplitude S and another one for its conjugate $\bar{S}$. In the end of our computation we set $S = \bar{S}$.

$$\mathcal{O}_Y[S, \bar{S}] \;=\; \langle\, P_v|\, \Omega_Y^\dagger\, (1 - \hat{S}^\dagger)\, \Omega_Y\ \hat{\mathcal{O}}\ \Omega_Y^\dagger\, (1 - \hat{\bar{S}})\, \Omega_Y\, |P_v\, \rangle$$
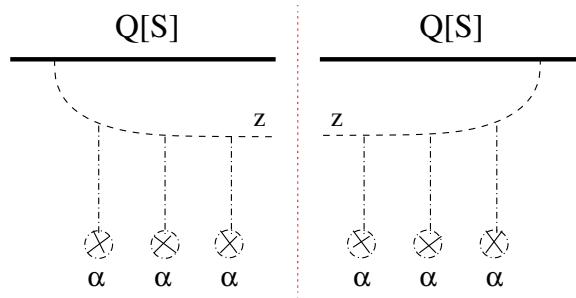
Figure 3: The operator Q

High energy evolution of the observable is given by the following equation

$$\frac{d\mathcal{O}_Y[S,\bar{S}]}{dY} \; = \; \lim_{\Delta y \to 0} \frac{\mathcal{O}_{Y+\Delta y}[S,\bar{S}] \; - \; \mathcal{O}_Y[S,\bar{S}]}{\Delta y} \; = \; -H_3[S,\bar{S}] \; \mathcal{O}_Y[S,\bar{S}]$$

Here the Hamiltonian H_3 (first introduced in [5])

$$H_3[S,\bar{S}] \; \equiv \; H_1[S] \; + \; H_1[\bar{S}] \; + \; 2 \int_z Q_i^a(z,[S]) \, Q_i^a(z,[\bar{S}])$$

$$H_1[S] \; \equiv \; H^{JIMWLK}[S] \; = \; \int_z Q_i^a(z,[S]) \, Q_i^a(z,[S]), \qquad\qquad H_2[S,\bar{S}] \; \equiv \; H_1[S] + H_1[\bar{S}]$$

is illustrated in Fig. 4. The Hamiltonian H_2 appears in diffractive processes and is responsible for evolution through a rapidity gap. The Hamiltonian H_3 is presumably the answer to properly formulated question of generalization of AGK cutting rules to QCD.
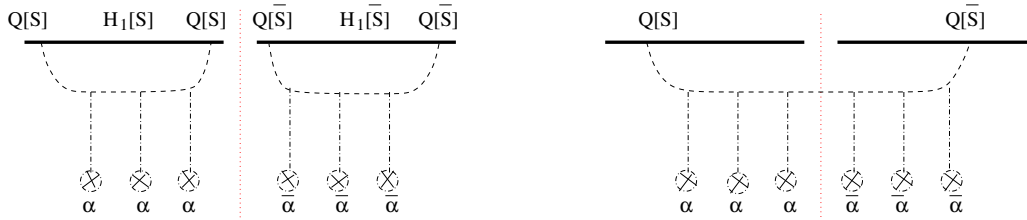

Figure 4: The Hamiltonian H_3

Having introduced the Hamiltonians we can introduce associated evolution operators:

$$U_{Y_1-Y_2}^3 \; = \; Exp\,[-\,H_3\,(Y_1\,-\,Y_2)] \qquad\qquad U_{Y_1-Y_2}^2 \; = \; Exp\,[-\,H_2\,(Y_1\,-\,Y_2)]$$

Thus a formal solution for inclusive diffraction with multiple gaps and multiple rescatterings (Fig. 5)

$$\sigma^{diff} \; \sim \; \int DS \, D\bar{S} \, W^t[S] \, \delta(S-\bar{S}) \, U_{Y_0-Y_n}^3 \; U_{Y_n-Y_{n-1}}^2 \cdots U_{Y_1-Y_2}^2 \, U_{Y-Y_1}^3 \; \Sigma^p[S,\bar{S}]$$

This expression is quite complex and of little use. Things become less formal and more useful when passing to the dipole degrees of freedom

$$s_{x,y} = \frac{1}{N} tr[S_F(x) S_F^\dagger(y)]$$

We need to remember, however, that the factorization

$$\langle s(x,y) s(u,v) \rangle_T = \langle s(x,y) \rangle_T \langle s(u,v) \rangle_T$$

is not always valid. This is very important in order to include target diffractive states.

For processes involving transverse momentum transfer,a quadrupole operator is also in need

$$q_{x,y,u,v} = \frac{1}{N} tr[S_F(x) S_F^\dagger(y) S_F(u) S_F^\dagger(v)]$$

Fortunately, no other higher multiplet operators emerge, if the projectile at rest is made only out of dipoles.
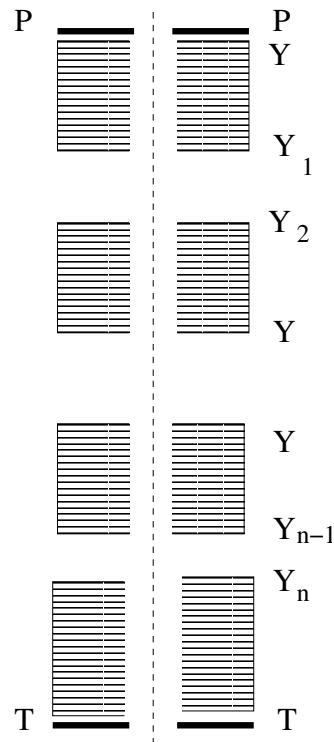
Figure 5: Diffraction with multiple gaps

Acknowledgments

References

[1] Slides:
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[2] A. Kovner and M. Lublinsky, JHEP **0611** 083 (2006); A. Kovner, M. Lublinsky and H. Weigert, Phys. Rev. **D74** 114023 (2006).
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