Status of $e^+e^- \to 3$ Jets at NNLO

Aude Gehrmann

Institute for Theoretical Physics, ETH, CH-8093 Zürich, Switzerland

We present the essential ingredients of the calculation of the next-to-next-to-leading order QCD corrections to three-jet production observables in electron-positron annihilation. Infrared singularities due to double real radiation at tree level and single real radiation at one-loop are extracted using the antenna subtraction method applied at next-to-next-to-leading order. All contributions to the three-jet cross section are implemented in a parton level generator. First results obtained with this generator concerning the NNLO contribution to the thrust event shape distribution are given here.

1 Introduction

Jet production cross sections in e^+e^- annihilation processes are classical observables which can be measured very accurately. These observables can be seen as a testing ground for the applicability of perturbative QCD. Furthermore, by comparing the measured 3-jet rate with the theoretical predictions for this rate, one can determine the strong coupling constant α_s. It turns out that the current error on α_s from jet observables [2] is dominated by the theoretical uncertainty. This uncertainty is related to the renormalisation scale dependence introduced by truncating the perturbative series at a given order in α_s. So far the 3-jet rate had been calculated up to the next-to-leading order (NLO)[3]. Clearly, to improve the determination of α_s, the calculation of the NNLO corrections to the 3-jet rate becomes mandatory. This calculation is now completed. We shall here briefly report on its essential ingredients and present first phenomenologically significant results concerning the thrust distribution.

2 The $e^+e^- \to 3$ jet cross section

Three-jet production at tree-level is induced by the decay of a virtual photon (or other neutral gauge boson) into a quark-antiquark-gluon final state. At higher orders, this process receives corrections from extra real or virtual particles.The individual partonic channels that contribute through to NNLO are shown in Table 1. All of the tree-level and loop amplitudes associated with these channels are known in the literature [4, 5, 6].

Partons are combined into jets using the same jet algorithm as in experiments such that the measured jet cross sections can be directly compared with the jet cross sections predicted theoretically. For the 3-jet rate, at leading order, each parton forms a jet on its own. At NLO, up to four partons can be present in the final state, in which case two of them are combined into one jet whereas at NNLO, up to five partons can be present in the final state such that three partons are clustered in one jet. The more partons are included in the jet, the more the parton-level jets resemble the hadron level jets seen experimentally and the better the matching between theory and experiments is.

LO	$\gamma^* \to q\,\bar{q}g$	tree level	NNLO	$\gamma^* \to q\,\bar{q}g$	two loop
NLO	$\gamma^* \to q\,\bar{q}g$	one loop		$\gamma^* \to q\,\bar{q}\,gg$	one loop
	$\gamma^* \to q\,\bar{q}\,gg$	tree level		$\gamma^* \to q\,\bar{q}\,q\,\bar{q}$	one loop
	$\gamma^* \to q\,\bar{q}\,q\bar{q}$	tree level		$\gamma^* \to q\,\bar{q}\,q\,\bar{q}\,g$	tree level
				$\gamma^* \to q\,\bar{q}\,g\,g\,g$	tree level

Table 1: The partonic channels contributing to $e^+e^- \to 3$ jets.

3 Infrared subtraction terms

To build 3-jet final states at a given order, a jet algorithm has to be applied separately to each partonic channel contributing at this order and all partonic channels have to be summed. However, each partonic channel contains infrared singularities which, after summation, cancel among each other. Consequently, these infrared singularities have to be extracted before the jet algorithm can be applied. While explicit infrared singularities from purely virtual contributions are obtained immediately after integration over the loop momenta, their extraction is more involved for real radiation. The singularities associated with the real emission of soft and/or collinear partons in the final state become only explicit after integrating the real radiation matrix elements over the appropriate phase space.

The infrared singularities of the real radiation contributions can be extracted using infrared subtraction terms. These terms must be constructed such that they approximate the full real radiation matrix elements in all singular limits while still being integrable analytically.

At NNLO, m-jet production is induced by final states containing up to $(m+2)$ partons, including the one-loop virtual corrections to $(m+1)$-parton final states. One introduces subtraction terms for the $(m+1)$- and $(m+2)$-parton contributions such that schematically the NNLO m-jet cross section reads,

$$\begin{aligned} \mathrm{d}\sigma^m_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^R_{NNLO} - \mathrm{d}\sigma^S_{NNLO}\right) \\ &\quad + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\ &\quad + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^S_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO}\,. \end{aligned} \tag{1}$$

$\mathrm{d}\sigma^S_{NNLO}$ denotes the real radiation subtraction term. It has the same unintegrated singular behaviour as $\mathrm{d}\sigma^R_{NNLO}$ in all appropriate limits. Likewise, $\mathrm{d}\sigma^{VS,1}_{NNLO}$ is the one-loop virtual subtraction term coinciding with the one-loop $(m+1)$-parton cross section $\mathrm{d}\sigma^{V,1}_{NNLO}$ in all singular limits. Finally, the two-loop correction to the m-parton cross section is denoted by $\mathrm{d}\sigma^{V,2}_{NNLO}$. Each line in the above formula is finite, free of any ϵ-poles, and can be implemented in a numerical program evaluating the jet rate.

To construct the subtraction terms, various methods exist at next-to-leading order [7] and general algorithms are available for the construction of one-particle subtraction terms [8, 9]. All of these algorithms derived in the framework of perturbative QCD are based on the factorisation properties of matrix element and phase spaces in kinematical regions corresponding to one parton becoming soft or collinear. One of these methods is antenna subtraction [9], which derives the one-particle subtraction terms at NLO from physical three-parton matrix elements. We extended this method to NNLO level [10], deriving one- and two-particle subtraction terms from three- and four-parton matrix elements [11]. These one and two-particle subtraction terms have been integrated analytically using the results given in [12].

Using this method, to evaluate the 3-jet rate at NNLO, we obtain numerically finite contributions from five-parton and four-parton processes. Furthermore we observe an explicit analytic cancellation of infrared poles in the four-parton and three-parton contributions, thus providing us with a powerful check of our method.

4 Numerical implementation and results

The different finite contributions have been implemented in a parton-level generator evaluating the 3-jet rate. Our starting point was the generator for $e^+e^- \to 4$ jets at NLO [13]. It contained already the 4-parton and 5-parton matrix elements and was based on the NLO antenna subtraction formalism. To this generator, we added the following contributions respectively in the 5-parton, 4 parton and 3-parton channels: the NNLO subtraction terms, the 1-loop real integrated subtraction term and the 2-loop matrix element. To embed the phase space present in the subtraction terms into the full phase space we used the mappings defined in [14]. The implementation is now completed, checked and first phenomenological results will soon be available. At this conference, we presented first results concerning the thrust distribution.

At NNLO, it takes the following form: (fixing $\mu_R = Q, \alpha_S = \alpha_s(Q)$)

$$\begin{aligned}(1-T)\,\frac{1}{\sigma_{\rm had}}\,\frac{{\rm d}\sigma}{{\rm d}T} \quad &= \quad \left(\frac{\alpha_s}{2\pi}\right) A(T) + \left(\frac{\alpha_s}{2\pi}\right)^2 (B(T) - 2A(T)) \\ &\quad + \left(\frac{\alpha_s}{2\pi}\right)^3 (C(T) - 2\,B(T) - 1.64\,A(T))\end{aligned}$$

The fixed order contributions to the thrust distribution are given in Fig. 1. We see that the NNLO corrections are of the order of 10-15% of the total result. Although the corrections are sizeable, the perturbative expression converges for this observable.

5 Conclusions

We have discussed the essential ingredients of the calculation of the $e^+e^- \to 3$ jets at NNLO. To perform this computation, the antenna subtraction method extended up to the NNLO level was required. All contributions to the 3-jet rate were implemented into a parton-level generator and the calculation is now finalised. As a first phenomenological result, we found that the NNLO contribution to the thrust distribution yield an enhancement of $10-15\%$, demonstrating the perturbative stability of the fixed order approach for this observable.

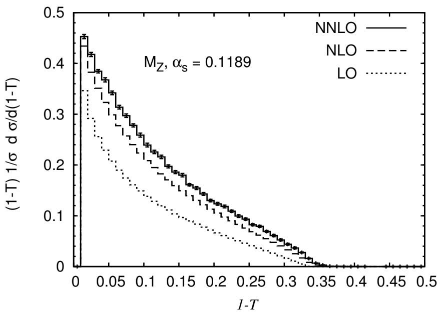

Figure 1: Thrust distribution at $\sqrt{s} = M_Z$.

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