New Theoretical Ideas: Anomaly Induced Effects in Magnetic Field and at LHC

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Anomaly cancellation between different sectors of a theory may mediate new interactions between the gauge boson of a theory. This may lead to effects, observable both at precision laboratory experiments and at accelerators. Such experiments may reveal the presence of hidden sectors or hidden extra dimensions.

It is well known that theories in which fermions have chiral couplings with gauge fields suffer from *anomalies* – a phenomenon of breaking of gauge symmetries of the classical theory at one-loop level. Anomalies make a theory inconsistent (in particular, its unitarity is lost). The only way to restore consistency of such a theory is to arrange the exact *cancellation* of anomalies between various chiral sectors of the theory. This happens, for example, in the Standard Model (SM), where the cancellation occurs between quarks and leptons within each generation [1]. Another well studied example is the Green-Schwarz anomaly cancellation mechanism [2] in string theory. In this case the cancellation happens between the anomalous contribution of chiral matter of the closed string sector with that of the open string.[1]

Particles involved in anomaly cancellation may have very different masses – for example, the mass of the top quark in the SM is much higher than the masses of all other fermions. On the other hand, gauge invariance should pertain in the theory at all energies, including those which are smaller than the mass of one or several particles involved in anomaly cancellation. The usual logic of renormalizable theories tells that the interactions, mediated by heavy fermions running in loops, are generally suppressed by the masses of these fermions [4]. The case of anomaly cancellation presents a notable counterexample to this famous "decoupling theorem" – the contribution of *a priori* arbitrary heavy particles should remain unsuppressed at arbitrarily low energies. As it was pointed out by D'Hoker and Farhi [5], this is possible because anomalous (i.e. gauge-variant) terms in the effective action have topological nature and therefore they are scale independent. As a result, they are not suppressed even at energies much smaller than the masses of the particles producing these terms via loop effects. This gives a hope to see at low energies some signatures generated by new high energy physics.

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[1] Formally, the Green-Schwarz anomaly cancellation occurs due to the anomalous Bianchi identity for the field strength of the Neveu-Schwarz 2-form. However, this modification of Bianchi identity arises from the 1-loop contribution of chiral fermions in the open string sector. A toy model, describing microscopically Green-Schwarz mechanism was studied e.g. in [3].

If a non-trivial anomaly cancellation involves the electromagnetic $U(1)$ gauge group (c.f. e.g. [6–11]), observable effects may be present in optical experiments. Indeed, the 4-dimensional electromagnetic anomaly is related to the quantity $\tilde{F}_{\mu\nu}F^{\mu\nu} = 4\vec{E}\cdot\vec{H} \neq 0$, where $F_{\mu\nu}$ is the electromagnetic field strength and $\tilde{F}_{\mu\nu}$ its dual. The high precision optical experiments (e.g. those measuring the change of polarization of light propagating in a strong magnetic field) could in principle see the anomalous terms, proportional to $\tilde{F}\cdot F$. There exists a significant experimental activity searching for such signals (see e.g. [14]), as various axion-like particles (ALPs) are expected to couple to $\tilde{F}_{\mu\nu}F^{\mu\nu}$ and produce interesting signatures in parallel electric and magnetic fields. A different type of experiment using static fields, which may test effects caused by non-trivial anomaly cancellation in the electromagnetic sector, was suggested in [**?**].

However, to generate an anomaly involving the electromagnetic (EM) group, some fermions should have chiral couplings with EM fields. If these particles are *massless*, their existence is severely constrained experimentally (see e.g. [15] or the book [16]). If the particles are massive, they can acquire mass via Higgs mechanism. Such an (electrically charged) Higgs field will necessarily give mass to the photon which is strongly constrained experimentally. Current experimental bound is $m_\gamma < 6\times10^{-17}$ eV as quoted by [17]. It is based on the work [18] which uses a magnetohydrodynamics argument based on survival of the Sun's field to the radius of the Earth's orbit. The most robust, model-independent constrain $m_\gamma < 10^{-14}$ eV comes from direct measurements of deviations of Coulomb law from r^{-2} dependence [19]. There also exist much stronger experimental restrictions, $m_\gamma < 3\times10^{-27}$ eV [20], which are however model-dependent [21]. Therefore, such theories will necessarily involve a small parameter (mass of the photon or charge of milli-charged particles), which in general strongly suppresses any possible effects [7, 8].

Another possibility is to realize non-trivial anomaly cancellation in the electroweak (EW) sector of the SM. Here the electromagnetic $U(1)$ subgroup is not anomalous by definition. However, the mixed triangular hypercharge $U_Y(1)\times SU(2)^2$ anomalies and gravitational anomalies are non-zero for generic choice of hypercharges. If one takes the most general choice of hypercharges, consistent with the structure of Yukawa terms, one sees that it is parametrized by two independent quantum numbers Q_e (shift of hypercharge of left-handed lepton doublet from its SM value) and Q_q (corresponding shift of quark doublet hypercharge). All the anomalies are then proportional to *one* particular linear combination: $\epsilon = Q_e + 3Q_q$. Interestingly enough, ϵ is equal to the sum of *electric charges* of the electron and proton. Experimental upper bound on the parameter ϵ, coming from checks of electro-neutrality of matter is rather small: $\epsilon < 10^{-21}e$ [17, 22]. If it is non-zero, the anomaly of the SM has to be cancelled by additional anomalous contributions from some physics beyond the SM, possibly giving rise to some non-trivial effects in the low energy effective theory.

A very interesting possibility may be realized in theories with extra dimensions. In this case, additional contribution to the anomaly may come from the higher-dimensional modes, which are separated from the 4-dimensional physics by a mass gap. Such a mechanism of anomaly cancellation is called "*anomaly inflow*" [23]. If an anomaly of EW gauge symmetry of the SM is cancelled by anomaly inflow, this may give rise to very interesting observable effects, making the 4-dimensional low energy effective theory non-local [6]. To see this, however, one needs to use an explicit mechanism of the localization of gauge fields in the presence of non-compact extra dimensions. This makes the theory rather complicated. For example, it may suffer from a strong coupling problem in the bulk at the "non-compact end" of an extra dimension. The model of Ref. [6] offers a way to overcome the strong coupling problem. It admits a nonperturbative solution in an external magnetic field. The solution is sharply localized in the 5th dimension so that it does not enter the strong coupling region. One can then build a perturbation theory around this localized solution and solve Maxwell equations in case of static charges or a stationary propagation (e.g. electromagnetic wave) in the external magnetic field. The analysis shows that the propagation of light, polarized parallel to the external magnetic field, acquires the "magnetic mass" $m_{\gamma H} \sim \sqrt{\epsilon |H_{\rm ext}|}$, similarly to the static case of [6]. The propagation of orthogonal polarization remains unchanged. This leads to the birefringence of light in the magnetic field. This effect can be probed in the experiments, searching for the ALPs [14]. However, as the model of [6] does not contain any new light degrees of freedom, the effect of dichroism and "shining light through the wall" will not be present in this case.

In all scenarios described above the anomaly-induced effects are proportional to a very small parameter, which makes their experimental detection very difficult. One may consider another situation, where anomalous charges and therefore, anomaly-induced effects, are $\mathcal{O}(1)$. To reconcile this with existing experimental bounds, such an anomaly cancellation should take place between SM and "hidden" sector, with corresponding new particles appearing at relatively high energies. Namely, many extensions of the SM add extra gauge fields to the SM gauge group (see e.g. [24] and refs. therein). For example, additional $U(1)$s naturally appear in models in which $SU(2)$ and $SU(3)$ gauge factors of the SM arise as parts of unitary $U(2)$ and $U(3)$ groups (as e.g. in D-brane constructions of the SM [25]). In this paper, we consider extensions of the SM with an additional $U_X(1)$ factor, so that the gauge group becomes $SU(3)_c \times SU(2)_W \times U_Y(1) \times U_X(1)$. As the SM fermions are chiral with respect to the EW group $SU(2)_W \times U_Y(1)$, even choosing the charges for the $U_X(1)$ group so that the triangular $U_X(1)^3$ anomaly vanishes, still this may easily give rise to the appearance of mixed anomalies: $U_X(1)U_Y(1)^2$, $U_X(1)^2U_Y(1)$, $U_X(1)SU(2)^2$. In this work we are interested in the situation when only (some of these) *mixed anomalies* with the electroweak group $SU(2) \times U_Y(1)$ are non-zero. A number of works have already discussed such theories and their signatures (see e.g. [7, 8, 25–27]).

The question of experimental signatures of such theories at LHC should be addressed differently, depending on whether or not the SM fermions are charged with respect to the $U_X(1)$ group:

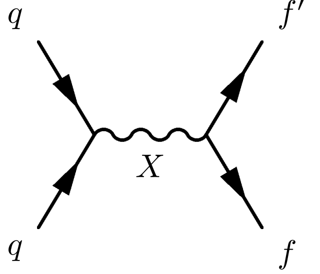

Figure 1: Direction production of neutral X boson in pp collision.

• If SM fermions are charged with respect to the $U_X(1)$ group, and the mass of the new X boson is around the TeV scale, we should be able to see the corresponding resonance in the forthcoming runs of LHC (in processes like those shown on Fig. 1). In this case an important question is to distinguish between theories with non-trivial cancellation of mixed anomalies, and those which are anomaly free.

• On the other hand, one is present with a completely different challenge if the SM fermions are *not charged* with respect to the $U_X(1)$ group. This makes direct production of the X boson impossible. Therefore, the question of whether an anomalous gauge boson with mass $M_X \sim 1$ TeV can be detected at LHC becomes especially interesting.

A theory in which the cancellation of the mixed $U_X(1)SU(2)^2$ anomaly occurs between some heavy fermions and Green-Schwarz (i.e. *tree-level gauge-variant*) terms was considered in [27]. The leading non-gauge invariant contributions from the triangular diagrams of heavy fermions, unsuppressed by the fermion masses, cancels the Green-Schwarz terms. The triangular diagrams also produce subleading (gauge-invariant) terms, suppressed by the mass of the fermions running in the loop. This leads to an appearance of dimension-6 operators in the effective action, having the general form $F^3_{\mu\nu}/\Lambda^2_X$, where $F_{\mu\nu}$ is the field strength of X, Z or $W^\pm$ bosons. Such terms contribute to the XZZ and XWW vertices. As the fermions in the loops are heavy, such vertices are in general strongly suppressed by their mass. However, motivated by various string constructions, [27] has assumed two things: *(a)* these additional massive fermions are above the LHC reach but not too heavy (e.g. have masses in tens of TeV); *(b)* there are many such fermions (for instance Hagedorn tower of states) and therefore the mass suppression can be compensated by the large multiplicity of these fermions.

In [28] we consider another possible setup, in which the anomaly cancellation occurs *only within a high-energy sector* (at scales not accessible by current experiments), but at low energies there remain terms XWW and XZZ unsuppressed by masses of heavy particles. Such a theory possess unique experimental signatures and can be tested at LHC. Similar setup (with completely different phenomenology) has been previously considered in [7, 8].

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