Photon Interactions and Chiral Dynamics

Wojciech Broniowski[1,2], *Alexandr E. Dorokhov*[3], *Enrique Ruiz Arriolo*[4]
[1]Institute of Physics, Jan Kochanowski University, PL-25406 Kielce, Poland
[2]The H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences,
PL-31342 Kraków, Poland
[3]Joint Institute for Nuclear Research, Bogoliubov Laboratory of Theoretical Physics,
114980, Moscow region, Dubna, Russia
[4]Departamento de Física Atómica, Molecular y Nuclear, Universidad de Granada,
E-18071 Granada, Spain

DOI: http://dx.doi.org/10.3204/DESY-PROC-2009-03/Broniowski

Twist-2 components of the real and virtual photon distribution amplitudes are evaluated in several chiral quark models. The results, obtained at the quark model scale, are then evolved to higher scales, probed in experiments or in lattice QCD. We also analyze the related form factors and coupling constants. Our results are a genuine dynamical prediction, following from the chiral dynamics.

1 Basics

This talk is based on Ref. [1], where more details and results can be found. Our approach is based on the fact that the spontaneously broken chiral symmetry provides the basic dynamics for the evaluation of soft matrix element involving the Goldstone bosons (pion, kaons) and gauge currents (photons, $W^{\pm}$, Z). That way one may evaluate in a genuinely dynamical way the soft quantities appearing in high-energy processes. A detailed presentation of the method and the compilation of predictions for the pion matrix elements can be found in Ref. [2].

A crucial ingredient of the method is the QCD evolution from the a priori unknown *quark model scale* to the scales relevant for the experiments or lattice calculations. Thus the scheme consists of two steps: 1) the evaluation of soft matrix elements in the chiral quark model and 2) the QCD evolution to a higher scale. The quark model scale may be estimated with the help of the momentum sum rule [2], and is found to be low, $Q_0 \simeq 320$ MeV (for the local chiral quark models). After the QCD evolution, a successful description of the available data for the pion is achieved for the parton distribution function (PDF) and the distribution amplitude (DA). There are numerous quark-model studies of these quantities as well as the more general pion generalized parton distributions (GPD's) in the literature [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. A related quantity, the pion-photon transition distribution amplitude (TDA) [22, 23] has also been evaluated in this framework [24, 25, 26].

The hadronic part of the photon wave-function consists, in the large-N_c limit, of a quark-antiquark pair. Since the chiral dynamics provides the quarks a large (constituent) mass, it influences the photon dynamics. Here we apply the methods developed and tested earlier for the pion to the photon case. We focus on the photon DA's, while the photon structure function

Table 1: The constants obtained in the quark model and evaluated to the reference scale of 1 GeV.

quantity at 1 GeV	non-local	SQM	QCD s.r.	VMD
$(-\langle 0 \lvert\overline{q}q\rvert 0\rangle)^{1/3}$ [GeV]	0.24	0.24	0.24 ± 0.02	-
χ_m [GeV^2]	2.73	1.37	3.15 ± 0.3	3.37
$f_{3\gamma}$ [GeV^{-2}]	-0.0035	-0.0018	-0.0039 ± 0.0020	-0.0046

is left for a separate study. The leading-twist photon distribution amplitudes (DA's) are defined via the matrix elements of quark bilinears delocalized along the light cone [27, 28, 29, 30],

$$\begin{aligned}
&\langle 0|\overline{q}(z)\sigma_{\mu\nu}[z,-z]q(-z)|\gamma^{\lambda}(q)\rangle = \\
&\quad ie_q\langle\bar{q}q\rangle\chi_{\rm m} f^{t}_{\perp\gamma}(q^2)\left(\epsilon^{(\lambda)}_{\perp\mu}p_\nu-\epsilon^{(\lambda)}_{\perp\nu}p_\mu\right)\int_0^1 dx e^{i(2x-1)q\cdot z}\phi_{\perp\gamma}(x,q^2)+h.t., \\
&\langle 0|\overline{q}(z)\gamma_{\mu}[z,-z]q(-z)|\gamma^{\lambda}(q)\rangle = \\
&\quad e_q f_{3\gamma} f^{v}_{\parallel\gamma}(q^2)p_\mu\left(\epsilon^{(\lambda)}\cdot n\right)\int_0^1 dx e^{i(2x-1)q\cdot z}\phi_{\parallel\gamma}(x,q^2)+h.t.,
\end{aligned}$$

where $\epsilon^{(\lambda)}\cdot q=0$ and $\epsilon^{(\lambda)}\cdot n=0$ (for real photons) and

$$p_\mu=q_\mu-\frac{q^2}{2}n_\mu,\ n_\mu=\frac{z_\mu}{p\cdot z},\ e^{(\lambda)}_\mu=\left(e^{(\lambda)}\cdot n\right)p_\mu+\left(e^{(\lambda)}\cdot p\right)n_\mu+e^{(\lambda)}_{\perp\mu}.$$

The quark magnetic susceptibility, $\chi_{\rm m}$, and $f_{3\gamma}$ are constants, $f^{t}_{\perp\gamma}(q^2)$ and $f^{v}_{\parallel\gamma}(q^2)$ are form factors, $\phi_{\perp\gamma}(x,q^2)$ and $\phi_{\parallel\gamma}(x,q^2)$ denote the DA's, while *h.t.* stands for the disregarded higher-twist contributions.

The leading-N_c quark model evaluation proceeds according to the one-loop diagram, where one of the vertices corresponds to the photon and the other to the probing operator, in our case $\sigma^{\mu\nu}$ and γ^{μ}. The quark propagators involve a constituent quark mass, due to spontaneous breaking of the chiral symmetry. We use a few variants of chiral quark models: the Nambu–Jona-Lasinio (NJL) (for reviews see, *e.g.*, [31, 15] and references therein) and the Spectral Quark model (SQM) [11, 32], which incorporates the vector-meson dominance, as well as the instanton-motivated non-local chiral quark model of Ref. [33, 34, 35, 36, 37]. In nonlocal models the quark mass depends on the virtuality, As a consequence, the vertices acquire corrections due to nonlocalities to consistently account for gauge and chiral Ward identities.

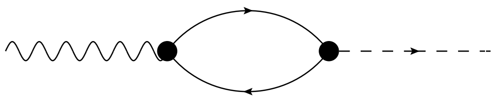

Figure 1: Feynman diagram for the evaluation of the photon DA's in chiral quark models.

2 Results

The result for the constants are presented in Table 1, where we also give the estimates of the QCD sum rules and the Vector Meson Dominance model [30]. QCD predicts the *scale*

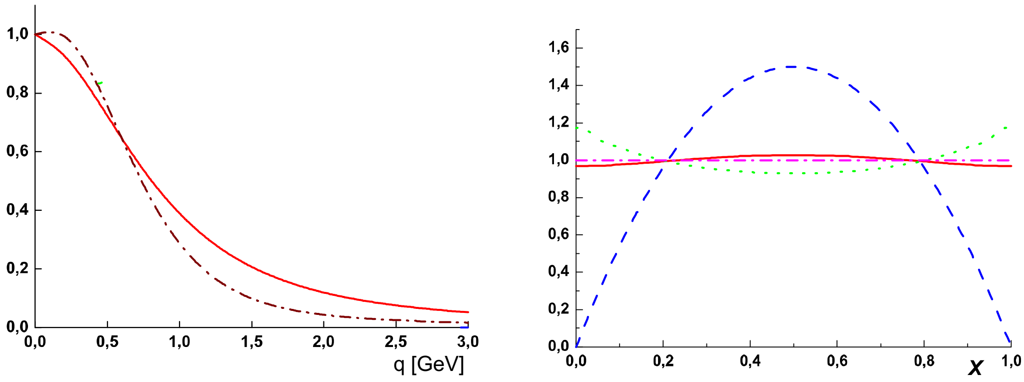

Figure 2: Left: the twist-2 tensor (dashed line) and vector (solid line) form factors in the non-local model. Right: the transverse DA of the photon, $\phi_{\perp\gamma}(x, q^2 = 0)$. Solid – non-local model, dot-dashed – local model, dotted – approximation of Ref. [38], dashed – the asymptotic form $6x(1-x)$.

dependence for the quark condensate $\langle 0\left|\overline{q}q\right|0\rangle$, its magnetic susceptibility χ_{m}, and $f_{3\gamma}$. At the leading order

$$\langle 0\left|\overline{q}q\right|0\rangle\big|_\mu = L^{-\gamma_{\overline{q}q}/b}\,\langle 0\left|\overline{q}q\right|0\rangle\big|_{\mu_0}, \quad \chi_m\big|_\mu = L^{-(\gamma_0-\gamma_{\overline{q}q})/b}\,\chi_m\big|_{\mu_0}, \quad f_{3\gamma}\big|_\mu = L^{-\gamma_f/b}\,f_{3\gamma}\big|_{\mu_0}$$

where $r = \alpha_s\left(\mu^2\right)/\alpha_s\left(\mu_0^2\right)$, $b = \left(11N_c - 2n_f\right)/3$, is the evolution ratio, with $\gamma_{\overline{q}q} = -3C_F$, $\gamma_0 = C_F$, $\gamma_f = 3C_A - C_F/3$, $C_F = 4/3$, and $C_A = 3$ for $N_c = 3$. We evolve from the quark model scale, $Q_0 = 320$ MeV, to the reference scale of 1 GeV. We note a similar magnitude and signs compared to the QCD sum rules or VMD estimates, with the local model producing smaller values than the nonlocal model.

The form factors from the non-local quark model are shown in the left panel of Fig. 2. For the local models (not displayed) the results are very similar. They exhibit the typical fall-off scale of $\sim m_\rho$ In particular, in SQM we recover the exact VMD formula

$$f_{\perp\gamma}^{t,\mathrm{SQM}}(q^2) = \frac{m_\rho^2}{m_\rho^2 + q^2}.$$

We note that the vector DA $\phi_{\perp\gamma}(x, q^2 = 0) = 1$ in local models and is very close to 1 in non-local models. For the virtual photon SQM gives the simple formulas:

$$\phi_{\|\gamma^*}(x, q^2) = \frac{1 + \frac{q^2}{m_\rho^2}}{\left(1 + \frac{4q^2}{m_\rho^2}x(1-x)\right)^{3/2}}.$$

In the limit of $q^2 \to -m_\rho^2$ it becomes $\delta\left(x - \frac{1}{2}\right)$, a quite reasonable result.

One may also study the photon light-cone wave function (a $k_\perp$-unintegrated object). It has a simple form in SQM (at the quark-model scale):

$$\Phi_{\perp\gamma}(x, \boldsymbol{k}_\perp) = \frac{6}{m_\rho^2(1 + 4\boldsymbol{k}_\perp^2/m_\rho^2)^{5/2}}$$

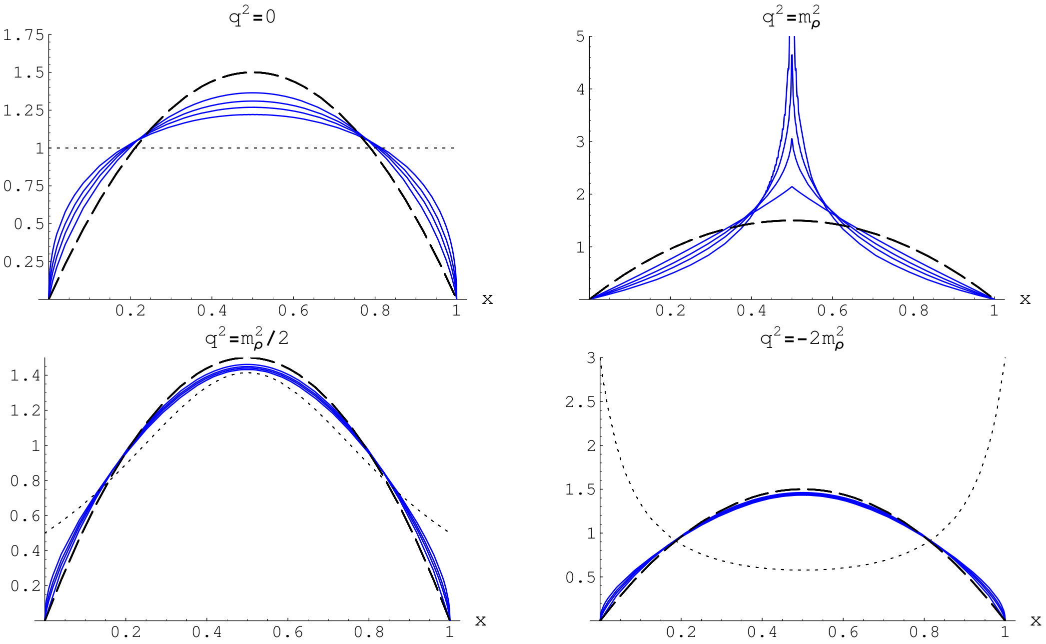

Figure 3: The leading order ERBL evolution of the leading-twist tensor DA, $\phi_{\perp\gamma}(x, q^2)$ evaluated in the local model at various virtualities: real photon (top left), ρ-meson (top right), virtual photon at $q^2 = -Q^2 = m_\rho^2/2$ (bottom left), and virtual photon at $q^2 = -Q^2 = -2m_\rho^2$ (bottom right). Initial conditions, indicated by dotted lines, are evaluated in SQM at the initial quark-model scale. The solid lines correspond to LO QCD evolution to the scales $Q = 1$, 2.4, 10, and 1000 GeV. With the larger the scale the evolved DA becomes closer to the asymptotic form $6x(1-x)$, plotted with the dashed line. The corresponding values of the evolution ratio r are given in the figures.

Note the power-law fall-off at large transverse momenta, $\Phi_{\perp\gamma}(x, \boldsymbol{k}_\perp) \sim 1/k_\perp^5$. In cross section this leads to tails $\sim 1/k_\perp^{10}$. For the virtual photon

$$\Phi_{\perp\gamma^*}(x, \boldsymbol{k}_\perp) = \frac{6\left(1 + \frac{q^2}{m_\rho^2}\right)}{m_\rho^2 \left(1 + 4\frac{\boldsymbol{k}_\perp^2 + q^2 x(1-x)}{m_\rho^2}\right)^{5/2}}$$

3 QCD evolution of DA's

Now we come to the QCD evolution, which, as already stressed, is crucial in bringing the results to the scales probed in experiments or lattices. We carry out the standard LO ERBL evolution with anomalous dimensions taken for the appropriate channels [39]. The method leads to simple expressions, diagonal in the Gegenbauer moments. In Fig. 3 we show the results for the tensor DA for the real photon, the ρ-meson, and the virtual photon. We note the large change caused by the evolution, which fairly fast brings the model predictions to the vicinity of the asymptotic

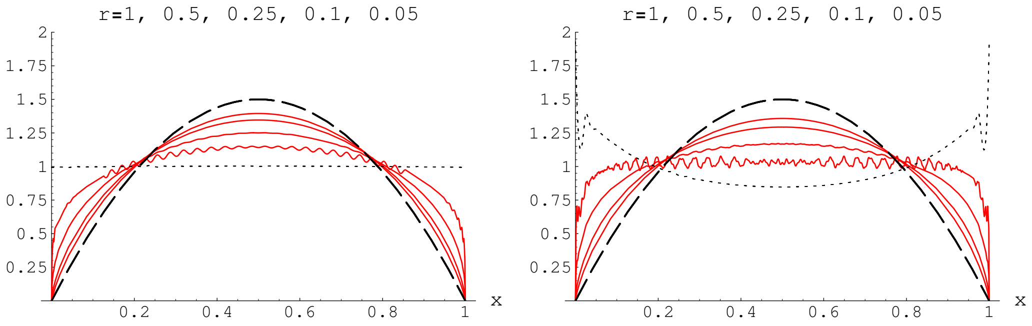

Figure 4: The LO ERBL evolution of the nonlocal model predictions for the leading-twist vector DAs of the photon, $\phi_{\|\gamma}(x, q^2)$. Left: real photon, right: the virtual photon at $q^2 = 0.25$ GeV^2. The dashed lines show the asymptotic DA, $6x(1-x)$. The initial conditions (dotted line) are evaluated in the nonlocal quark model at the initial scale $Q_0^{\rm inst} = 530$ MeV. The solid lines correspond to evolved DA'a at subsequent scales $Q = 1$, 2.4, 10, and 1000 GeV. The corresponding values of the evolution ratio r are given in the figures. Tiny wiggles in the evolved curves is a numerical effect.

limit. Similar results can be done in the nonlocal model, as well as for the vector DA [1]. We show the results in Fig. 4.

4 Conclusion

Chiral quark models provide a link between high- and low-energy analyses, allowing to compute various soft matrix elements for hadronic processes. They yield in a fully dynamical way the initial conditions for the QCD evolution, which is essential to bring the predictions up to the experimental or lattice scales. Numerous predictions for processes involving the Goldstone bosons and photons can be made. The scale in chiral quark models is low, about 320 MeV, hence the QCD evolution is “fast”. Simple analytic formulas – useful to understand the general properties, can be obtained in local quark models. For the pion, with the LO QCD evolution the overall agreement with the available data and lattice simulations is very reasonable (PDF, DA, GPD, TDA, generalized form factors [40]). While the presented results for the can be used in phenomenological analyses in high-energy reactions (see, *e.g.*, the recent work of Ref. [41]), the model predictions can be further tested also with future lattice simulations for the photon and ρ-meson.

Acknowledgments

This work has been supported in part by the JINR Bogolyubov-Infeld program, the Scientific School grant 4476.2006.2 the Polish Ministry of Science and Higher Education grants N202 034 32/0918 and N202 249235, and by the Spanish DGI and FEDER funds with grant FIS2008-01143/FIS, Junta de Andalucía grant FQM225-05, and EU Integrated Infrastructure Initiative Hadron Physics Project contract RII3-CT-2004-506078.

References

[1] Alexander E. Dorokhov, Wojciech Broniowski, and Enrique Ruiz Arriola. Photon distribution amplitudes and light-cone wave functions in chiral quark models. *Phys. Rev.*, D74:054023, 2006.

[2] Wojciech Broniowski, Enrique Ruiz Arriola, and Krzysztof Golec-Biernat. Generalized parton distributions of the pion in chiral quark models and their QCD evolution. *Phys. Rev.*, D77:034023, 2008.

[3] R. M. Davidson and E. Ruiz Arriola. Structure functions of pseudoscalar mesons in the su(3) njl model. *Phys. Lett.*, B348:163–169, 1995.

[4] A. E. Dorokhov and Lauro Tomio. Quark distribution function in pion within instanton liquid model. 1998.

[5] Maxim V. Polyakov and C. Weiss. Two-pion light-cone distribution amplitudes from the instanton vacuum. *Phys. Rev.*, D59:091502, 1999.

[6] Maxim V. Polyakov and C. Weiss. Skewed and double distributions in pion and nucleon. *Phys. Rev.*, D60:114017, 1999.

[7] Alexander P. Bakulev, Rusko Ruskov, Klaus Goeke, and N. G. Stefanis. Parton skewed distributions in the pion and quark hadron duality. *Phys. Rev.*, D62:054018, 2000.

[8] A. E. Dorokhov and Lauro Tomio. Pion structure function within the instanton model. *Phys. Rev.*, D62:014016, 2000.

[9] I. V. Anikin, A. E. Dorokhov, A. E. Maksimov, and L. Tomio. Off-diagonal quark distributions in pions in the effective single-instanton approximation. *Phys. Atom. Nucl.*, 63:489–498, 2000.

[10] I. V. Anikin, A. E. Dorokhov, A. E. Maksimov, L. Tomio, and V. Vento. Nonforward parton distributions of the pion within an effective single instanton approximation. *Nucl. Phys.*, A678:175–186, 2000.

[11] E. Ruiz Arriola. Parton distributions for the pion in a chiral quark model. 2001.

[12] R. M. Davidson and E. Ruiz Arriola. Parton distributions functions of pion, kaon and eta pseudoscalar mesons in the njl model. *Acta Phys. Polon.*, B33:1791–1808, 2002.

[13] Michal Praszalowicz and Andrzej Rostworowski. Pion light cone wave function in the non-local NJL model. *Phys. Rev.*, D64:074003, 2001.

[14] Enrique Ruiz Arriola and Wojciech Broniowski. Pion light-cone wave function and pion distribution amplitude in the nambu-jona-lasinio model. *Phys. Rev.*, D66:094016, 2002.

[15] E. Ruiz Arriola. Pion structure at high and low energies in chiral quark models. ((v)). *Acta Phys. Polon.*, B33:4443–4479, 2002.

[16] B. C. Tiburzi and G. A. Miller. Generalized parton distributions for q anti-q pions. *Phys. Rev.*, D67:013010, 2003.

[17] B. C. Tiburzi and G. A. Miller. Generalized parton distributions and double distributions for q anti-q pions. *Phys. Rev.*, D67:113004, 2003.

[18] L. Theussl, S. Noguera, and V. Vento. Generalized parton distributions of the pion in a bethe- salpeter approach. *Eur. Phys. J.*, A20:483–498, 2004.

[19] Michal Praszalowicz and Andrzej Rostworowski. Pion generalized distribution amplitudes in the nonlocal chiral quark model. *Acta Phys. Polon.*, B34:2699–2730, 2003.

[20] Wojciech Broniowski and Enrique Ruiz Arriola. Impact-parameter dependence of the generalized parton distribution of the pion in chiral quark models. *Phys. Lett.*, B574:57–64, 2003.

[21] Adam Bzdak and Michal Praszalowicz. An attempt to construct pion distribution amplitude from the pcac relation in the nonlocal chiral quark model. *Acta Phys. Polon.*, B34:3401–3416, 2003.

[22] B. Pire and L. Szymanowski. Hadron annihilation into two photons and backward vcs in the scaling regime of qcd. *Phys. Rev.*, D71:111501, 2005.

[23] B. Pire and L. Szymanowski. Qcd analysis of anti-p n –¿ gamma* pi in the scaling limit. *Phys. Lett.*, B622:83–92, 2005.

[24] Wojciech Broniowski and Enrique Ruiz Arriola. Pion photon transition distribution amplitudes in the spectral quark model. *Phys. Lett.*, B649:49, 2007.

[25] A. Courtoy and S. Noguera. The Pion-Photon Transition Distribution Amplitudes in the Nambu-Jona Lasinio Model. *Phys. Rev.*, D76:094026, 2007.

[26] Piotr Kotko and Michal Praszalowicz. Pion-to-photon transition distribution amplitudes in the non-local chiral quark model. e-Print: arXiv:0803.2847 [hep-ph].

[27] Ahmed Ali and Vladimir M. Braun. Estimates of the weak annihilation contributions to the decays B → rho + gamma and B → omega + gamma. *Phys. Lett.*, B359:223–235, 1995.

[28] Patricia Ball and Vladimir M. Braun. The ρ Meson Light-Cone Distribution Amplitudes of Leading Twist Revisited. *Phys. Rev.*, D54:2182–2193, 1996.

[29] Patricia Ball, Vladimir M. Braun, Y. Koike, and K. Tanaka. Higher twist distribution amplitudes of vector mesons in QCD: Formalism and twist three distributions. *Nucl. Phys.*, B529:323–382, 1998.

[30] Patricia Ball, V. M. Braun, and N. Kivel. Photon distribution amplitudes in QCD. *Nucl. Phys.*, B649:263–296, 2003.

[31] Chr. V. Christov et al. Baryons as non-topological chiral solitons. *Prog. Part. Nucl. Phys.*, 37:91–191, 1996.

[32] Enrique Ruiz Arriola and Wojciech Broniowski. Spectral quark model and low-energy hadron phenomenology. *Phys. Rev.*, D67:074021, 2003.

[33] John Terning. Gauging nonlocal Lagrangians. *Phys. Rev.*, D44:887–897, 1991.

[34] B. Holdom. Approaching low-energy QCD with a gauged, nonlocal, constituent quark model. *Phys. Rev.*, D45:2534–2541, 1992.

[35] Robert S. Plant and Michael C. Birse. Meson properties in an extended non-local NJL model. *Nucl. Phys.*, A628:607–644, 1998.

[36] Wojciech Broniowski. Gauging non-local quark models. Talk presented at the Miniworkshop on Hadrons as Solitons, Bled, Slovenia, 6-17 Jul 1999. Published in Bled 1999, Hadrons as solitons, p. 17-26, e-print: hep-ph/9909438

[37] I. V. Anikin, A. E. Dorokhov, and L. Tomio. Pion structure in the instanton liquid model. *Phys. Part. Nucl.*, 31:509–537, 2000.

[38] V. Yu. Petrov, Maxim V. Polyakov, R. Ruskov, C. Weiss, and K. Goeke. Pion and photon light-cone wave functions from the instanton vacuum. *Phys. Rev.*, D59:114018, 1999.

[39] A. V. Belitsky and A. V. Radyushkin. Unraveling hadron structure with generalized parton distributions. *Phys. Rept.*, 418:1–387, 2005.

[40] Wojciech Broniowski and Enrique Ruiz Arriola. Gravitational and higher-order form factors of the pion in chiral quark models. *Phys. Rev.*, D78:094011, 2008.

[41] B. Pire and L. Szymanowski. “Probing the nucleon’s transversity and the photon’s distribution amplitude in lepton pair photoproduction. e-print: arXiv:0905.1258 [hep-ph].